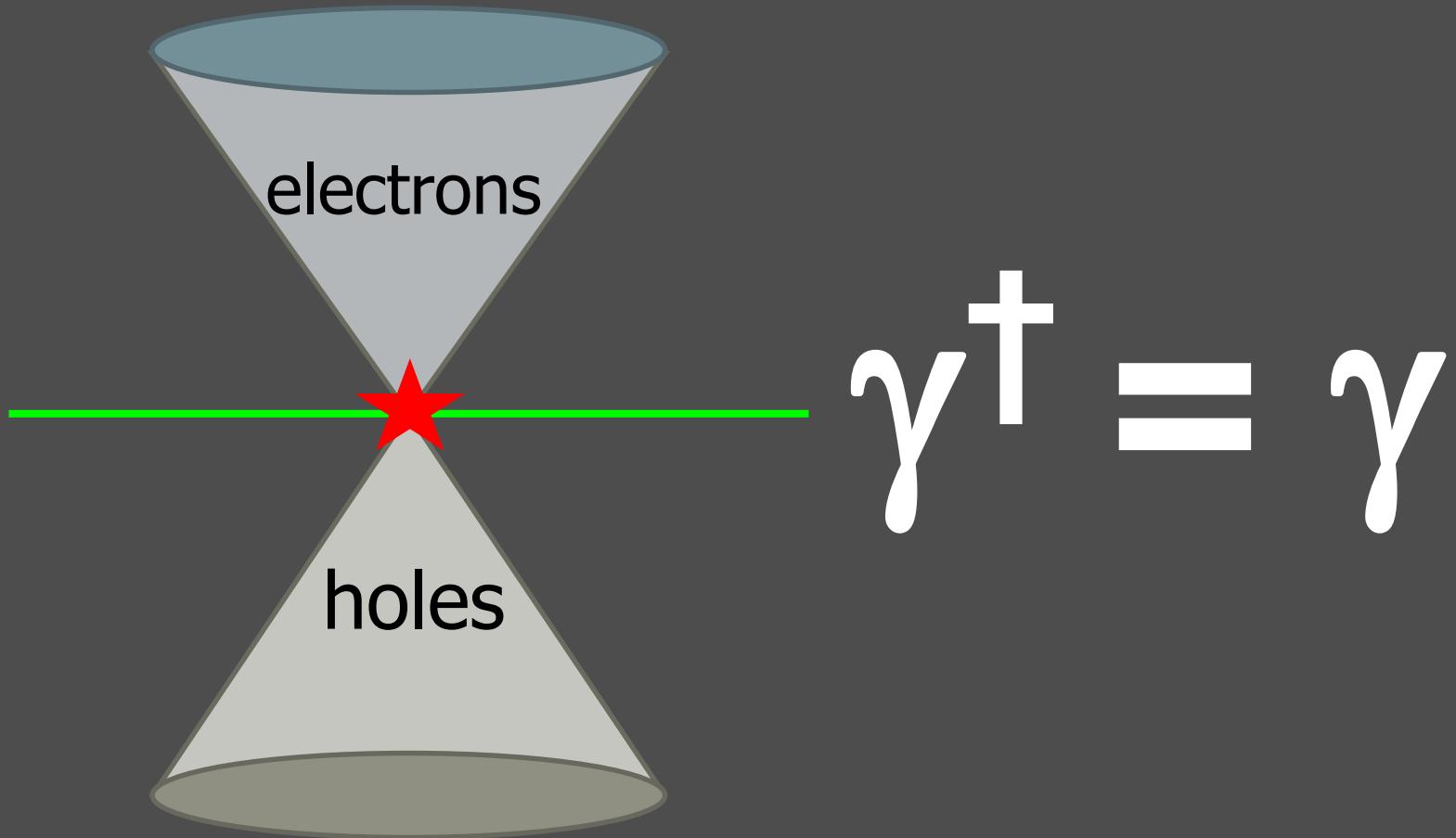


Majorana in nanowires

Lecture I (braiding)

Sergey Frolov
University of Pittsburgh

a particle that is its own antiparticle



an equal superposition of electron and hole

Majorana fermion $\gamma^+ = \gamma$

single electron, hole

c^\dagger, c



$$c = \gamma_1 + i\gamma_2$$

$$c^\dagger = \gamma_1 - i\gamma_2$$

 γ_1

γ_2 

Solid Theoretical Foundation

Toy model:

$$H_{\text{Kitaev}} = \sum_i \left(-t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu(c_i^\dagger c_i - \frac{1}{2}) + \Delta c_i c_{i+1} + h.c. \right)$$

A. Kitaev, Physics Uspekhi (2001)

First proposal based on existing materials – topological insulators
L. Fu and C. Kane PRL 2008

Semiconductor nanowire proposals:

$$\mathcal{H} = [p^2/2m - \mu(y)] \tau_z + u(y)p \sigma_z \tau_z + B(y)\sigma_x + \Delta(y)\tau_x.$$

Lutchyn, Sau, Das Sarma, PRL 2010
Oreg, Refael, von Oppen, PRL 2010

single fermion:



$$C = \gamma_1 + i\gamma_2$$

chain of fermions:

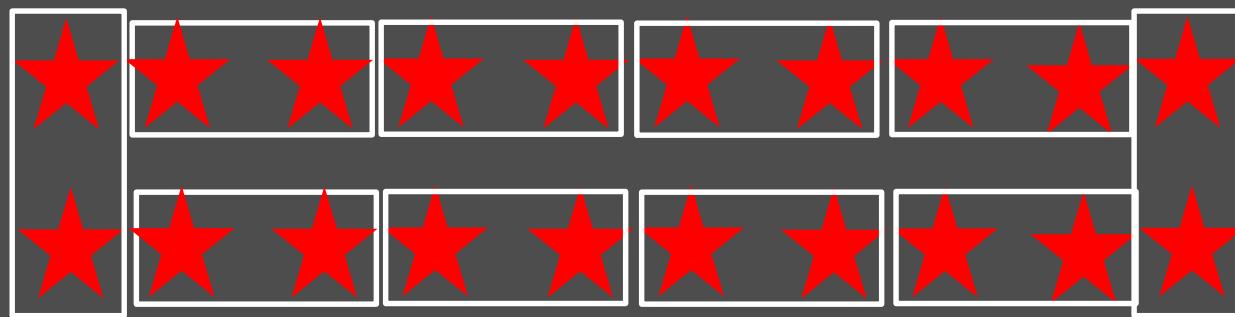


p-wave coupling:



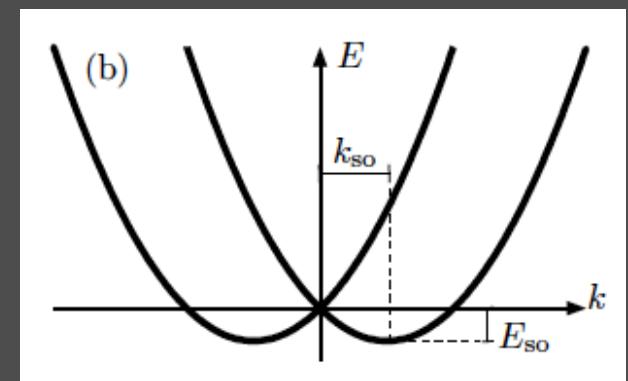
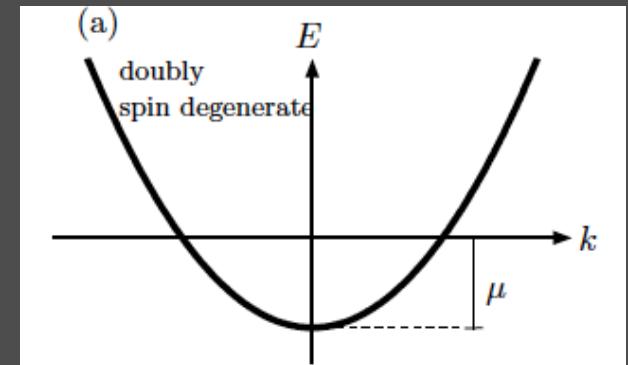
$$H_{\text{Kitaev}} = \sum_i \left(-t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu(c_i^\dagger c_i - \frac{1}{2}) + \Delta c_i c_{i+1} + h.c. \right)$$

Spinful case – no Majorana



Majorana recipe:

1. One-dimensional wire
2. Spin-orbit interaction
3. Superconductivity
4. Magnetic field

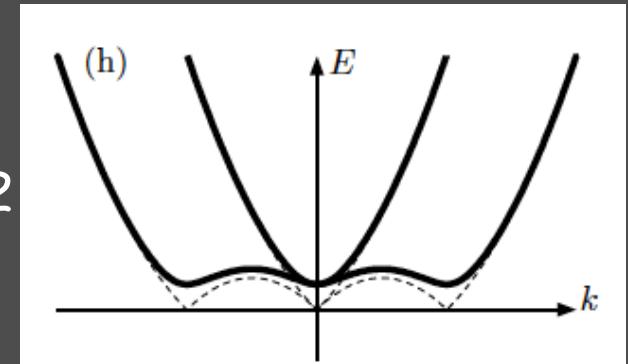


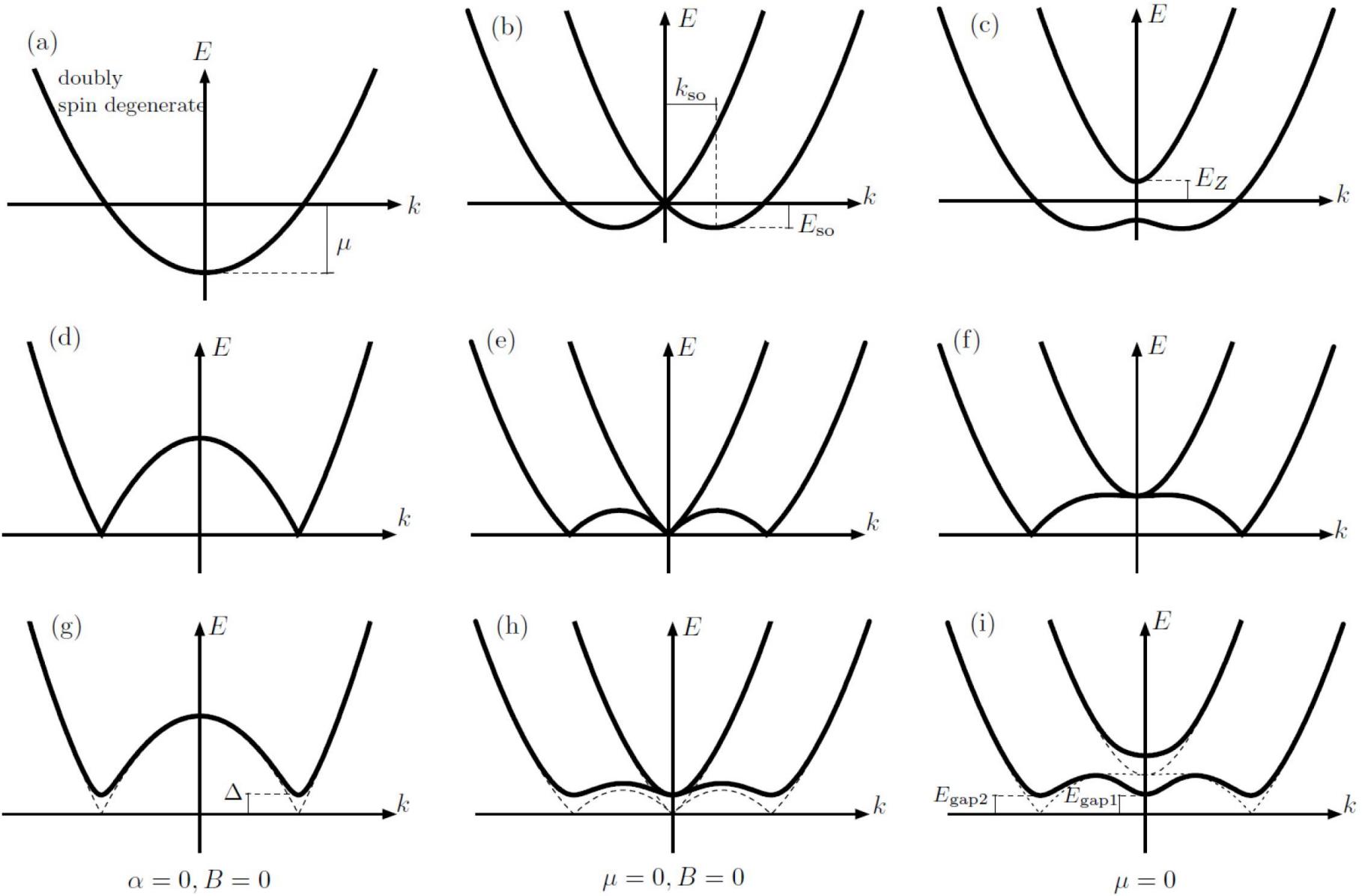
$$\mathcal{H} = [p^2/2m - \mu(y)] \tau_z + u(y)p \sigma_z \tau_z + B(y)\sigma_x + \Delta(y)\tau_x.$$

$$\gamma_1^\dagger = \gamma_1$$

$$\gamma_2^\dagger = \gamma_2$$

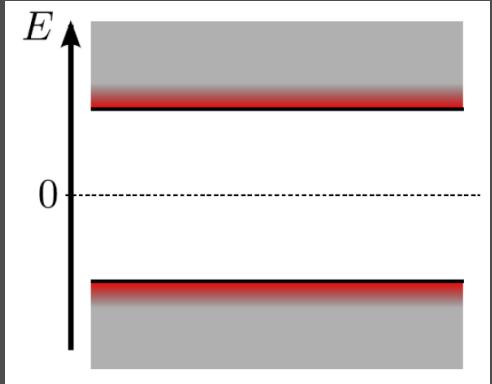
Lutchyn, Sau, Das Sarma, PRL 2010
Oreg, Refael, von Oppen, PRL 2010



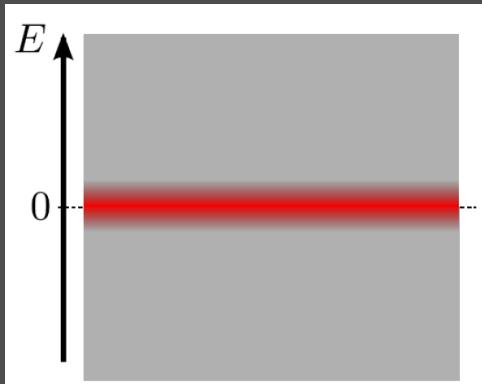


Sim by Fabian Hassler

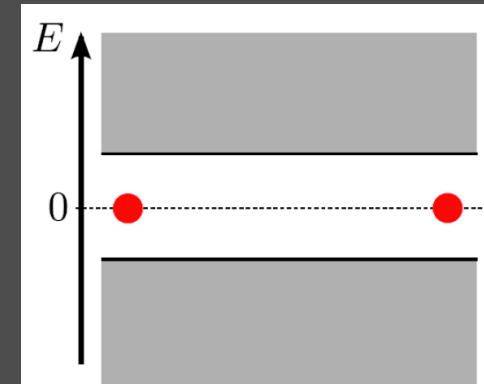
Topological Transition



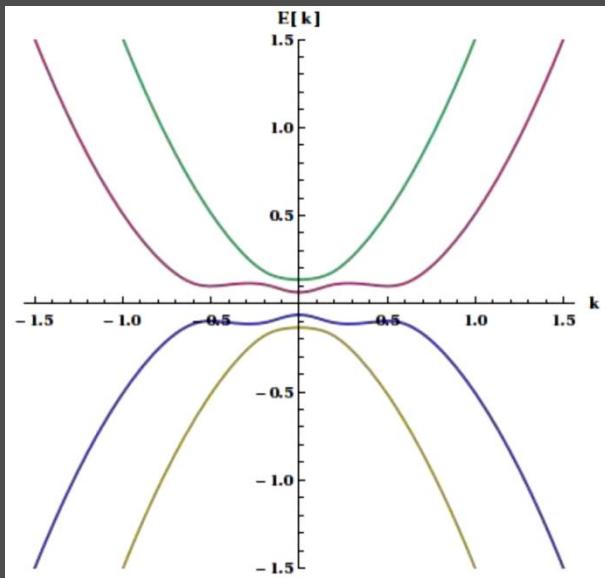
$$E_z < \Delta$$



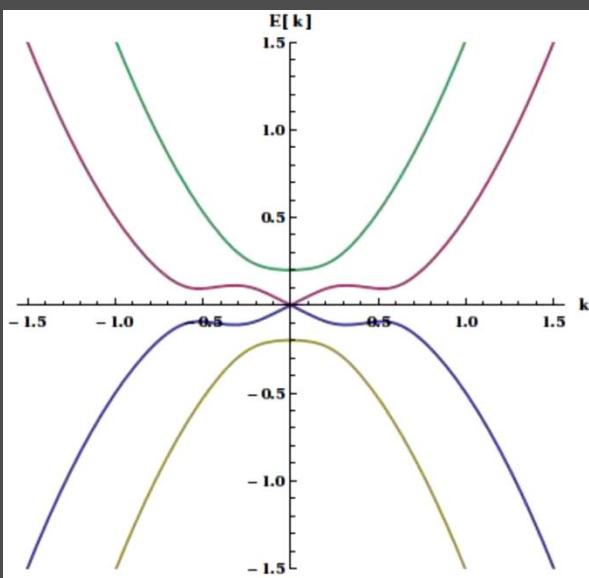
$$E_z = \Delta$$



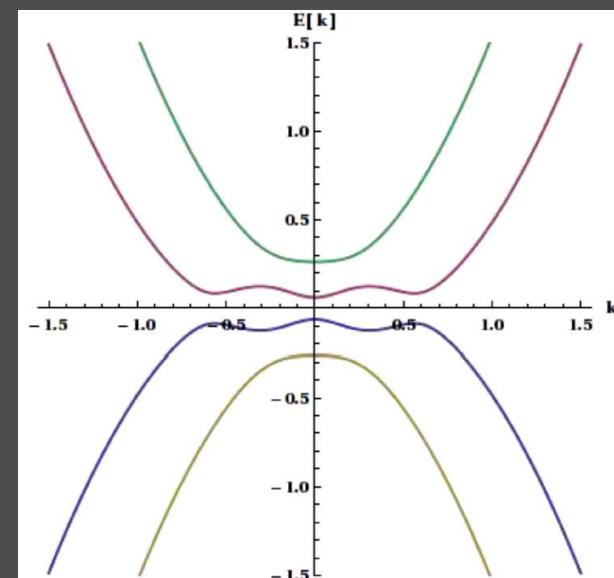
$$E_z > \Delta$$



Trivial Superconductor
“positive gap”

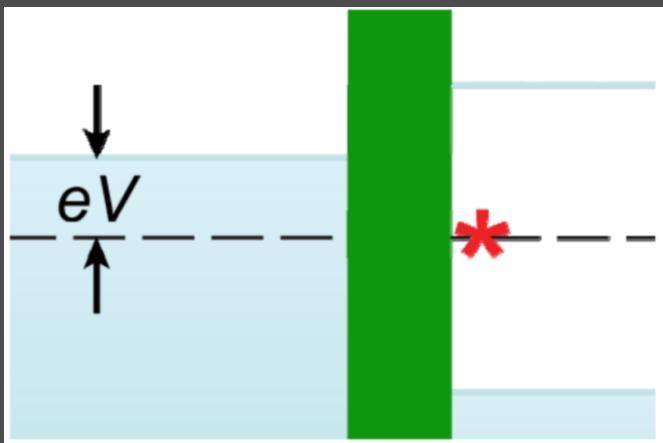
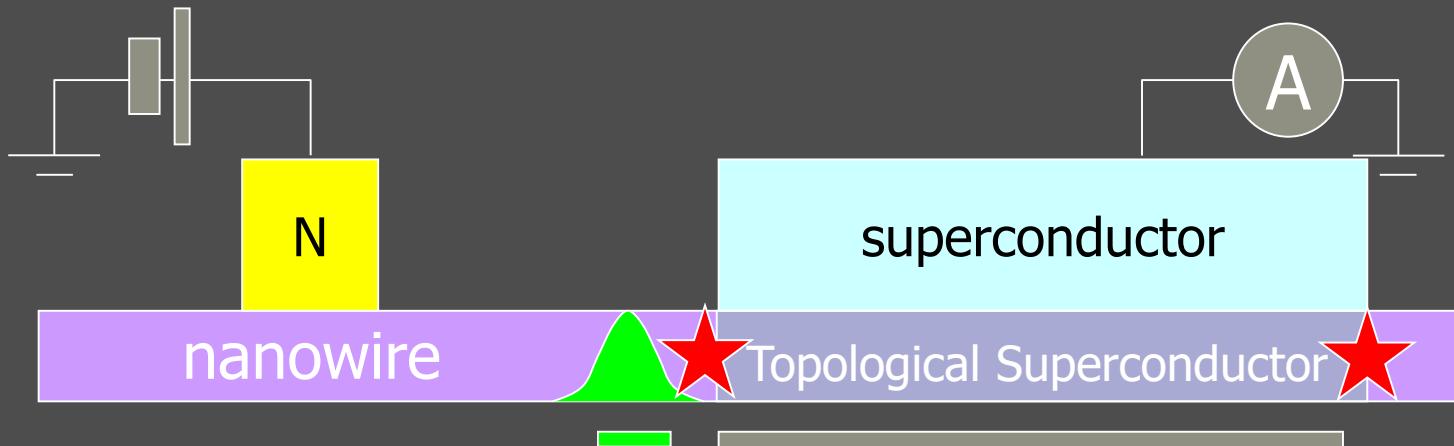


Majorana
“zero gap”



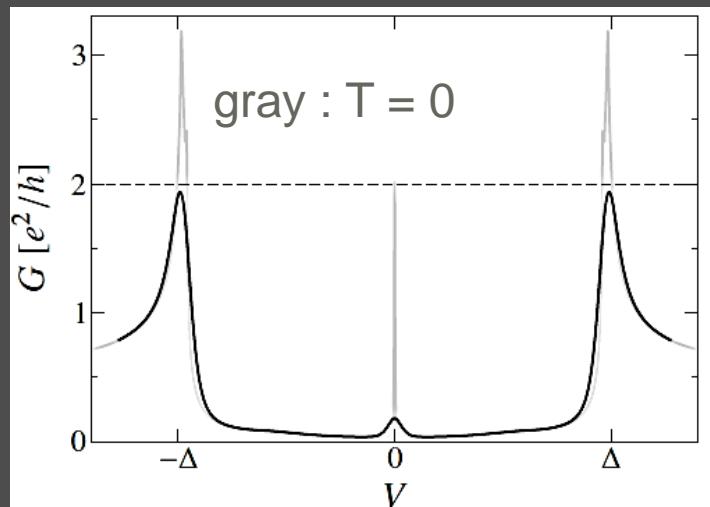
Topological Superconductor
“negative gap”

Tunneling experiment



Tunneling into a Majorana bound state:
Resonant Andreev current!

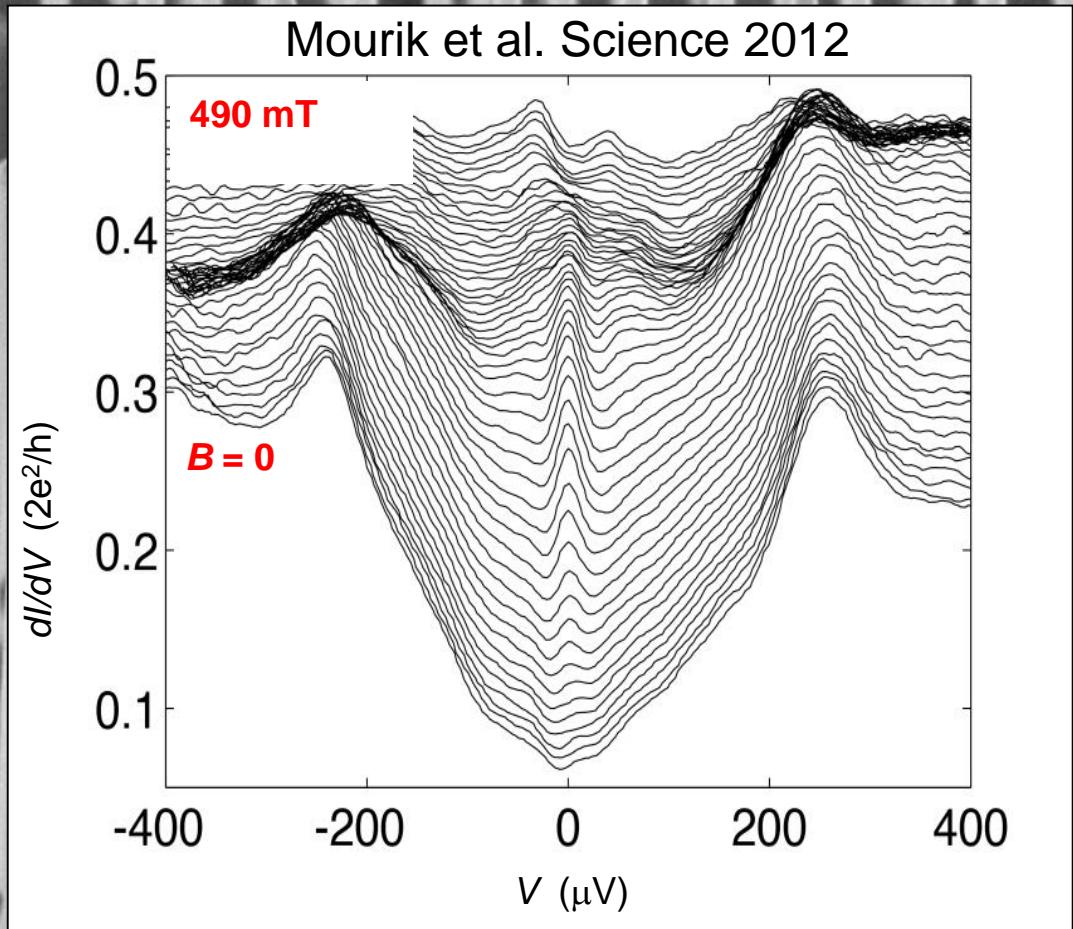
Zero bias peak, $2e^2/h$ conductance
Theory: Law, Lee & Ng PRL 2009
Claimed by Zhang et al Nature 2019
(but I see problems with that paper.)



Majorana recipe:

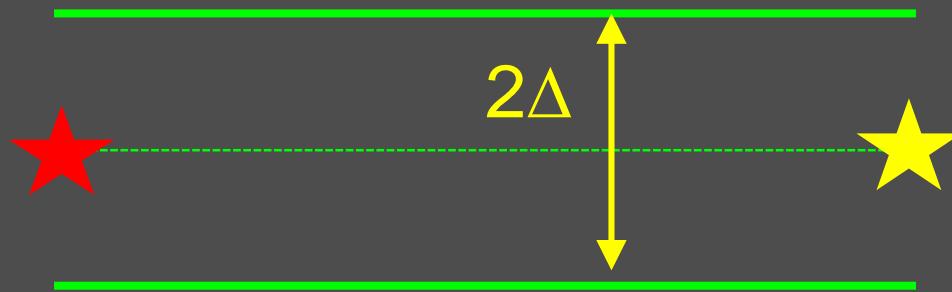
Lutchyn, Sau, Das Sarma, PRL 2010
Oreg, Refael, von Oppen, PRL 2010

1. Nanowire
2. Spin-orbit interaction
3. Superconductivity
4. Magnetic field

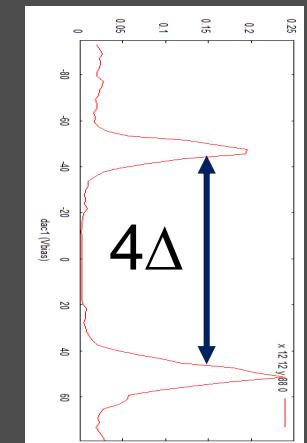
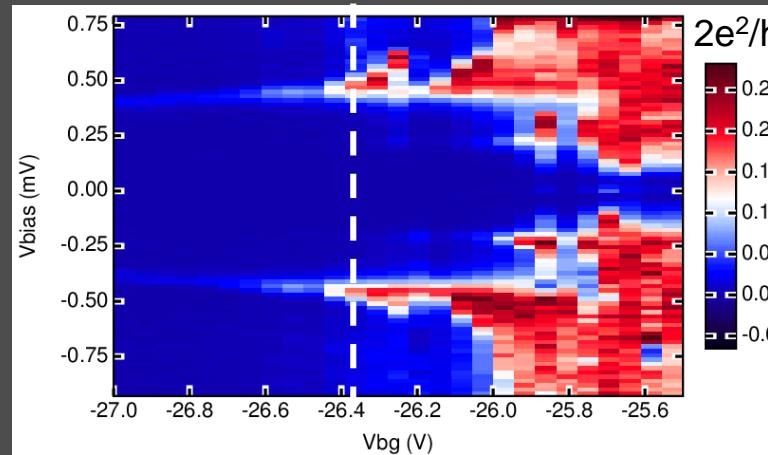
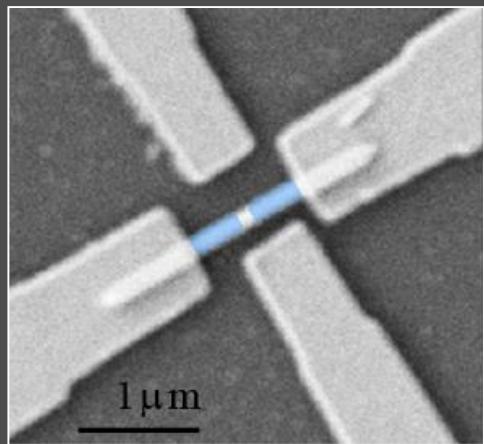


InSb nanowires: length 3 μm , diameter 100 nm
Plissard et al, Nano Letters 2012

Induced superconducting gap is one measure of topological protection



Hard gap realized at zero field,
Generically ‘softens’ when field is applied to induce Majorana



$$c^\dagger = \gamma_1 - i\gamma_2$$

$|1\rangle$

$$\gamma_1 \rightarrow \gamma_2$$

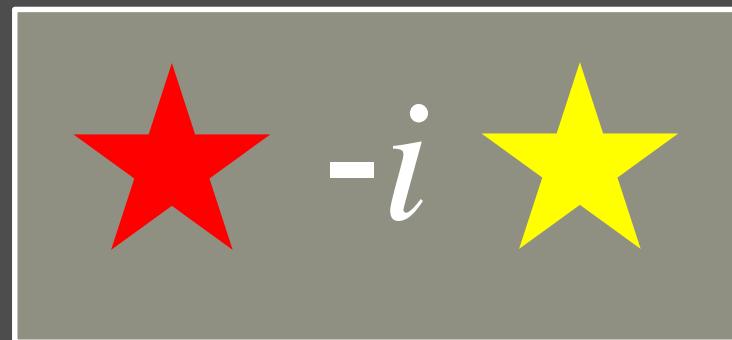
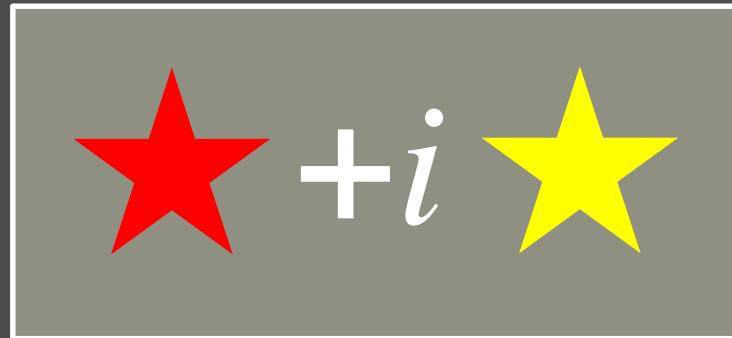
$$\gamma_2 \rightarrow -\gamma_1$$

Non-abelian anyons!
Moore, G. & Read, N. (1991)

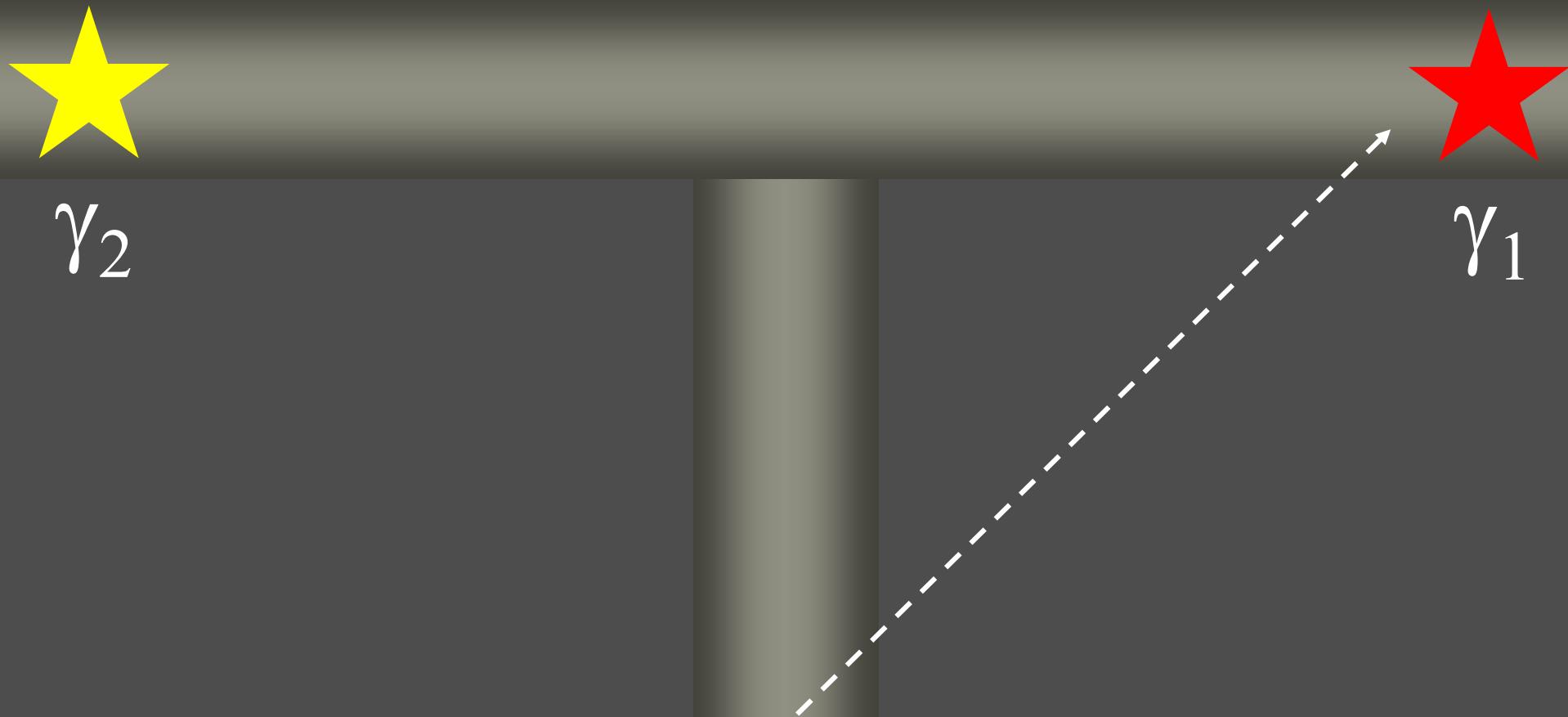
$$c = \gamma_1 + i\gamma_2$$

$|0\rangle$

2-Majorana system is a fermion box



Qubit flip (braiding) in a nanowire T-junction



No need to move anything in space
(except information)



Majorana qubit readout: fusion and parity measurement

$$|1\rangle \quad c^\dagger = \gamma_1 - i\gamma_2$$



γ_3

$\gamma_1 \gamma_2$

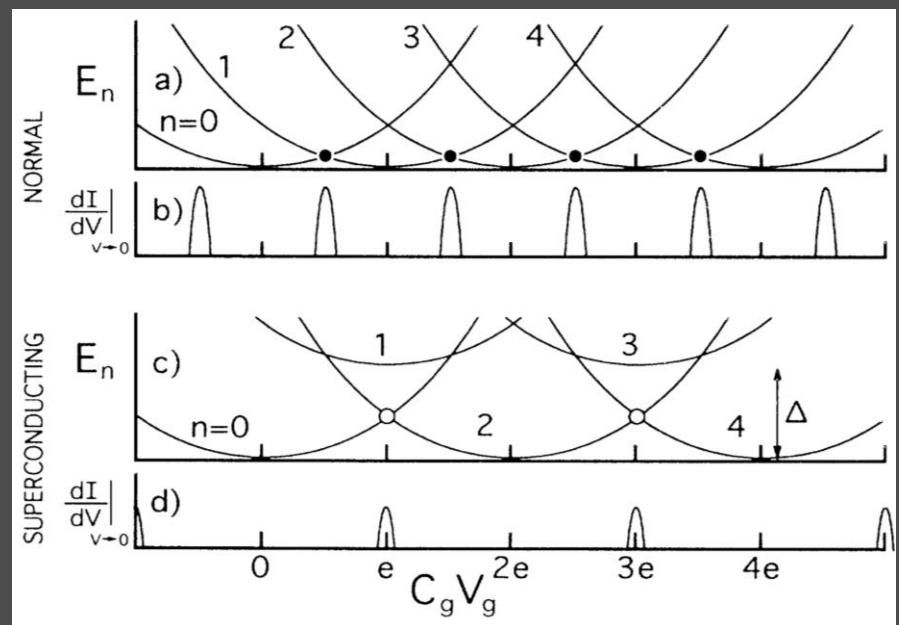
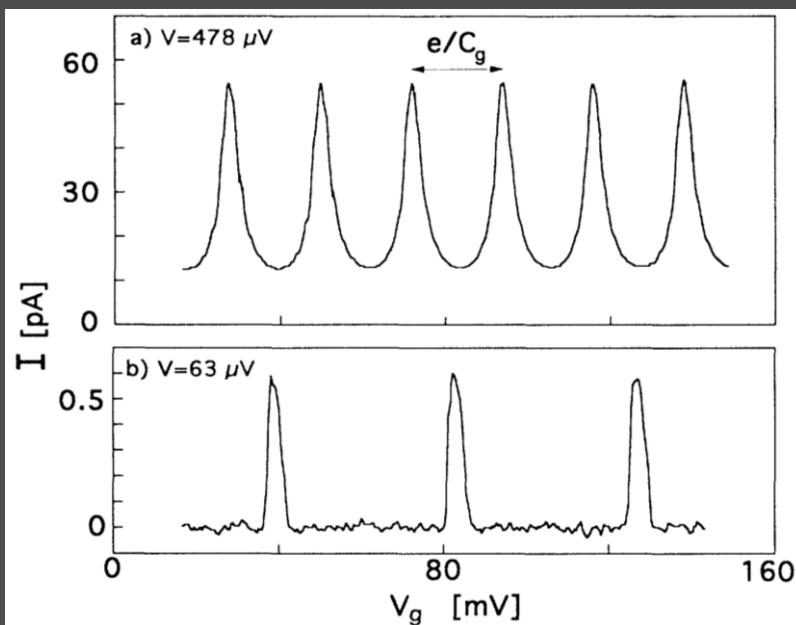
γ_4

$$|0\rangle \quad c = \gamma_1 + i\gamma_2$$

Even-odd effect in superconducting islands

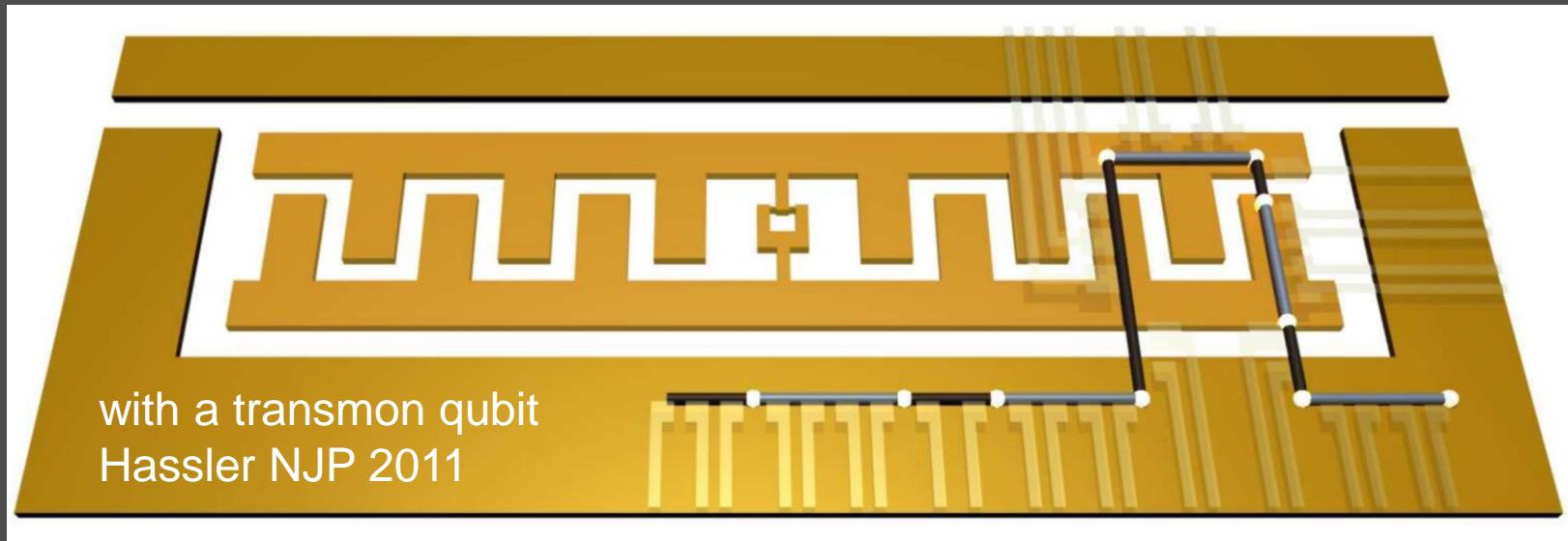
Adding single electrons to an island of superconductor costs extra energy Δ
(only observed in aluminum islands)

Adding single electrons to a **Majorana superconductor island**
costs no extra energy
(the island remains a superconductor!)



Eiles, Martinis, Devoret PRL 1993

Readout Majorana qubit be detecting charge on an island

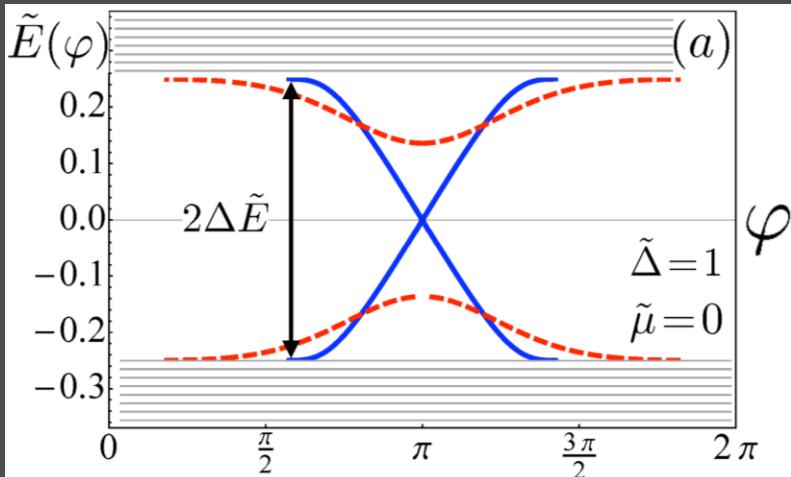
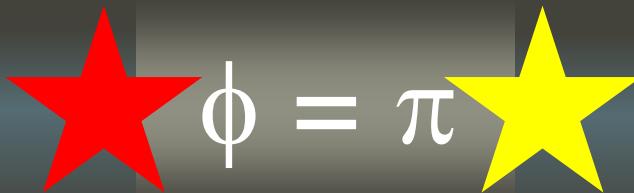


with quantum dots
Karzig PRB 2017

4π Josephson effect



$$\phi \neq \pi$$

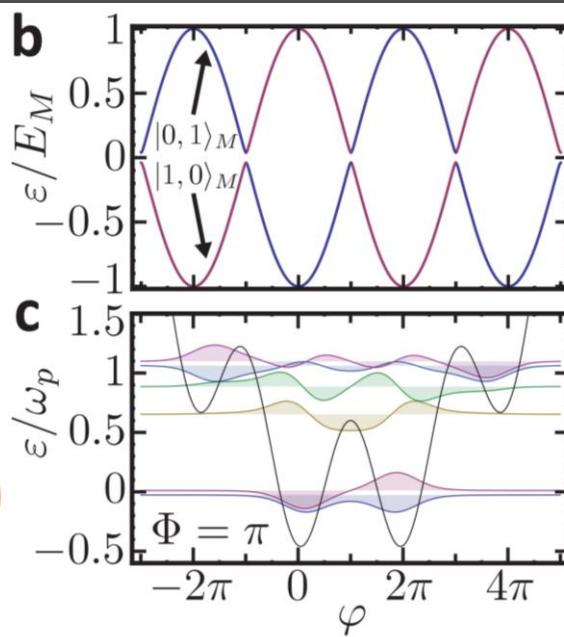
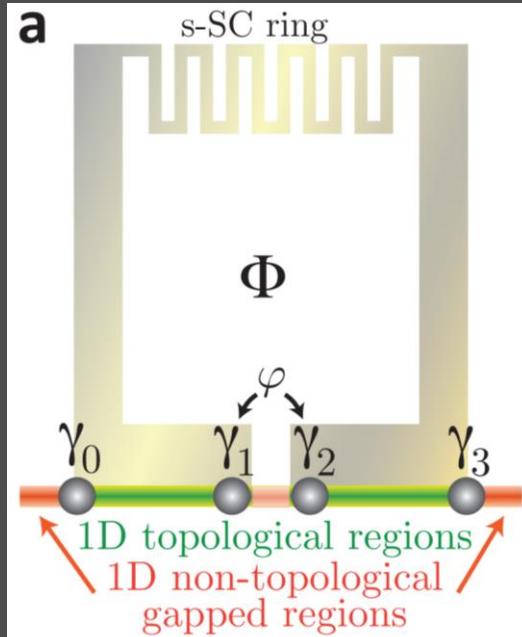


1e-periodic Josephson effect

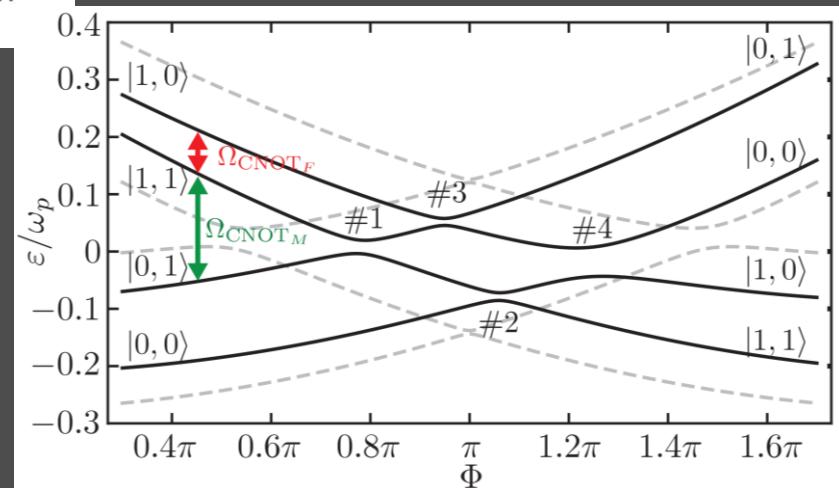
Dynamical signatures
(Shapiro, Josephson radio)
can manifest without Majorana

Majorana-fluxonium

Pekker, Hou, Manucharyan, Demler – PRL 2013



Two qubits in one device
Readout and initialization
No braiding
CNOT gate:



Braidonium Qubit

with John Stenger, Michael Hatridge, David Pekker – PRB 2019

