Departments of Physics
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Introduction to Circuit QED
Lecture 2

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http://quantuminstitute.yale.edu/

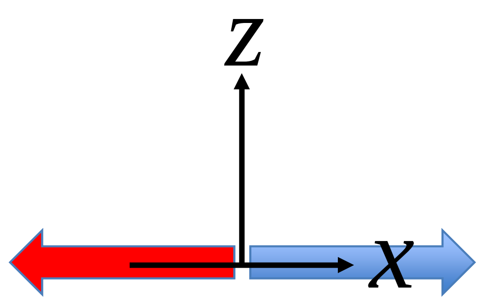
Reminder from Lecture 1: Measuring Photon Number Parity

- use quantized light shift of qubit frequency

$$\frac{\omega_{\mathrm{q}} + 2\chi a^{\dagger} a}{2} \sigma^{z}$$

$$e^{-i2\chi\hat{n}t\frac{\sigma^z}{2}} = \mathrm{e}^{-i\pi\hat{n}\frac{\sigma^z}{2}}$$

$\hat{n} = 1, 3, 5, \ldots$



$\hat{n} = 0, 2, 4, \ldots$

Lecture 2:
Quantum State Manipulation
and
Measurement in Circuit QED

The ability to measure photon number parity <u>without</u> measuring photon number is an incredibly powerful tool.

- Quantum Optics at the Single Photon Level
- Measuring Wigner Functions
- Creating and Verifying Schrödinger Cat States
- Cat in Two Boxes

Quantum optics at the single photon level

- Photon state engineering

Goal: arbitrary photon Fock state superpositions

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + \ldots$$

Use the coupling between the cavity (harmonic oscillator) and the two-level qubit (anharmonic oscillator) to achieve this goal.

Dispersively coupled cavity-qubit system is fully <u>controllable</u>.

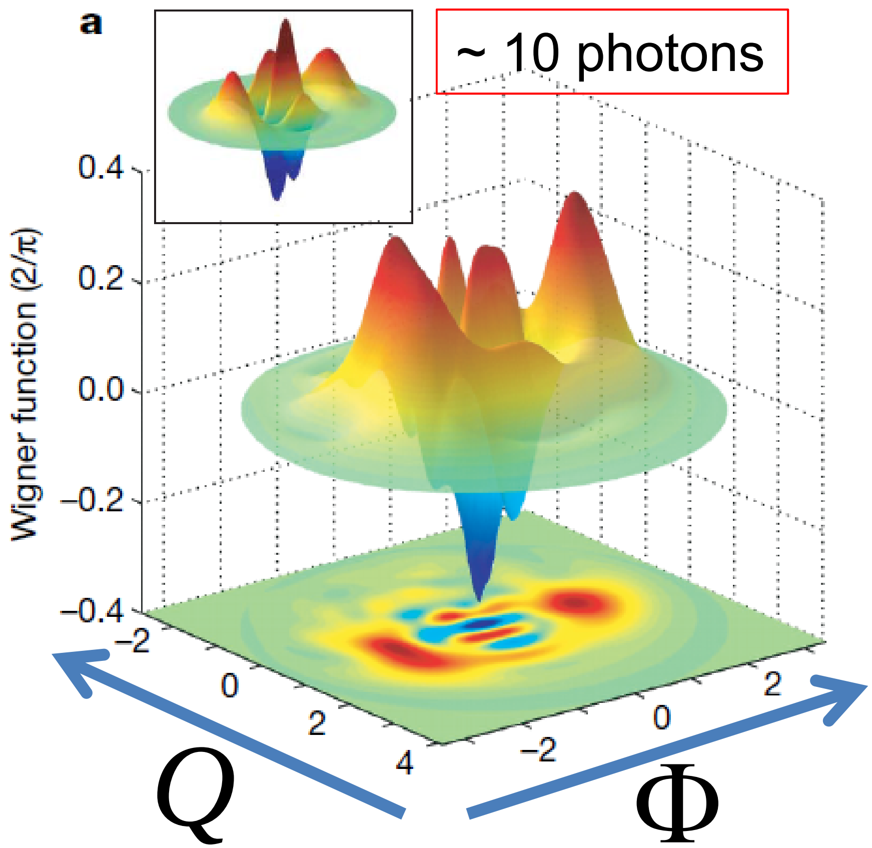
Previous State of the Art for Complex Oscillator States

Expt'l. Wigner tomography: Leibfried et al., 1996 ion traps (NIST – Wineland group)

Rydberg atom cavity QED

Haroche/Raimond, 2008 Rydberg (ENS)

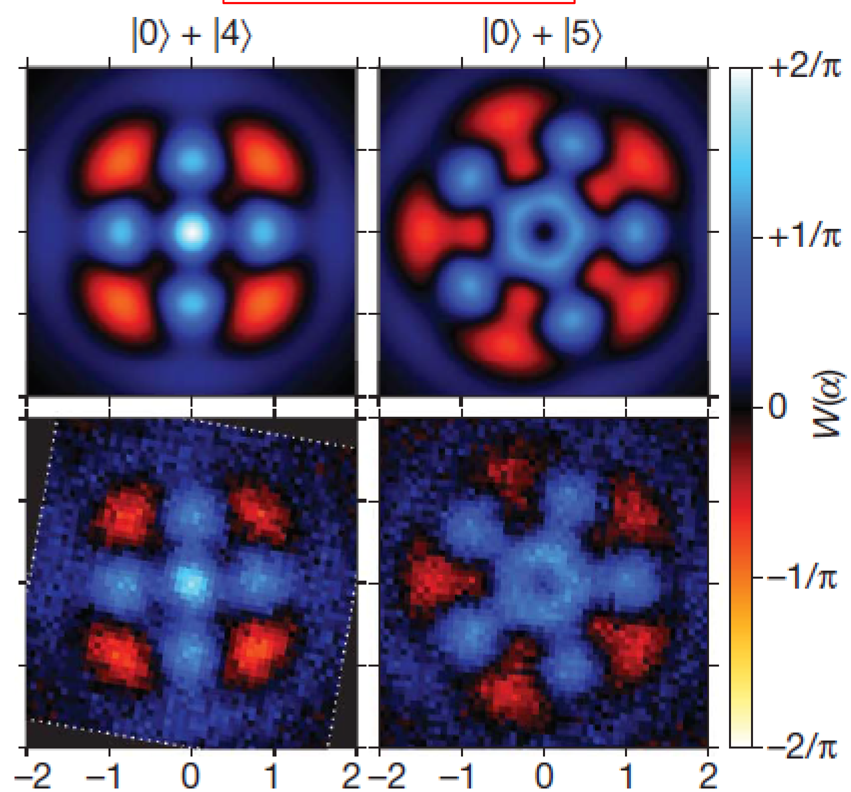
~ 10 photons



Phase qubit circuit QED

Hofheinz et al., 2009
(UCSB – Martinis/Cleland)

~ 10 photons

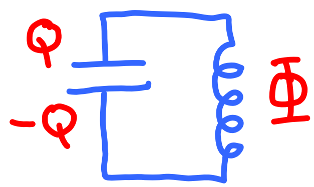


What concepts do we need to know to understand a

Schrödinger Cat State?

Photons in First Quantization

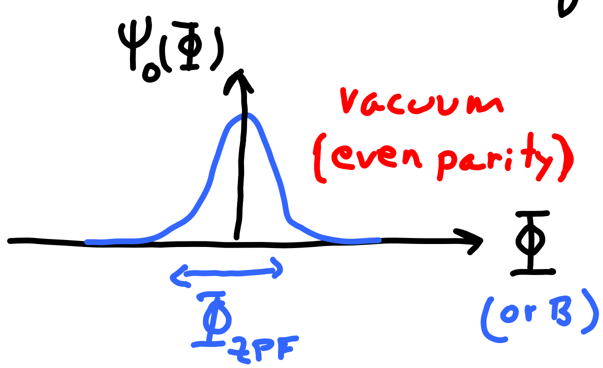
Circuit QED

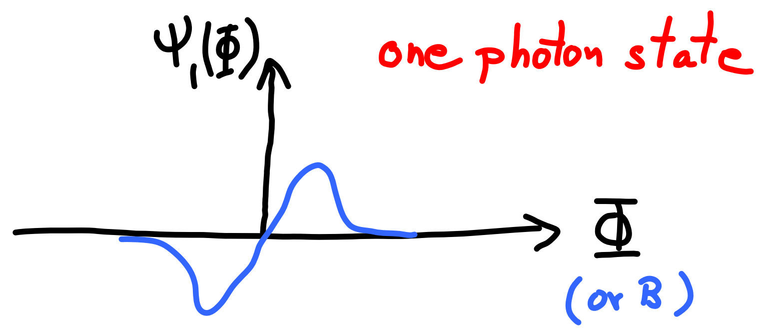


$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}; \quad [\hat{\Phi}, \hat{Q}] = -i\hbar$$

$$\omega_R = \frac{1}{\sqrt{LC}}$$

Photons and first quantization



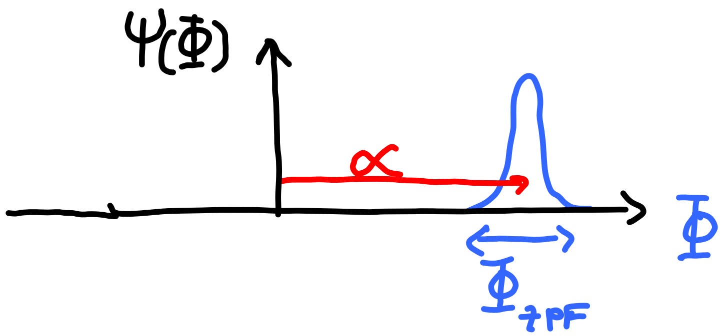


$$P(\Phi) = |\Psi(\Phi)|^2$$

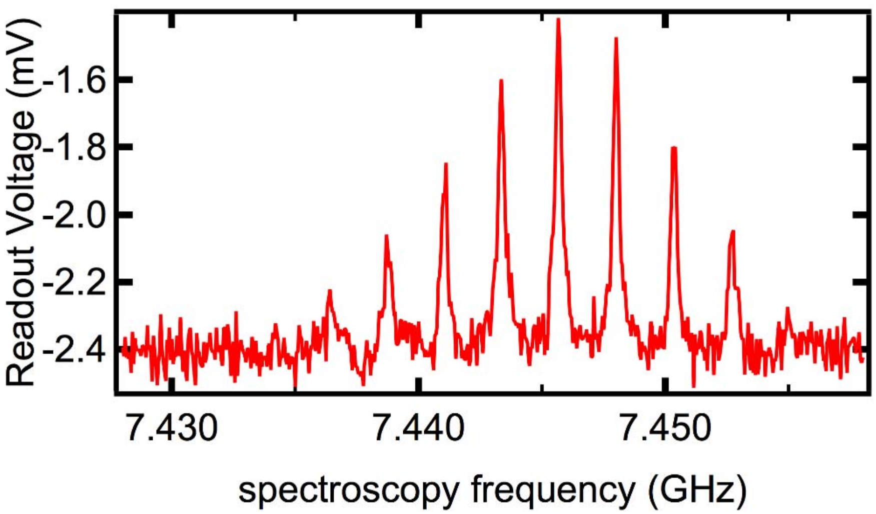
Coherent state is closest thing to a classical sinusoidal RF signal

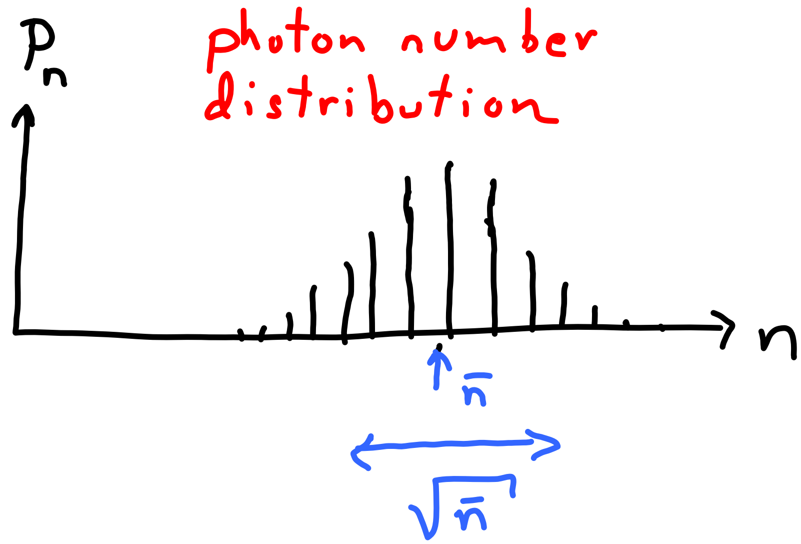
$$\psi(\Phi) = \psi_0(\Phi - \alpha)$$

Coherent state = displaced vacuum

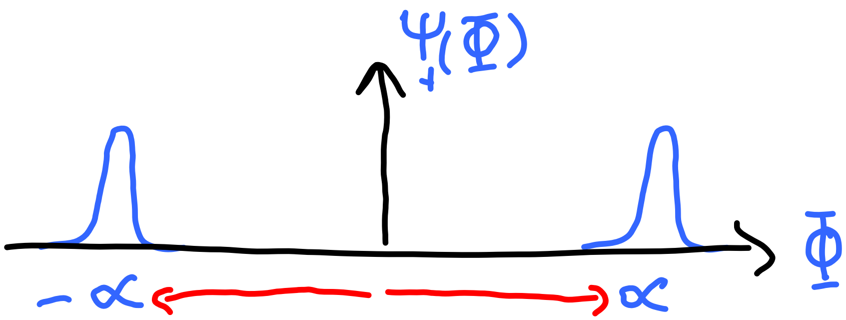


$$\bar{n} = \langle \alpha | a^\dagger a | \alpha \rangle = |\alpha|^2$$



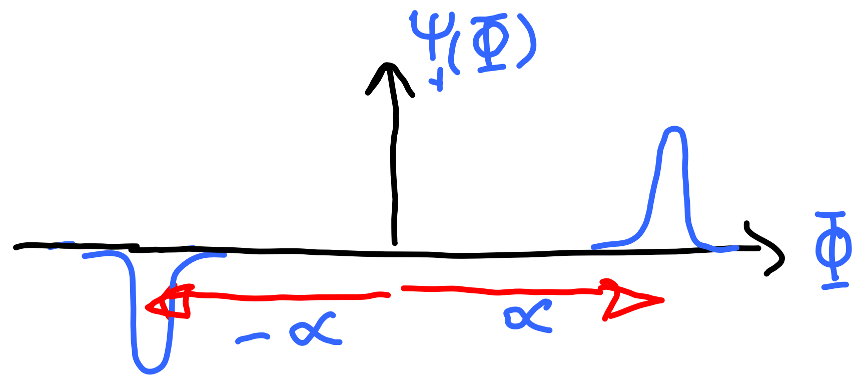


'Schrödinger Cat State'



$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\left\{|\alpha\rangle + |-\alpha\rangle\right\}$$

'even cat' only <u>even</u> n's



$$|\psi_-\rangle = \frac{1}{\sqrt{2}}\left\{|\alpha\rangle - |-\alpha\rangle\right\}$$

'odd cat' only <u>odd</u> n's

Superposition of two different 'macroscopic' states

$$\text{"size"} = \text{"distance}^2\text{"} = |2\alpha|^2 = 4\bar{n}$$

(normalization is only approximate)

$$|\text{even}\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle + |-\alpha\rangle\right)$$

$$|\text{odd}\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle - |-\alpha\rangle\right)$$

(normalization approx. only)

Novel property:

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

$$a|-\alpha\rangle = -\alpha|-\alpha\rangle$$

$$\Rightarrow$$

How cats die:

$$a|\text{even}\rangle = \alpha|\text{odd}\rangle$$

$$a|\text{odd}\rangle = \alpha|\text{even}\rangle$$

$$\Gamma_{\varphi} = 2\bar{n}\kappa$$

$$= 2\kappa\,|\,\alpha\,|^2 = \frac{\kappa}{2}(4\bar{n})$$

How do we create a cat?

‘Classical’ signal generators only displace the vacuum and create coherent states.

We need some non-linear coupling to the cavity via a qubit.

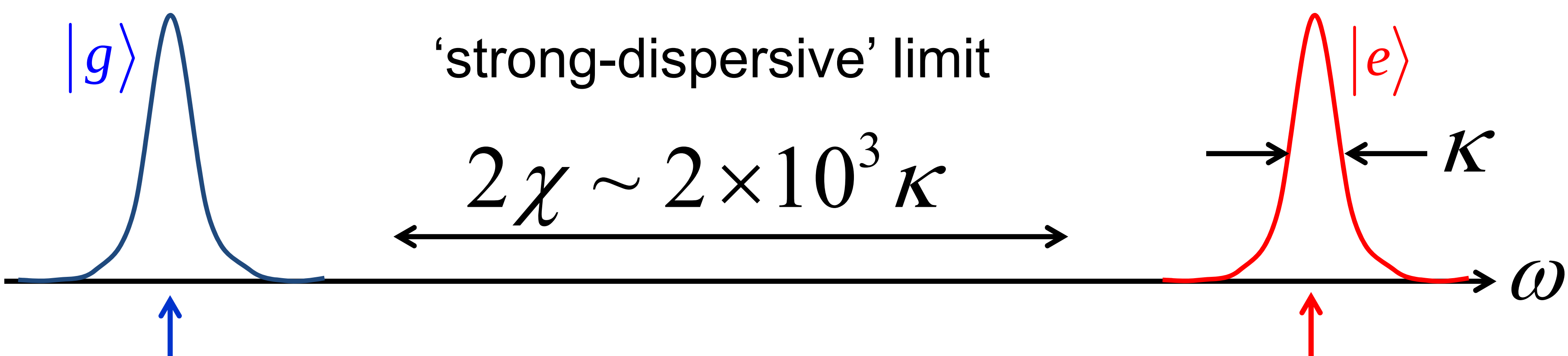
Strong Dispersive Hamiltonian

$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2} \sigma^{z} + \chi \sigma^{z} a^{\dagger} a + H_{\mathrm{damping}}$$

$$\chi >> \kappa, \Gamma$$

resonator qubit dispersive coupling

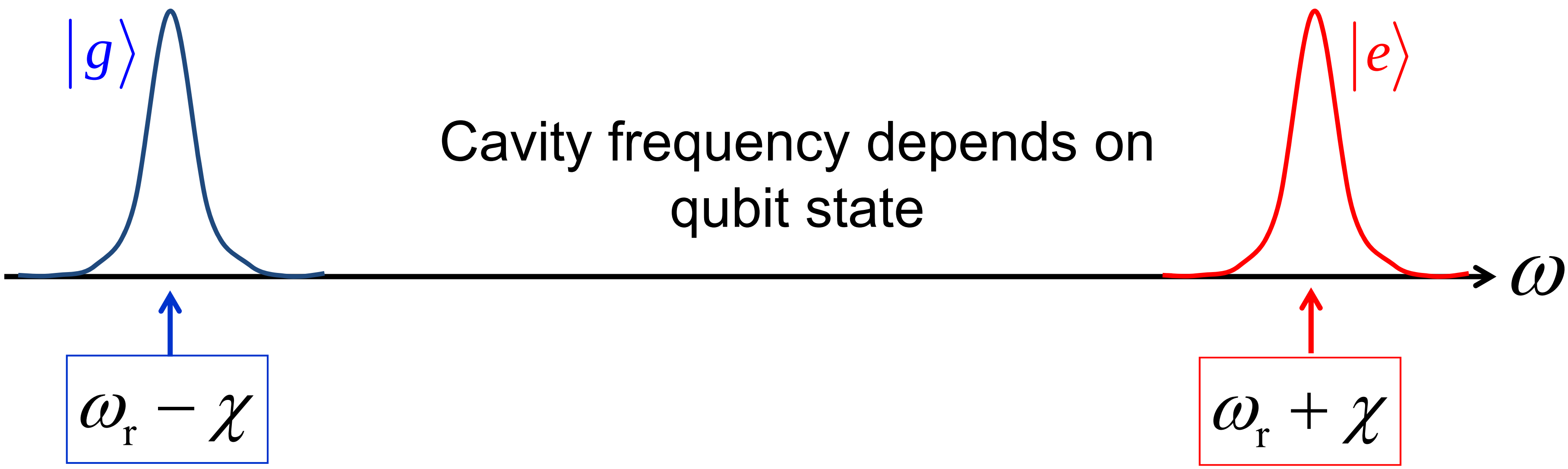
$$\text{cavity frequency} = \omega_{\mathrm{r}} + \chi \sigma^{z}$$



$\omega_{\mathrm{r}} - \chi$

$\omega_{\mathrm{r}} + \chi$

Strong-Dispersive Limit yields a powerful toolbox



Cavity frequency depends on qubit state

Microwave pulse at this frequency excites cavity only if qubit is in ground state

Microwave pulse at this frequency excites cavity only if qubit is in excited state

$$D_\alpha^g$$

Engineer's tool #1:

Conditional displacement of cavity

$$\pi_n$$

Engineer's tool #2:

Conditional flip of qubit if exactly n photons

$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2} \sigma^{z} + \chi \sigma^{z} a^{\dagger} a + H_{\mathrm{damping}}$$

resonator   qubit   dispersive coupling

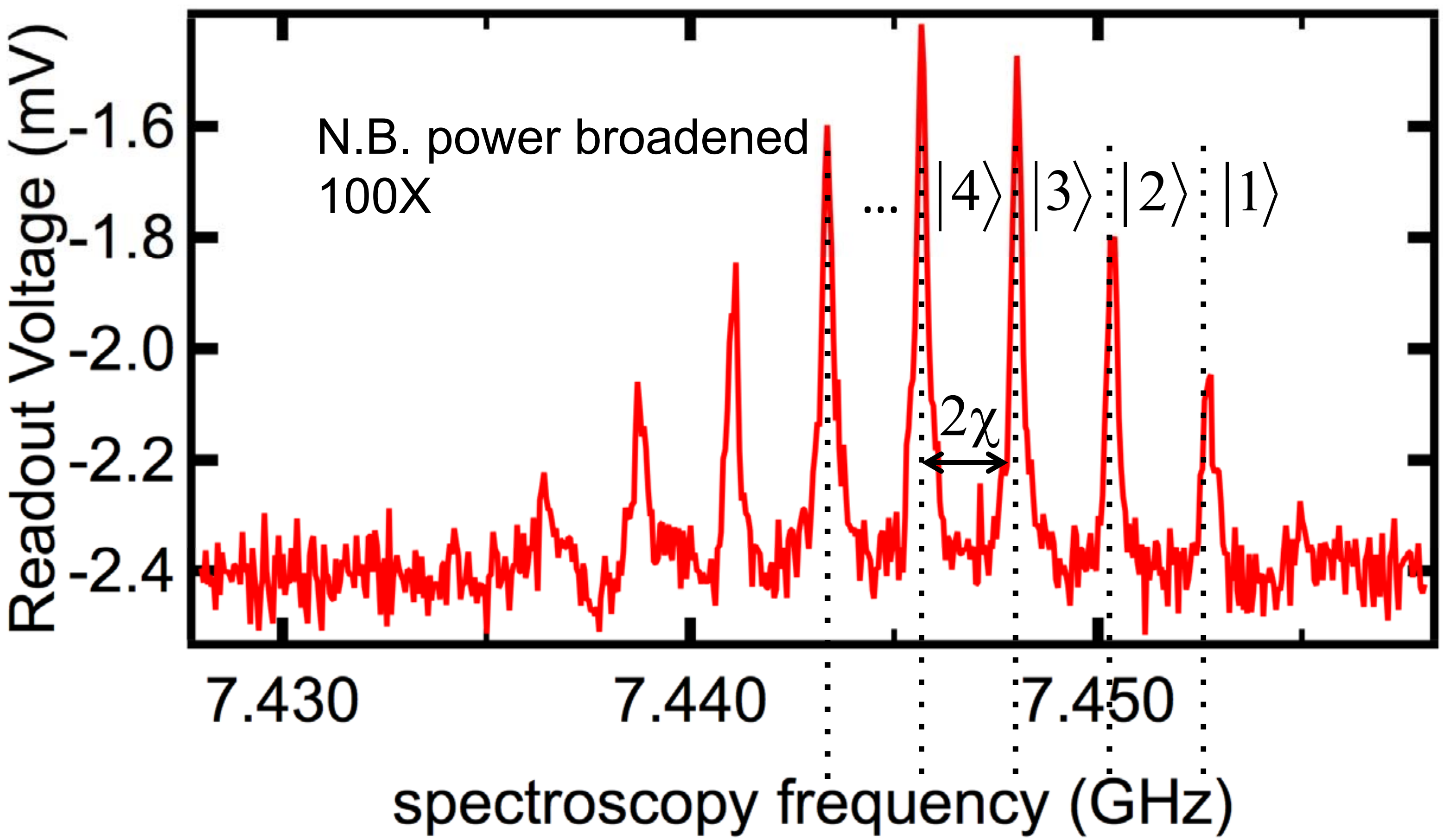
Reinterpret dispersive term:

- quantized light shift of qubit frequency

$$\frac{\omega_{\mathrm{q}} + 2\chi a^{\dagger} a}{2} \sigma^{z}$$

- quantized light shift of qubit frequency
  (coherent microwave state)

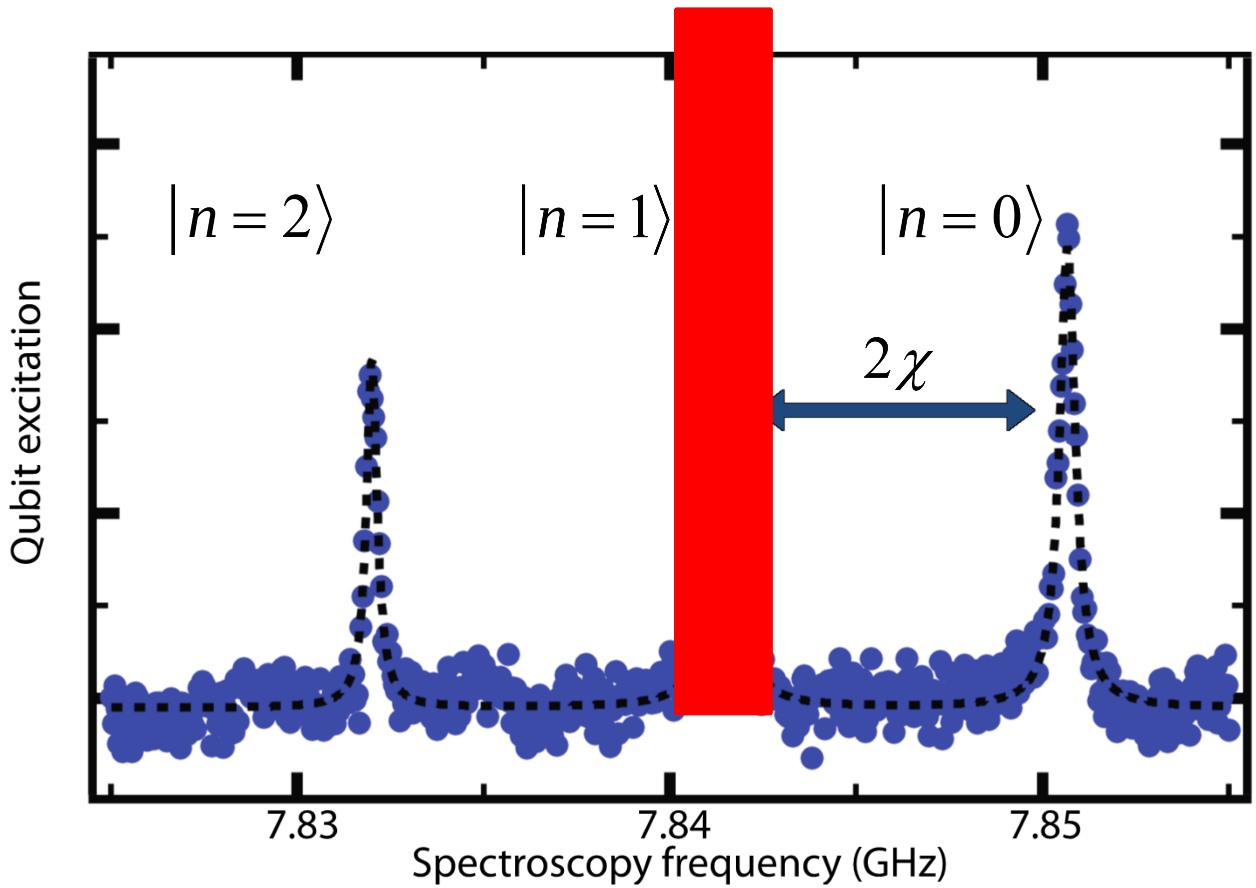
$$\frac{\omega_q + 2\chi a^\dagger a}{2}\sigma^z$$



strong dispersive coupling I

$$V_{\mathrm{DISPERSIVE}} \approx \chi a^{\dagger} a \sigma^{z}$$

Qubit Spectroscopy



Coherent state in the cavity

Conditional bit flip π_n

Strong Dispersive Coupling Gives Powerful Tool Set

Cavity conditioned bit flip π_n

Qubit-conditioned cavity displacement D_α^g

- multi-qubit geometric entangling phase gates (Paik et al.)
- Schrödinger cats are now 'easy' (Kirchmair et al.)

Photon Schrödinger cats on demand

experiment	theory
G. Kirchmair	M. Mirrahimi
B. Vlastakis	Z. Leghtas
A. Petrenko	

Deterministic Cat State Production

Vlastakis et al. *Science* ***342****, 607 (2013)*

$$|\psi\rangle = \frac{1}{\sqrt{2}}|g\rangle\left(|\alpha\rangle \pm |-\alpha\rangle\right)$$

Will skip over details of cat state production;
Focus on proving the cat is not an incoherent mixture:

- measure photon number parity in the cat

- measure the Wigner function
  (phase space distribution of cat)

Proving phase coherence via photon number distribution

Coherent state: $\left|\psi\right\rangle = \left|\alpha = 2\right\rangle$

Mean photon number: 4

Even parity cat state: $\left|\psi\right\rangle = \left|\alpha\right\rangle + \left|-\alpha\right\rangle$

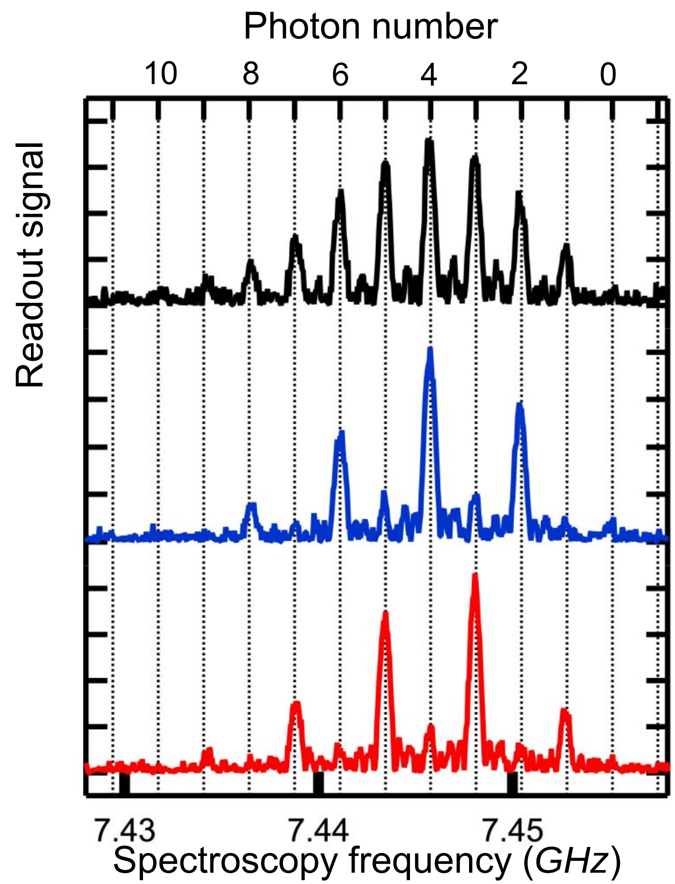
Only photon numbers: 0, 2, 4, …

$\hat{P}\left|\psi\right\rangle = +\left|\psi\right\rangle$

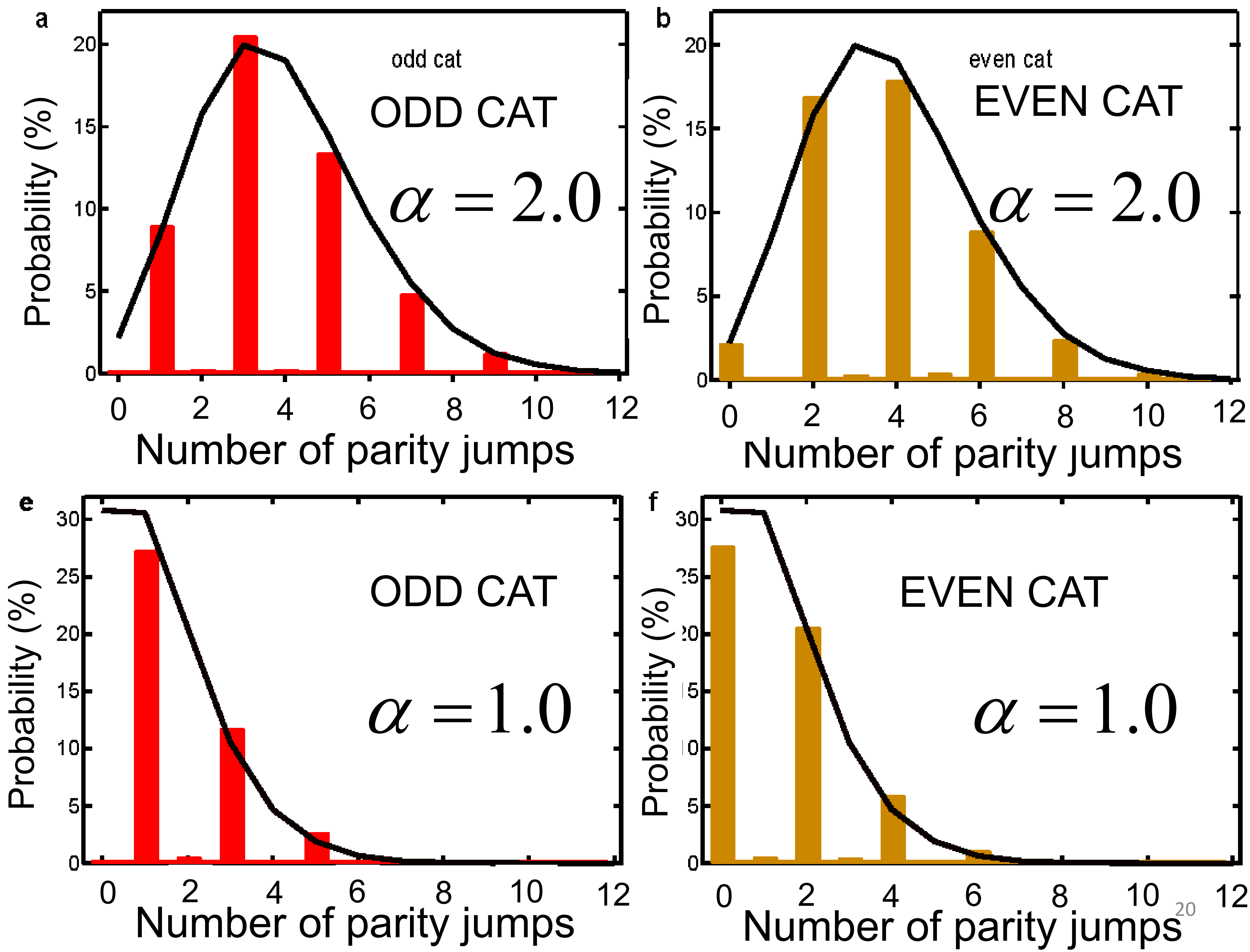
Odd parity cat state: $\left|\psi\right\rangle = \left|\alpha\right\rangle - \left|-\alpha\right\rangle$

Only photon numbers: 1, 3, 5, …

$\hat{P}\left|\psi\right\rangle = -\left|\psi\right\rangle$



Qubit Spectrum


a
odd cat
ODD CAT
$\alpha = 2.0$
Probability (%)
Number of parity jumps
b
even cat
EVEN CAT
$\alpha = 2.0$
Probability (%)
Number of parity jumps
e
ODD CAT
$\alpha = 1.0$
Probability (%)
Number of parity jumps
f
EVEN CAT
$\alpha = 1.0$
Probability (%)
Number of parity jumps

We have proven our states have the correct parity and photon number distribution.

We have not (strictly) verified all the phases are correct.

Need full state tomography via measurement of the Wigner Function.

Wigner Function Measurement

Vlastakis, Kirchmair, et al., *Science* (2013)

Density Matrix:

$$\rho(\Phi',\Phi) = \langle \Phi' | \Psi \rangle \langle \Psi | \Phi \rangle = \Psi(\Phi')\Psi^*(\Phi)$$

Define center of mass and relative coordinates:

$$\bar{\Phi} \equiv \frac{\Phi + \Phi'}{2}, \quad \mathrm{r} \equiv \frac{\Phi - \Phi'}{2}$$

Wigner Function (definition):

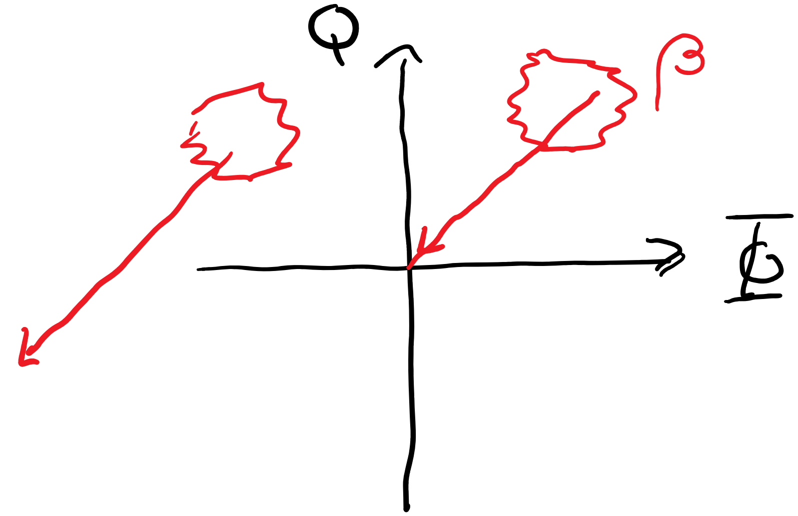
$$W(\bar{Q},\bar{\Phi}) = \int dr\, e^{i\bar{Q}r} \rho(\bar{\Phi} - \frac{r}{2}, \bar{\Phi} + \frac{r}{2})$$

Combines position and momentum information by Fourier transforming relative coordinate

Wigner Function = "Displaced Parity"

Vlastakis, Kirchmair, et al., *Science* (2013)

Simple Recipe:
1. Apply microwave tone to displace oscillator in phase space.
2. Measure mean parity.

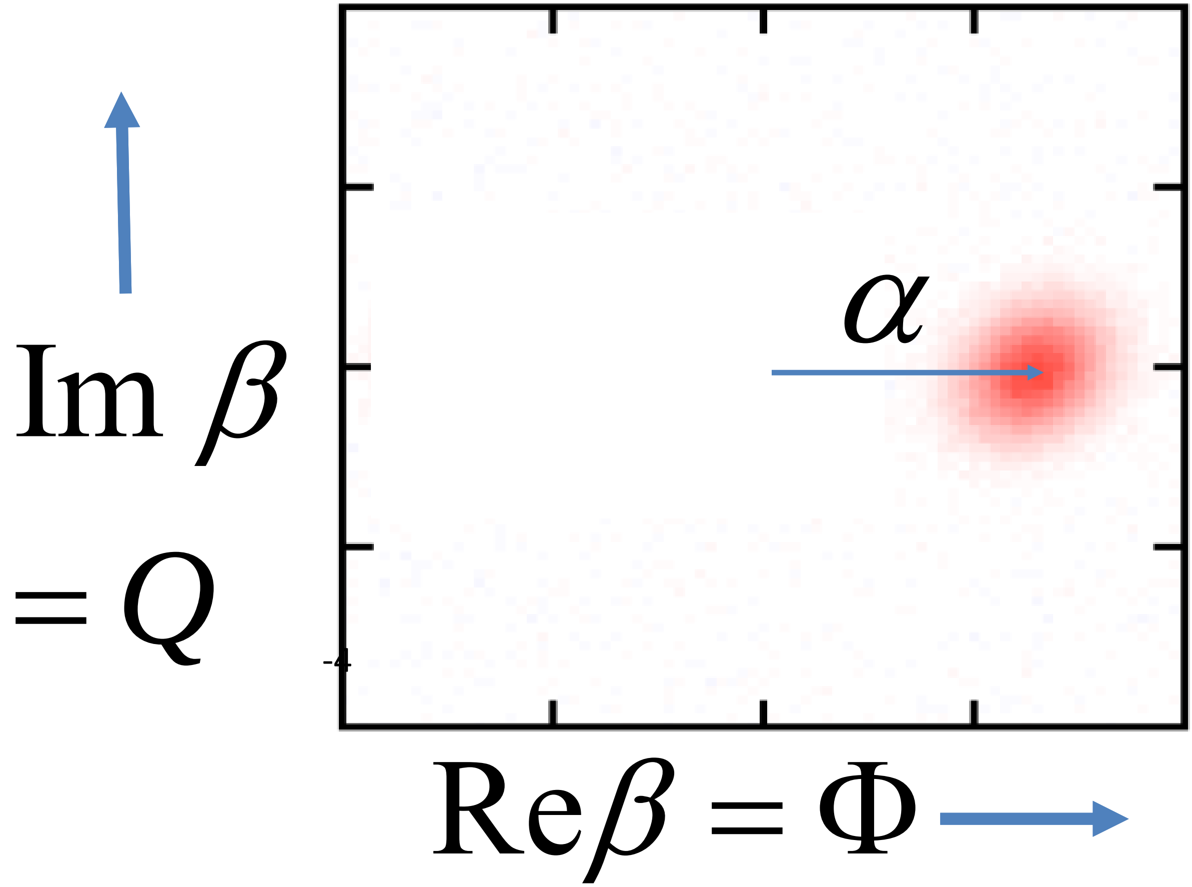


Handy identity (Luterbach and Davidovitch):

$$W(\beta) = \langle \Psi | D(+\beta)\hat{P} D(-\beta) | \Psi \rangle \qquad \hat{P} = (-1)^{\hat{N}} = \text{parity}$$

Full state tomography on large dimensional Hilbert space can be done very simply over a single input-output wire.

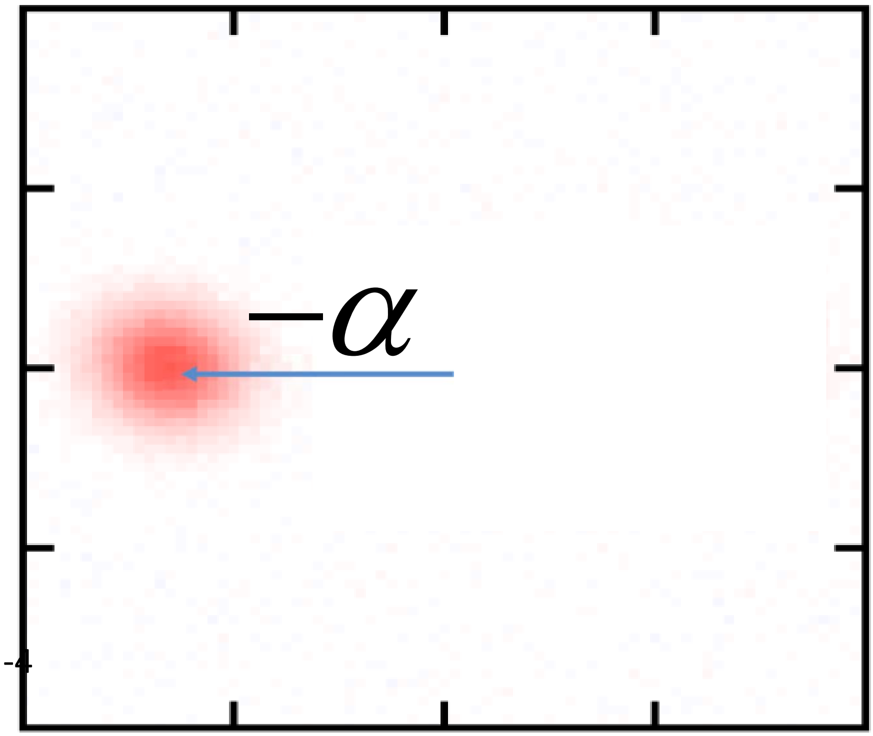
Wigner Function of a Coherent State $\left|\Psi\right\rangle = \left|\alpha\right\rangle$



$$W(\beta) = \left\langle \Psi \middle| D(+\beta)\hat{P}D(-\beta) \middle| \Psi \right\rangle$$

$$\hat{P} = (-1)^{\hat{N}} = \text{ parity}$$

Wigner Function of a Coherent State $\left|\Psi\right\rangle = \left|-\alpha\right\rangle$



$\mathrm{Im}\,\beta = Q$

$\mathrm{Re}\beta = \Phi$

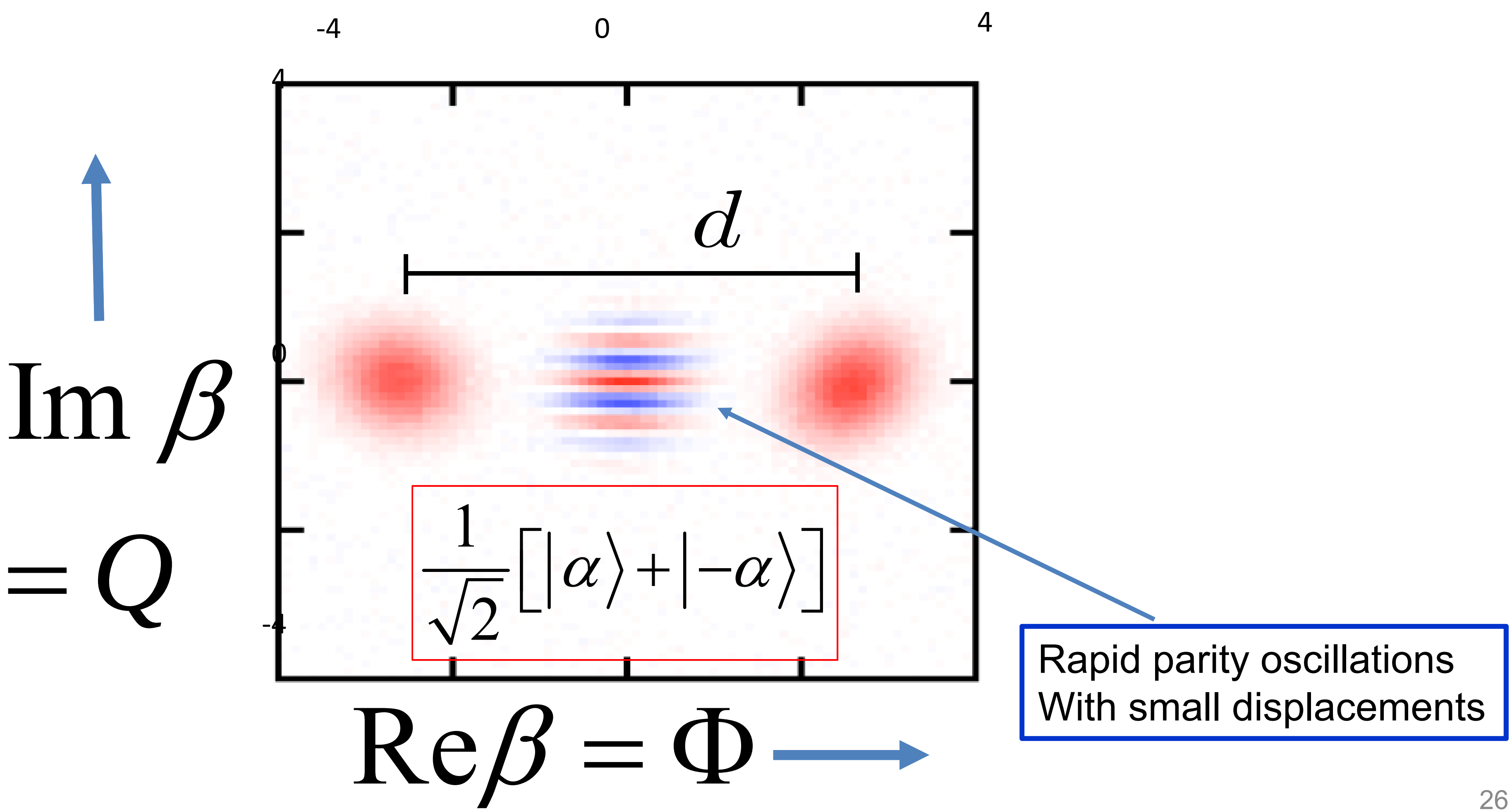
$$W(\beta) = \left\langle \Psi \middle| D(+\beta)\hat{P}D(-\beta) \middle| \Psi \right\rangle$$

$$\hat{P} = (-1)^{\hat{N}} = \text{ parity}$$

Wigner Function of a Cat State

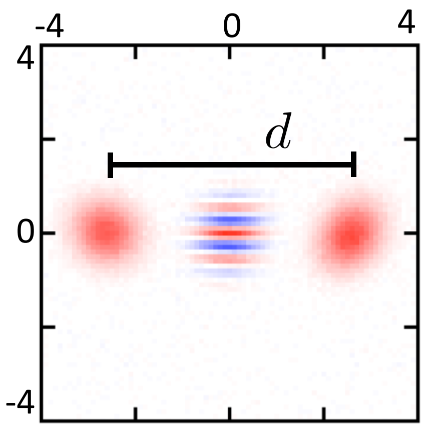
Vlastakis, Kirchmair, et al., *Science* (2013)

Interference fringes prove cat is coherent:



Deterministic Cat State Production

Vlastakis, Kirchmair, et al., *Science* (2013)

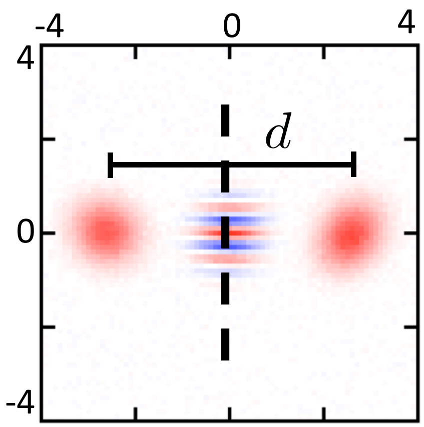


Data!

Expt'l Wigner function

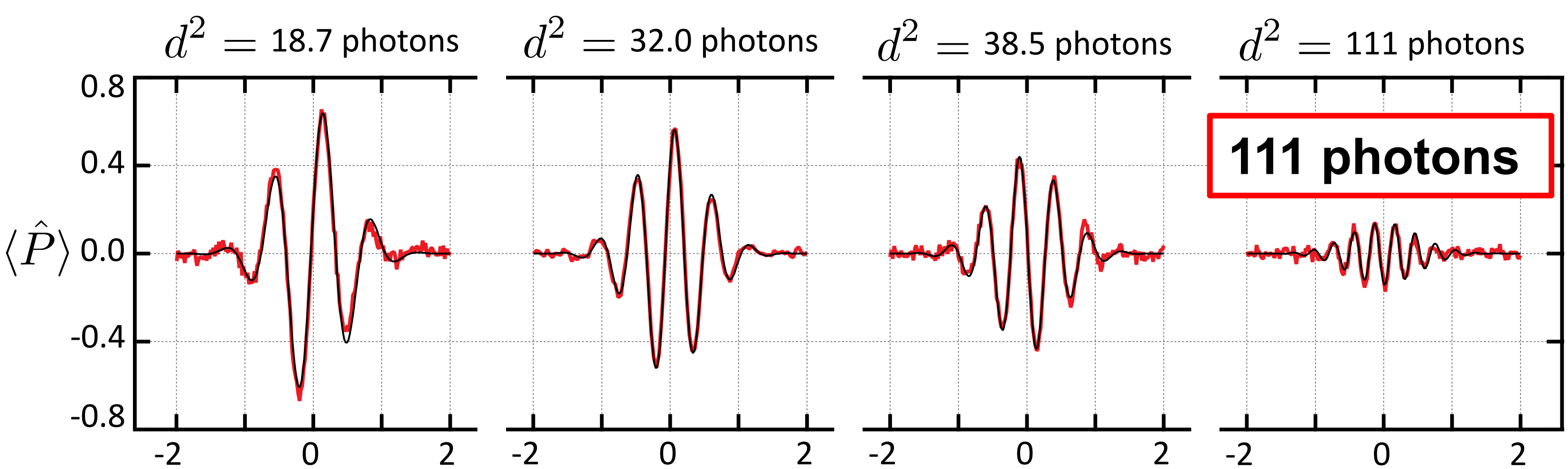
Deterministic Cat State Production

Vlastakis, Kirchmair, et al., Science (2013)



Data!

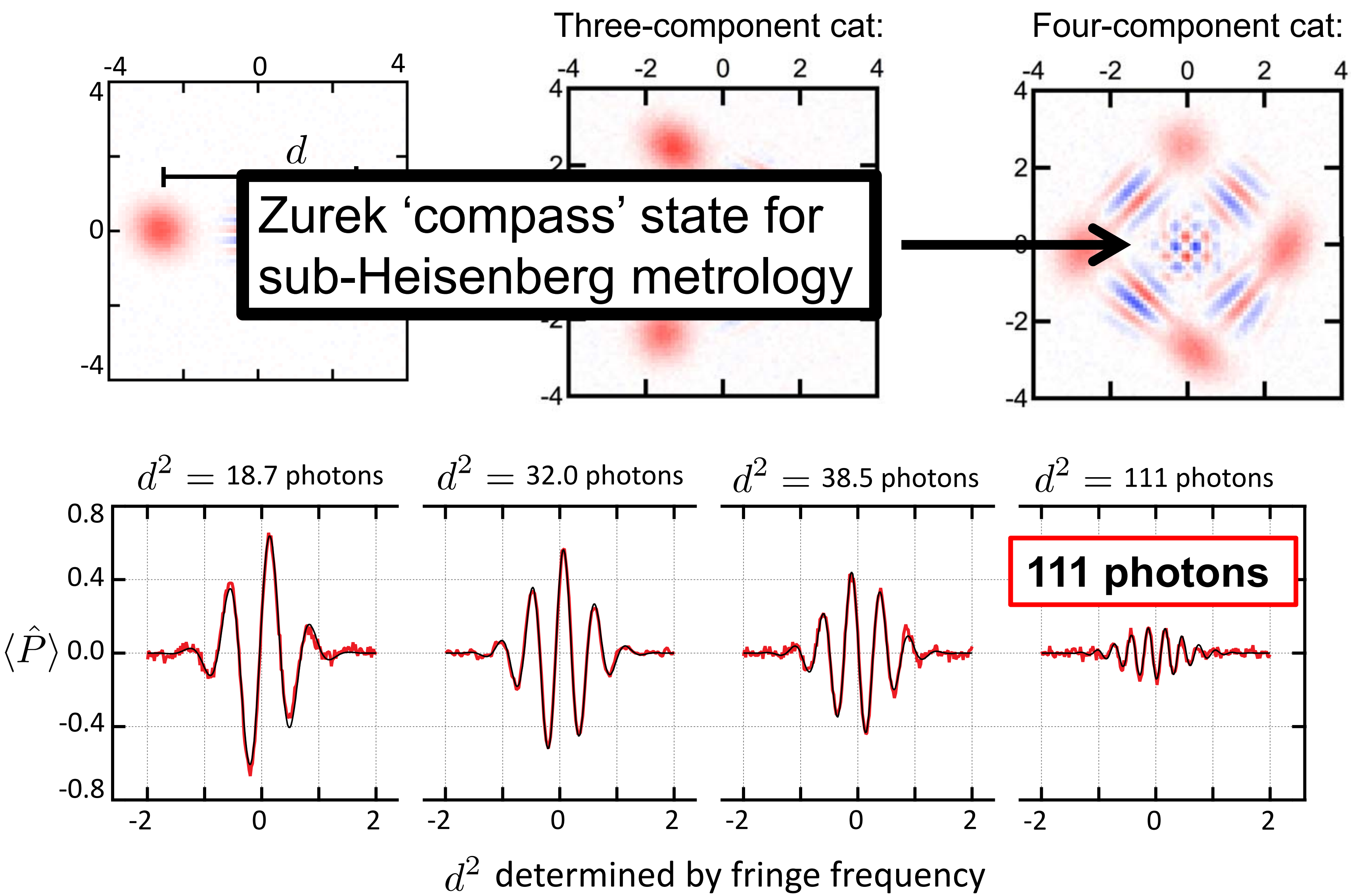
Expt'l Wigner function



Most macroscopic superposition ever created?

Deterministic Photon Cat Production

Vlastakis, Kirchmair, et al., *Science* (2013)



Non-Deterministic Cat State Production

Using Parity Measurement

Cat State = Coherent State Projected onto Parity

L. Sun et al., *Nature* (July 2014)

qubit cavity

$$|+x\rangle|\alpha\rangle = |+x\rangle\left[\frac{|\alpha\rangle+|-\alpha\rangle}{2}+\frac{|\alpha\rangle-|-\alpha\rangle}{2}\right] = |+x\rangle\left[\frac{|\text{even}\rangle}{\sqrt{2}}+\frac{|\text{odd}\rangle}{\sqrt{2}}\right]$$

time evolve to entangle spin with cat states:

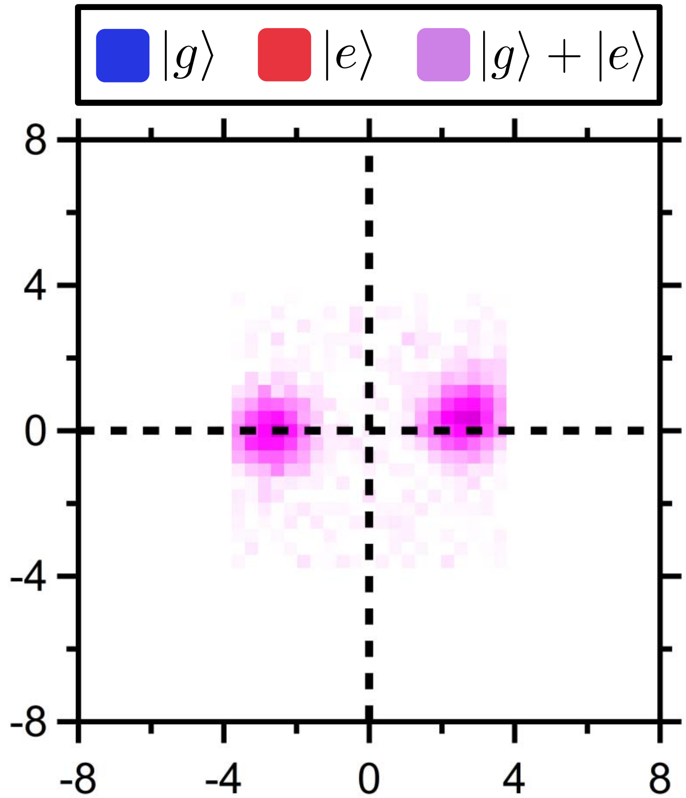
$$e^{-i2\chi\hat{n}t\frac{\sigma^z}{2}} = \mathrm{e}^{-i\pi\hat{n}\frac{\sigma^z}{2}}$$

$$\rightarrow\left[\frac{|+x\rangle|\text{even}\rangle}{\sqrt{2}}+\frac{|-x\rangle|\text{odd}\rangle}{\sqrt{2}}\right]$$

Wigner Tomography of cats entangled with qubit

L. Sun et al., *Nature* (July 2014)

$$\rightarrow \left[\frac{\left|+x\right\rangle \left|\text{even}\right\rangle}{\sqrt{2}} + \frac{\left|-x\right\rangle \left|\text{odd}\right\rangle}{\sqrt{2}} \right]$$

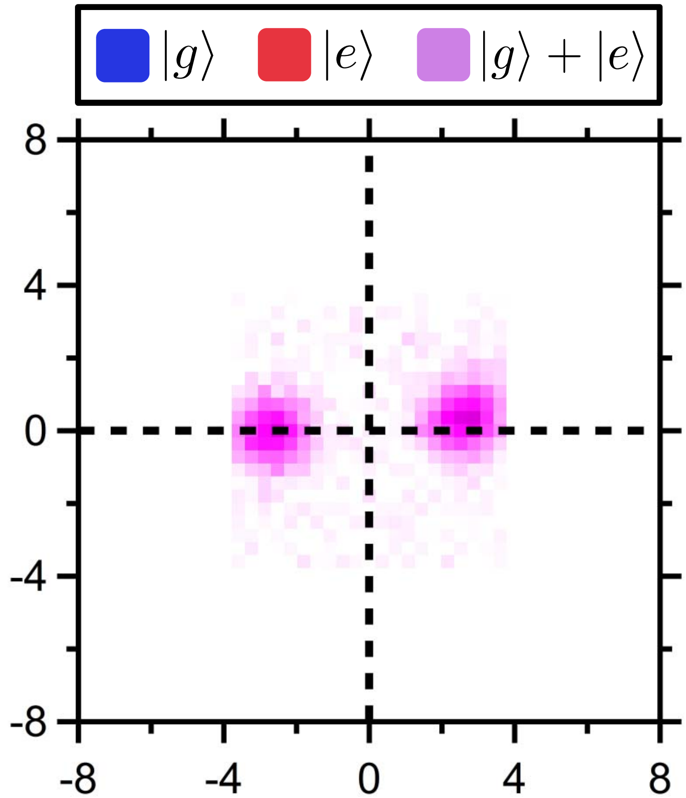


Wigner function of cavity (tracing out qubit) yields an incoherent MIXTURE of two coherent states and not a cat. (no fringes)

Equivalently: mixture of even and odd cats.

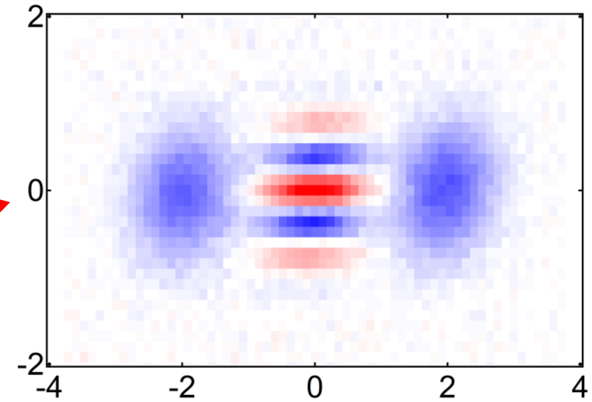
Wigner Tomography Conditioned on Qubit State

L. Sun et al., *Nature* (July 2014)

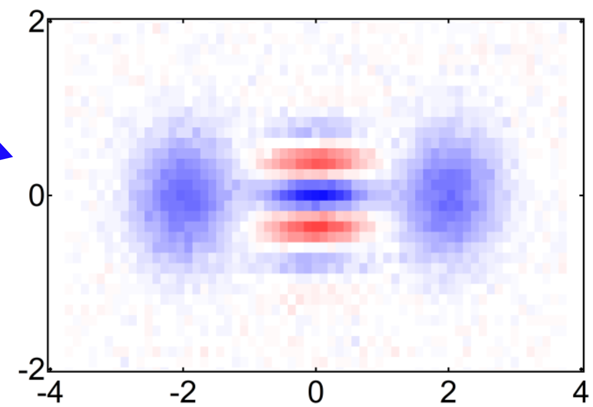


"qubit is in |+x>"

$$|\psi_{odd}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle)$$



"qubit is in |-x>"



$$|\psi_{even}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle)$$

Fidelity of produced cats:

$$F = \langle\psi_{cat}|\,\hat{\rho}\,|\psi_{cat}\rangle = 0.83$$

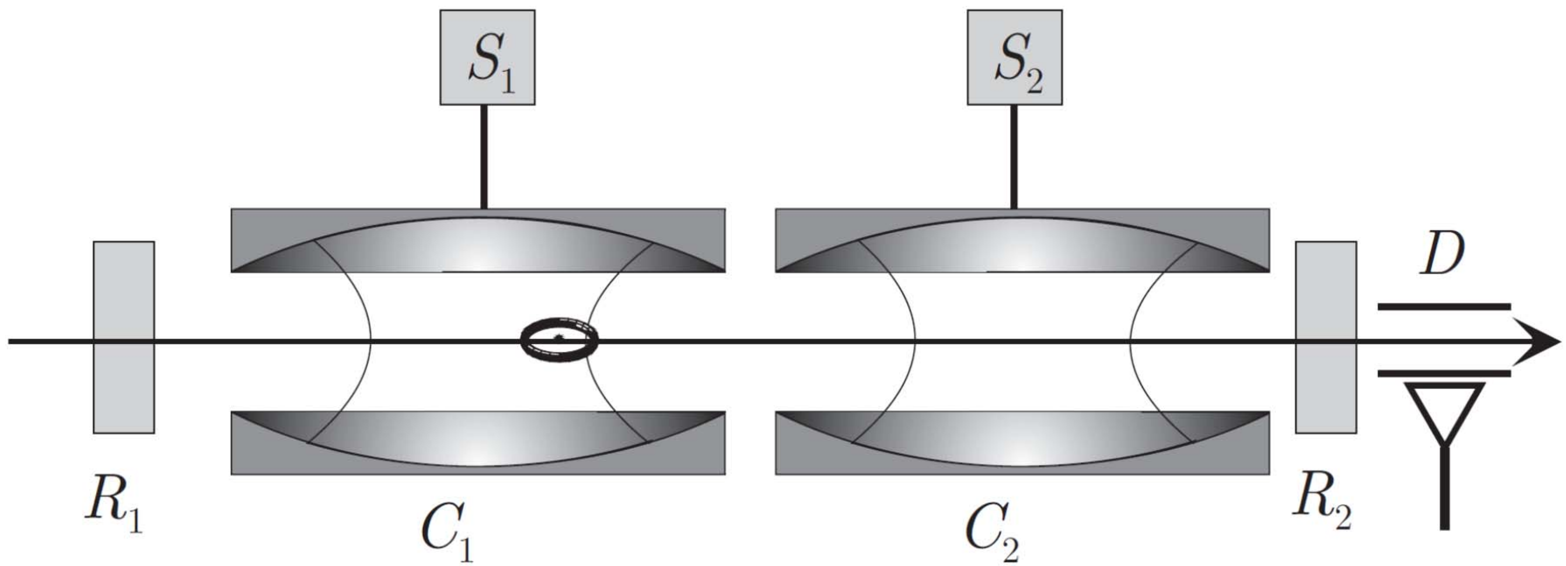
Cat In Two Boxes

Cat in Two Boxes

Qubit measures joint parity!

$$P_{12} = P_1 P_2 = e^{i\pi(\hat{n}_1+\hat{n}_2)}$$

Theoretical proposal by Paris group:
Eur. Phys. J. D **32**, 233–239 (2005)



$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|+\alpha\rangle|+\alpha\rangle \pm |-\alpha\rangle|-\alpha\rangle\right]$$

Cat in Two Boxes

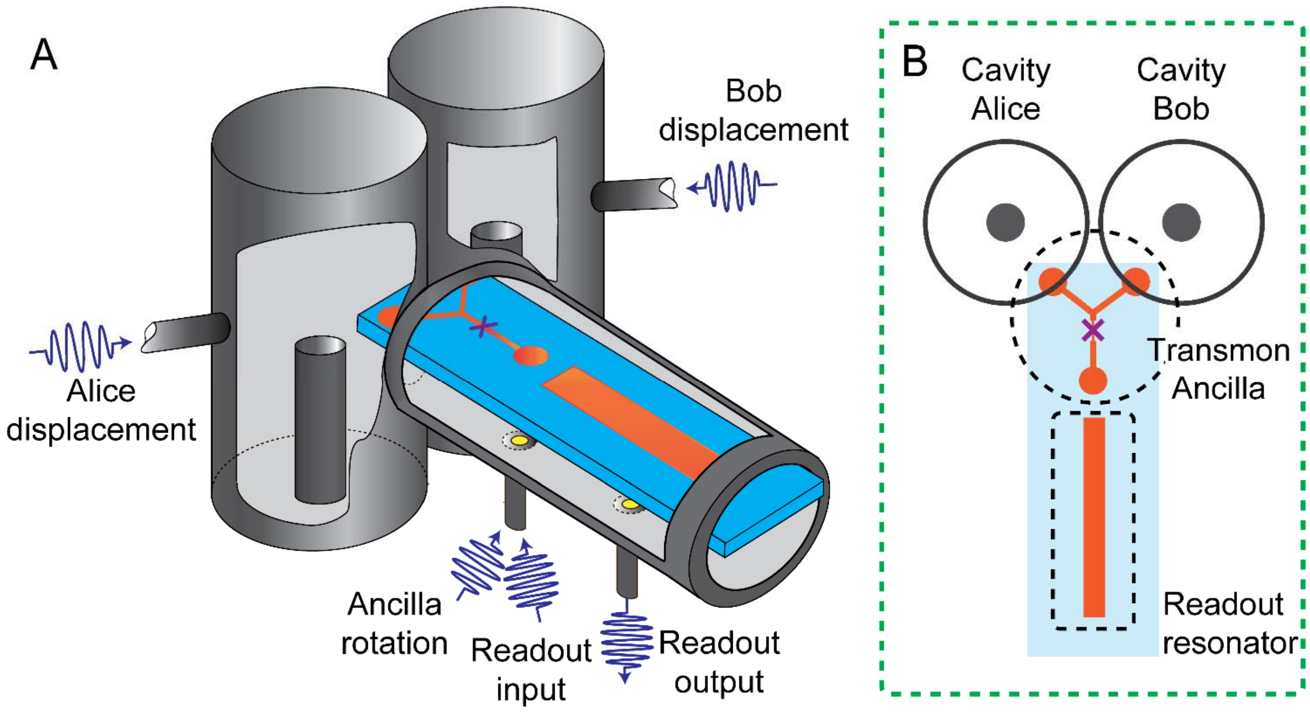
Qubit measures joint parity!

$$P_{12} = P_1 P_2 = e^{i\pi(\hat{n}_1 + \hat{n}_2)}$$

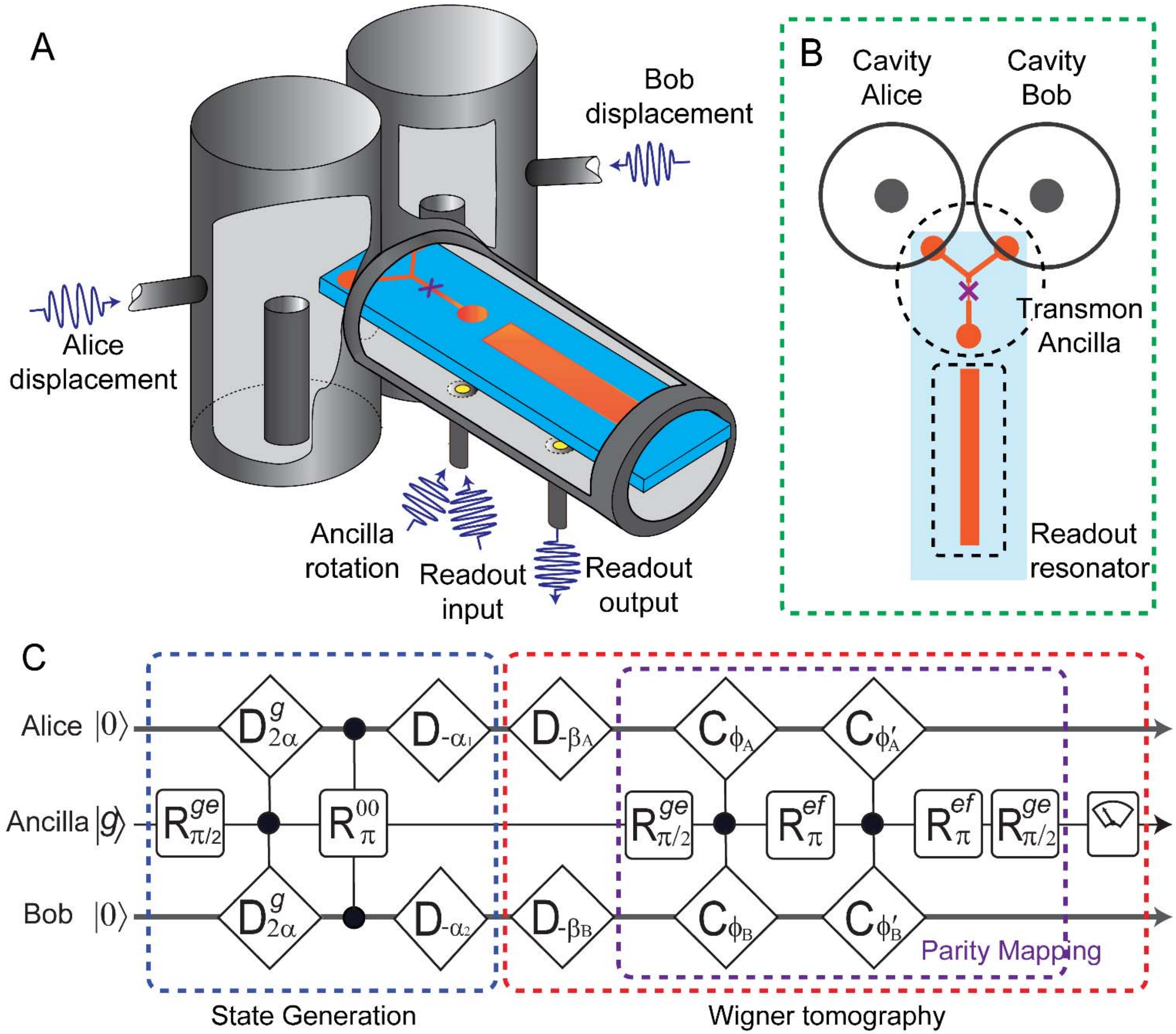
Experiment by Yale group:
Science **352,** 1087 (2016)

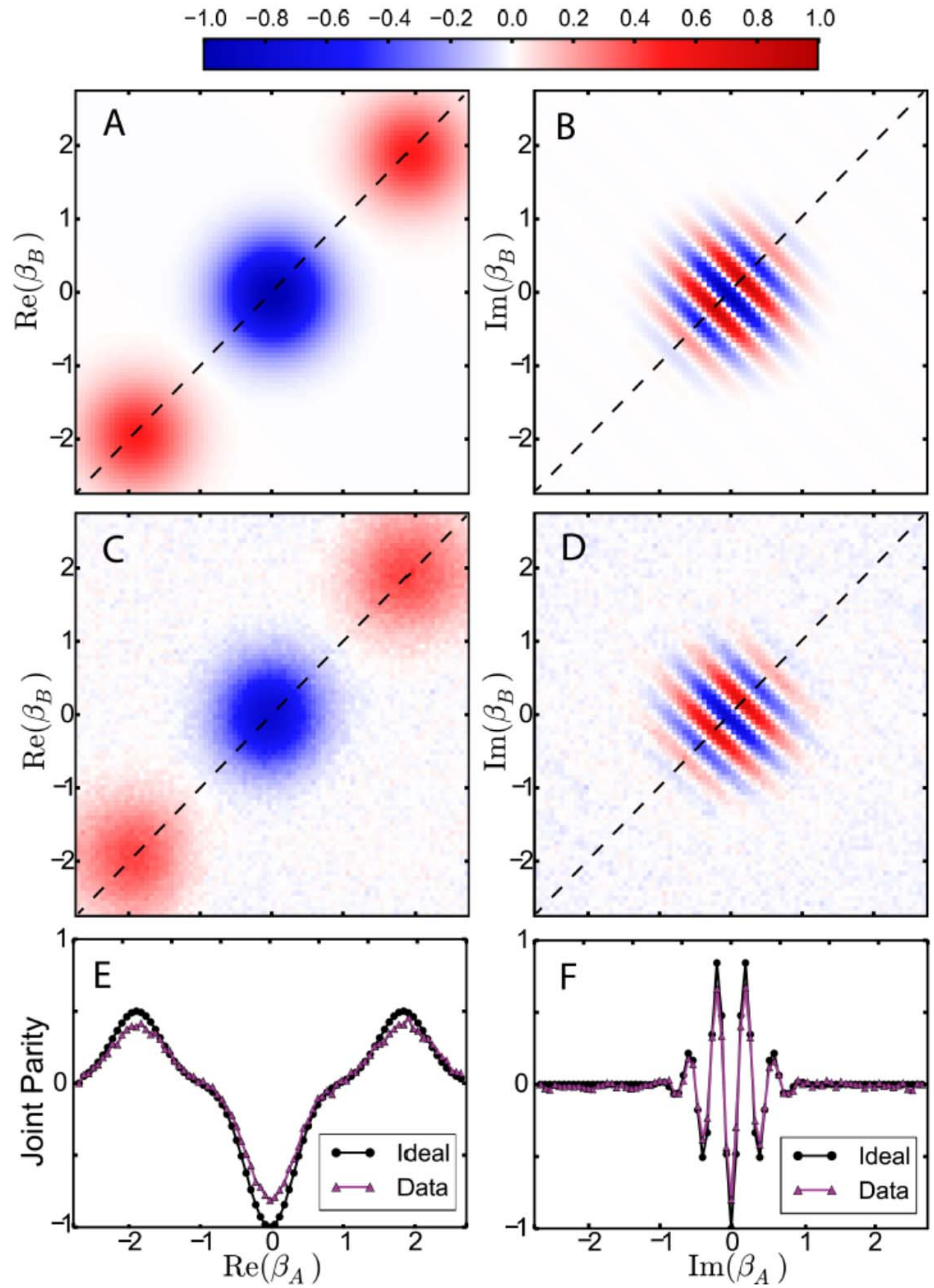
- Universal controllability
- 3-level qubit can measure

$$P_1, P_2, \text{ and } P_{12}$$



Cat in Two Boxes





Theory

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|+\alpha\rangle|+\alpha\rangle \pm |-\alpha\rangle|-\alpha\rangle\right]$$

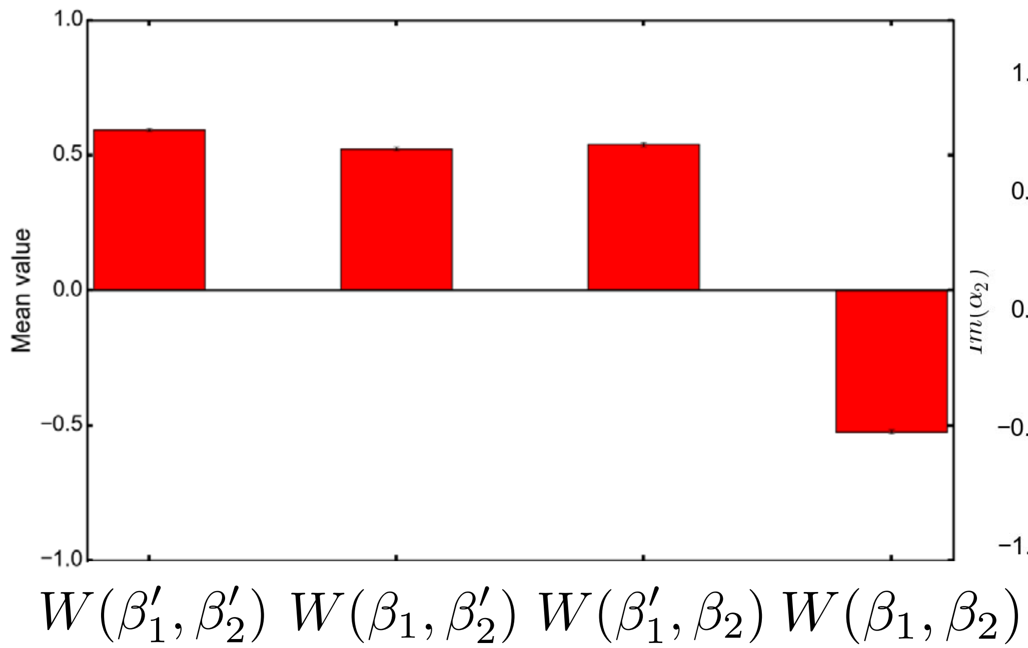
Experiment

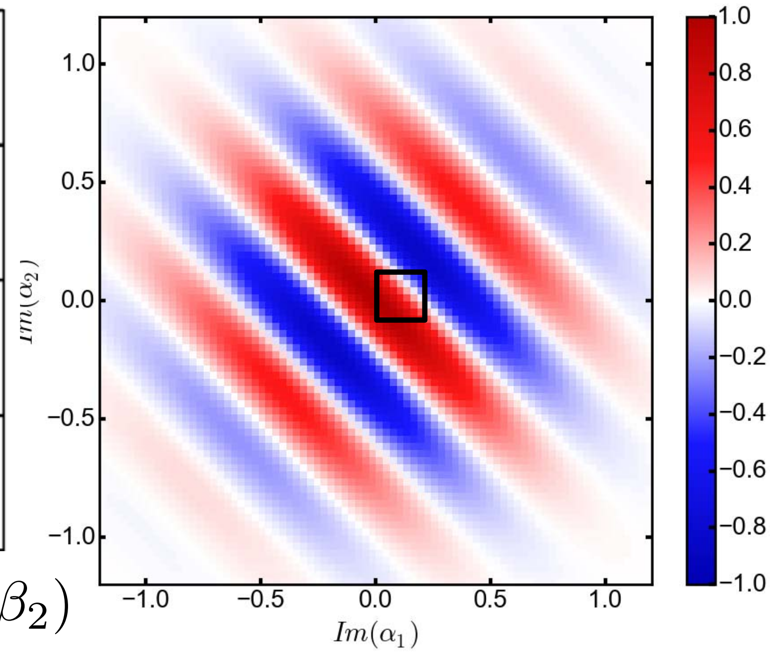
Two-cavities:
4-dimensional phase space and Wigner functions.

Entanglement of Two Logical Cat-Qubits

CHSH: (evaluate Wigner at 4 points in 4D phase space)

$$B = W(\beta_1', \beta_2') + W(\beta_1, \beta_2') + W(\beta_1', \beta_2) - W(\beta_1, \beta_2)$$





A proposal to test Bell's inequalities with mesoscopic non-local states in cavity QED

P. Milman[1], A. Auffeves[1], F. Yamaguchi[2], M. Brune[1], J.M. Raimond[1,a], and S. Haroche[1,3]

$$B = 2.18 \pm 0.02$$

$$\text{CHSH Bell: } 2 \le B \le 2\sqrt{2}$$

Summary of Lecture 2:

The ability to measure photon number parity without measuring photon number is an incredibly powerful tool.

Lecture 2: Using parity measurements for:

- Wigner Function Measurements
- Creation and verification of photon cat states

Lecture 3: Using parity measurements for:

- Continuous variable quantum error correction

For separate discussion offline:

Detailed Recipe to Make a

1. Schrödinger Cat
2. Schrödinger Cat State

Strong Dispersive Coupling Gives Powerful Tool Set

Cavity conditioned bit flip π_n

Qubit-conditioned cavity displacement D_α^g

- multi-qubit geometric entangling phase gates (Paik et al.)
- Schrödinger cats are now 'easy' (Kirchmair et al.)

Photon Schrödinger cats on demand

experiment	theory
G. Kirchmair	M. Mirrahimi
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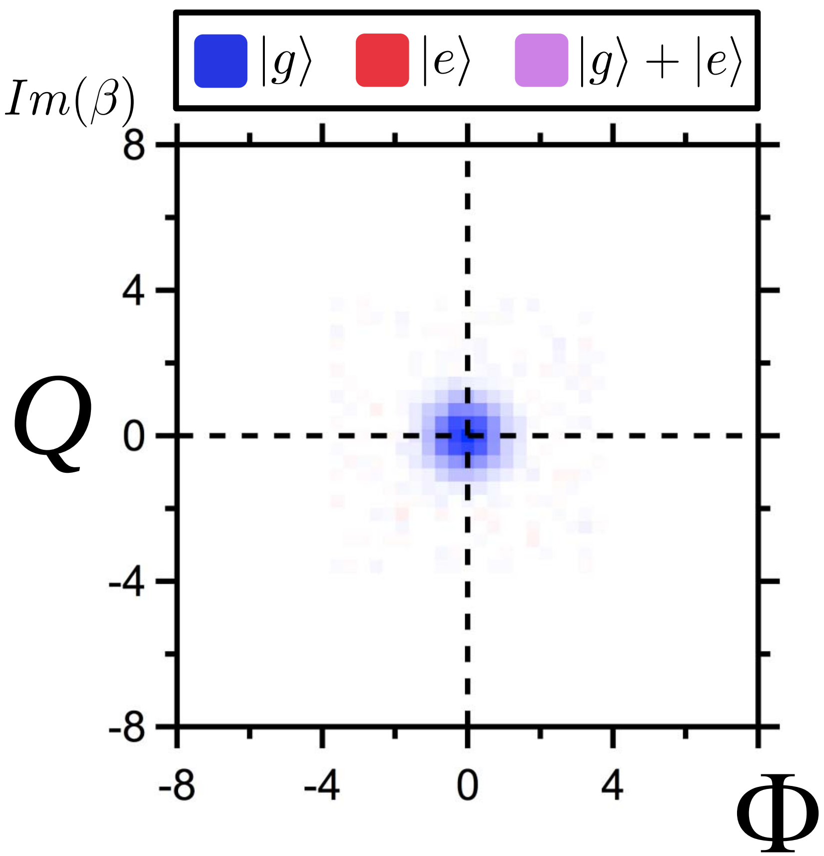
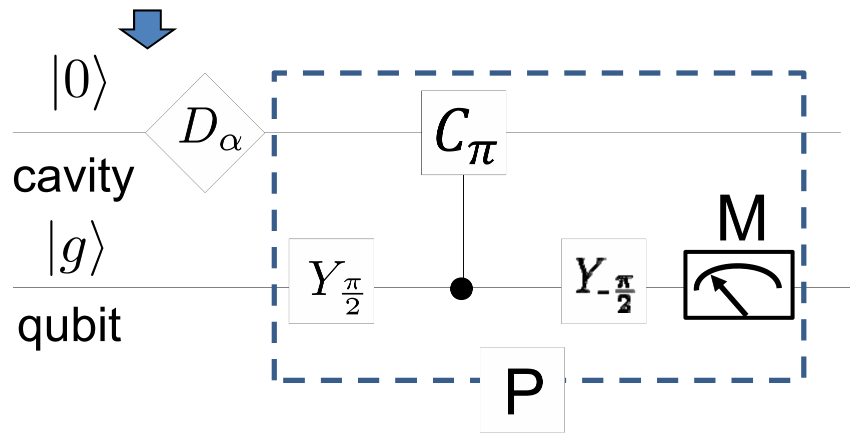
A. Petrenko	

Making a cat: the experiment

$$|\psi_{\text{ideal}}\rangle = |g\rangle \otimes |0\rangle$$

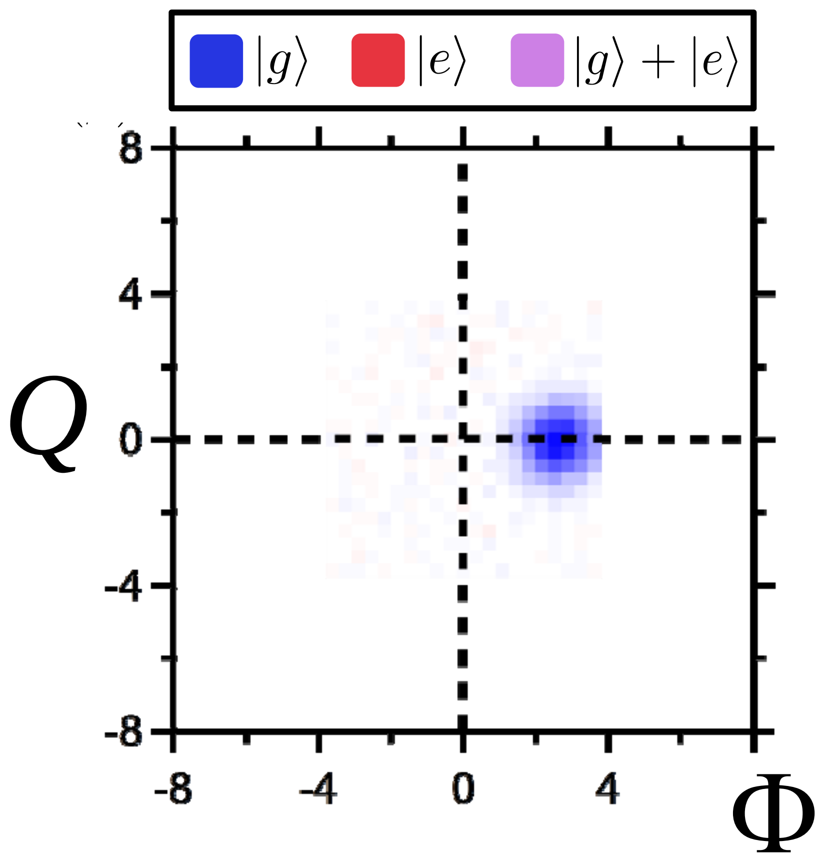


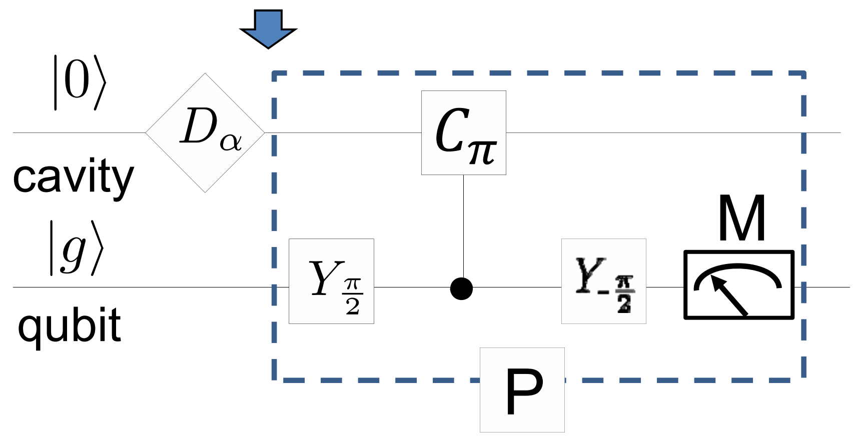


(*fine print for the experts: this is the Husimi Q function not Wigner)

Making a cat: the experiment

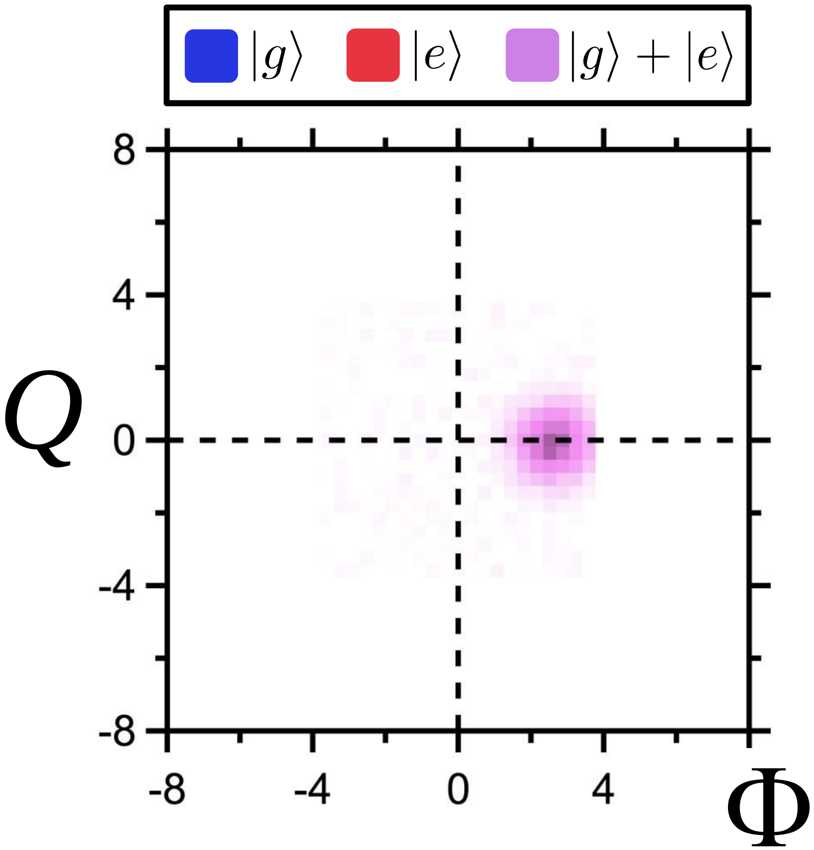
$$|\psi_{ideal}\rangle = |g\rangle \otimes |\alpha\rangle$$

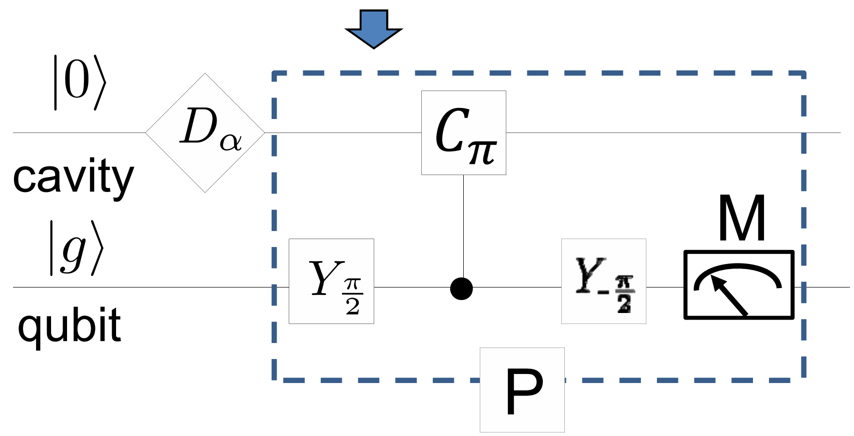




Making a cat: the experiment

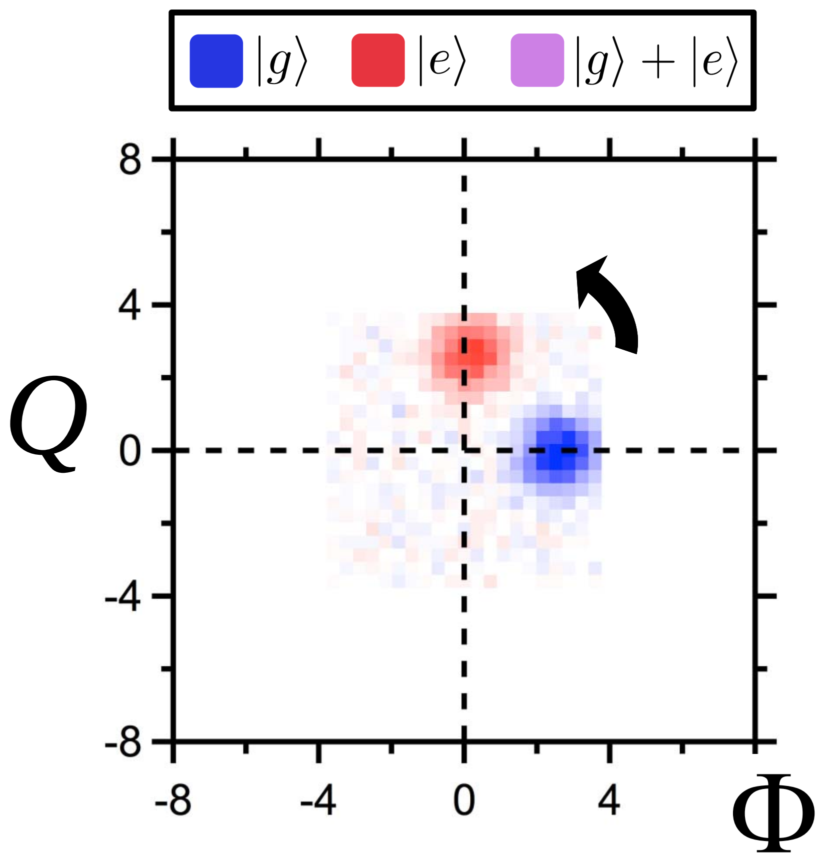
$$|\psi_{\text{ideal}}\rangle = \mathcal{N}\left(|g\rangle + |e\rangle\right) \otimes |\alpha\rangle$$

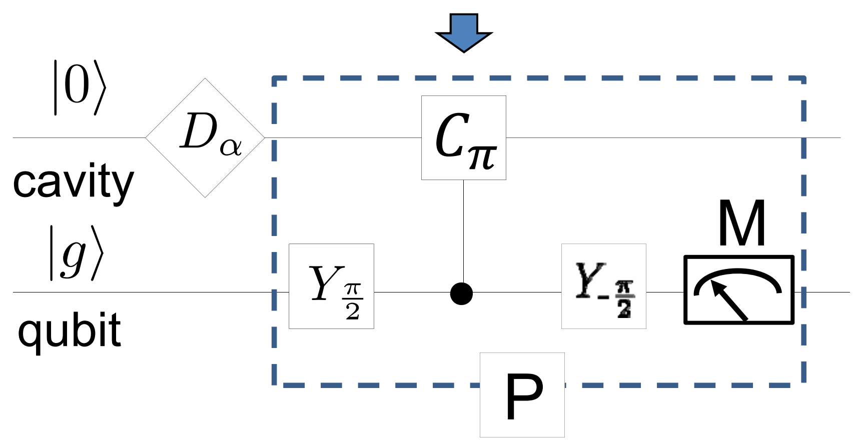




Making a cat: the experiment

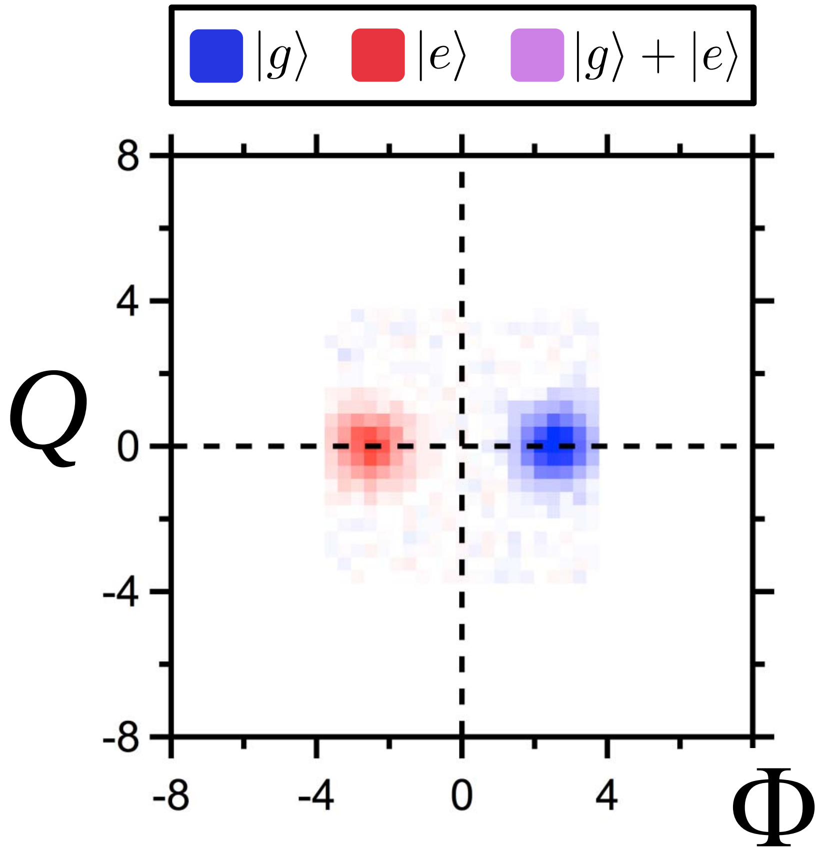
$$|\psi_{\text{ideal}}\rangle = \mathcal{N}\left(|g,\alpha\rangle + |e,\alpha e^{-i\chi t}\rangle\right)$$

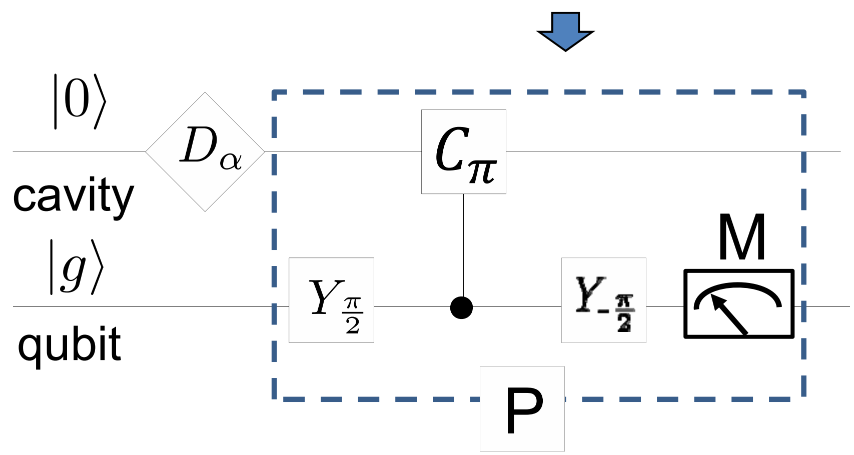




Making a cat:

$$|\psi_{\text{ideal}}\rangle = \mathcal{N}\left(|g,\alpha\rangle + |e,-\alpha\rangle\right)$$



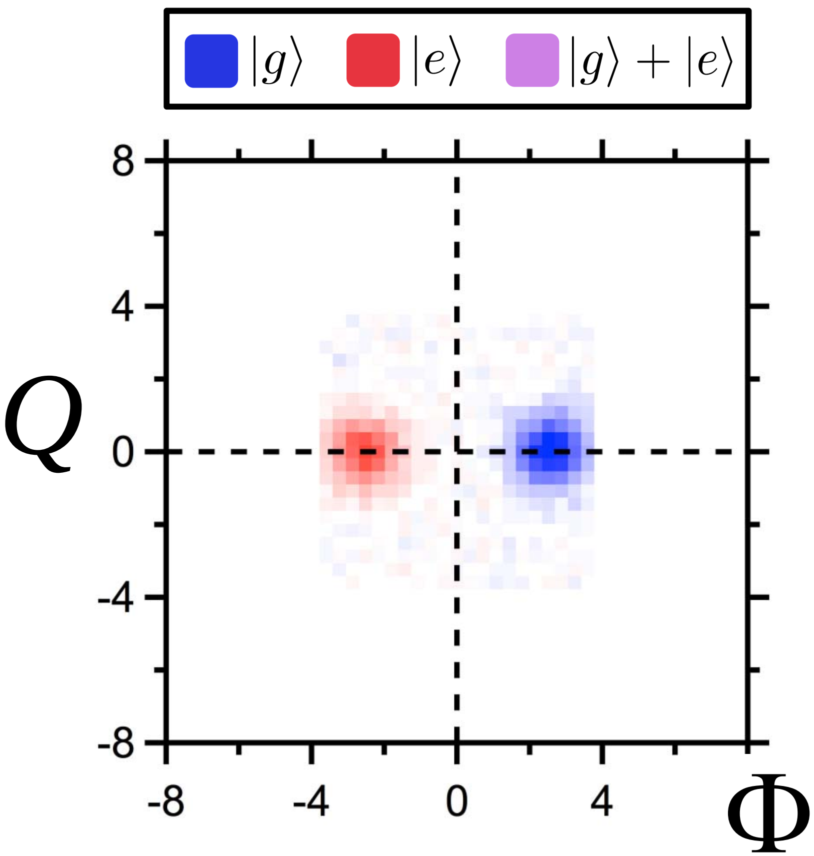


after time: $t = \pi/\chi$

qubit acquires π phase per photon…

Making a cat:

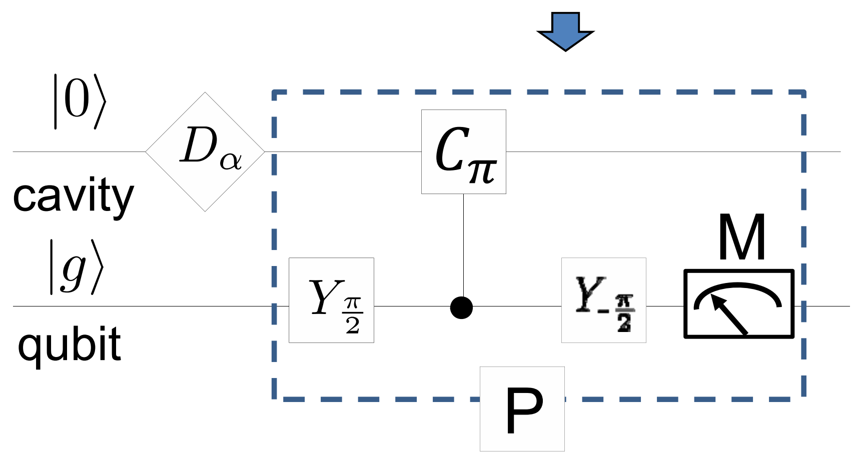
$$|\psi_{\text{ideal}}\rangle = \mathcal{N}\left(|g,\alpha\rangle + |e,-\alpha\rangle\right)$$



Qubit fully <u>entangled</u> with cavity

'cat is dead; poison bottle open'

'cat is alive; poison bottle closed'



after time: $t = \pi/\chi$

qubit acquires π phase per photon…

We have a 'cat'

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|\alpha\rangle + |e\rangle|-\alpha\rangle\right)$$

We want a 'cat state'

$$|\tilde{\psi}\rangle = \frac{1}{\sqrt{2}}|g\rangle\left(|\alpha\rangle + |-\alpha\rangle\right)$$

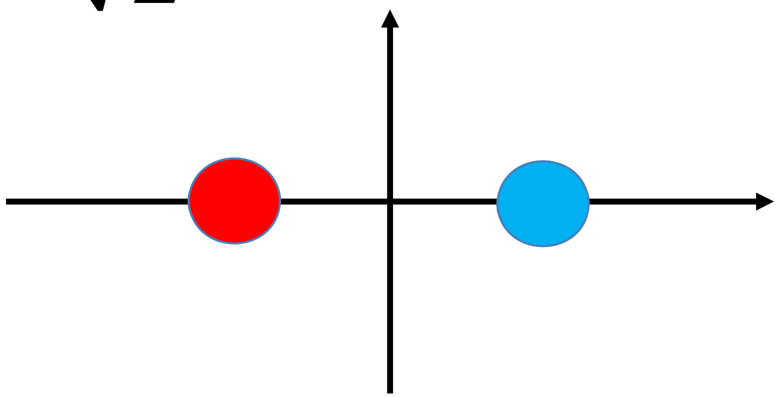
Qubit in ground state; cavity in **photon cat state**

How do we disentangle the qubit from the cavity?

Combining conditional cavity displacements with conditional qubit flips, one can disentangle the qubit from the photons

'cat'

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|\alpha\rangle + |e\rangle|-\alpha\rangle\right)$$

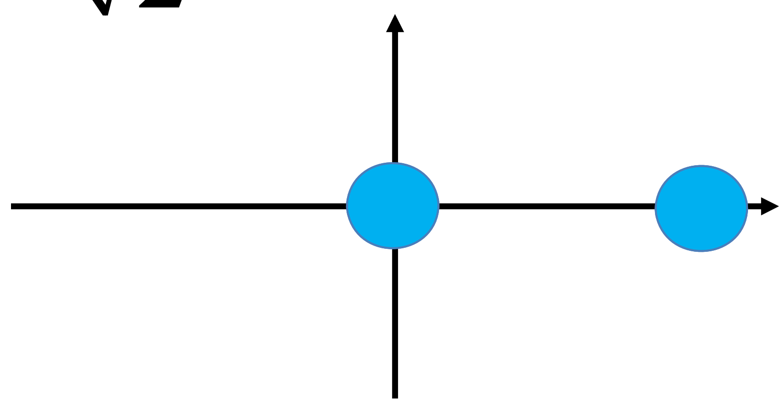


$$D_\alpha|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|2\alpha\rangle + |e\rangle|0\rangle\right)$$

Combining conditional cavity displacements with conditional qubit flips, one can disentangle the qubit from the photons

'cat'

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|\alpha\rangle + |e\rangle|-\alpha\rangle\right)$$

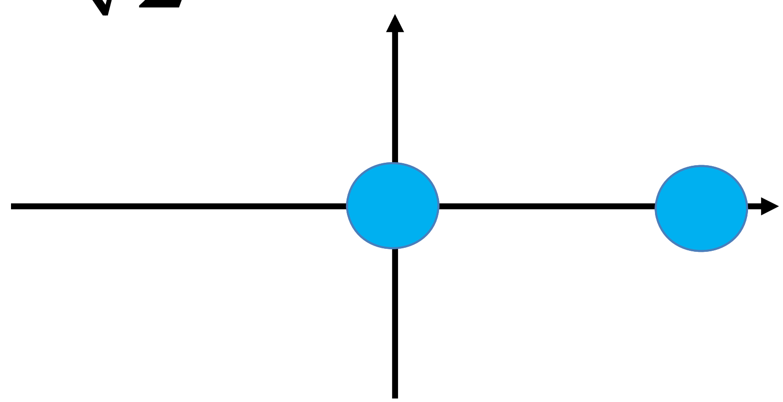


$$\pi_0 D_\alpha |\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|2\alpha\rangle + |g\rangle|0\rangle\right)$$

Combining conditional cavity displacements with conditional qubit flips, one can disentangle the qubit from the photons

'cat'

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|\alpha\rangle + |e\rangle|-\alpha\rangle\right)$$

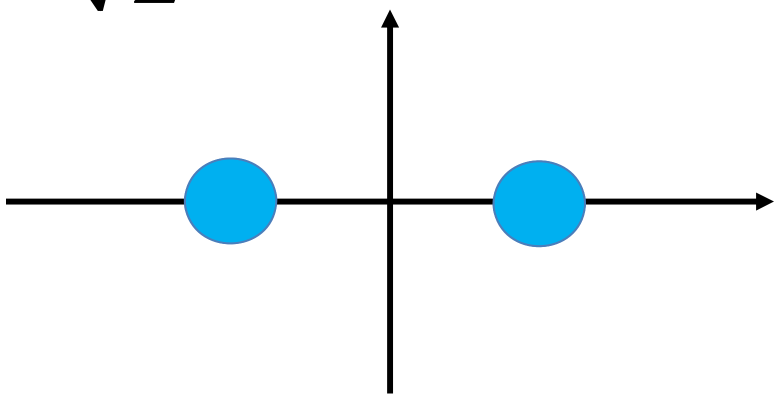


$$D_{-\alpha}\pi_0 D_\alpha|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|+\alpha\rangle + |g\rangle|-\alpha\rangle\right)$$

Combining conditional cavity displacements with conditional qubit flips, one can disentangle the qubit from the photons

'cat'

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle|\alpha\rangle + |e\rangle|-\alpha\rangle\right)$$



'cat state'

$$D^{g}_{-\alpha}\pi_0\, D_\alpha\, |\psi\rangle = \frac{1}{\sqrt{2}}|g\rangle\left(|\alpha\rangle + |-\alpha\rangle\right)$$