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Basic Concepts in Quantum Information: Quantum Memories Quantum Error Correction

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http://quantuminstitute.yale.edu/

Lecture I: Introduction to circuit QED

Lecture II: Quantum state engineering in the strong-dispersive limit: Cat States

Lecture III: Quantum Error Correction with Cat States

Experiment:

'Extending the lifetime of a quantum bit with error correction in superconducting circuits,' Ofek, et al., (*Nature*, 2016)

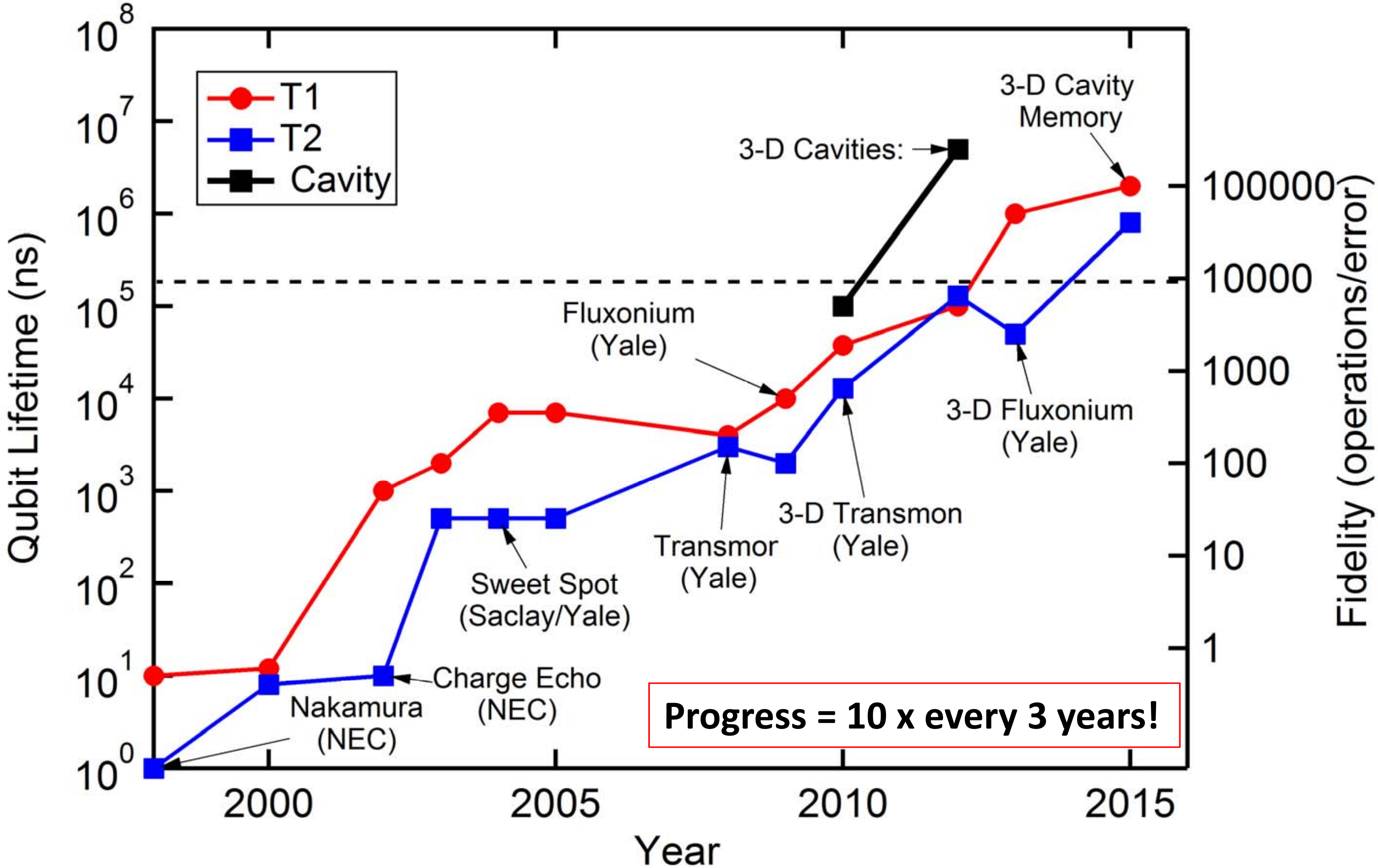
Theory:

Leghtas, Mirrahimi, et al., PRL **111**, 120501(2013).

M. Michael et al., Phys. Rev. X **6**, 031006 (2016)

'New class of error correction codes for a bosonic mode'

Remarkable Progress in Coherence

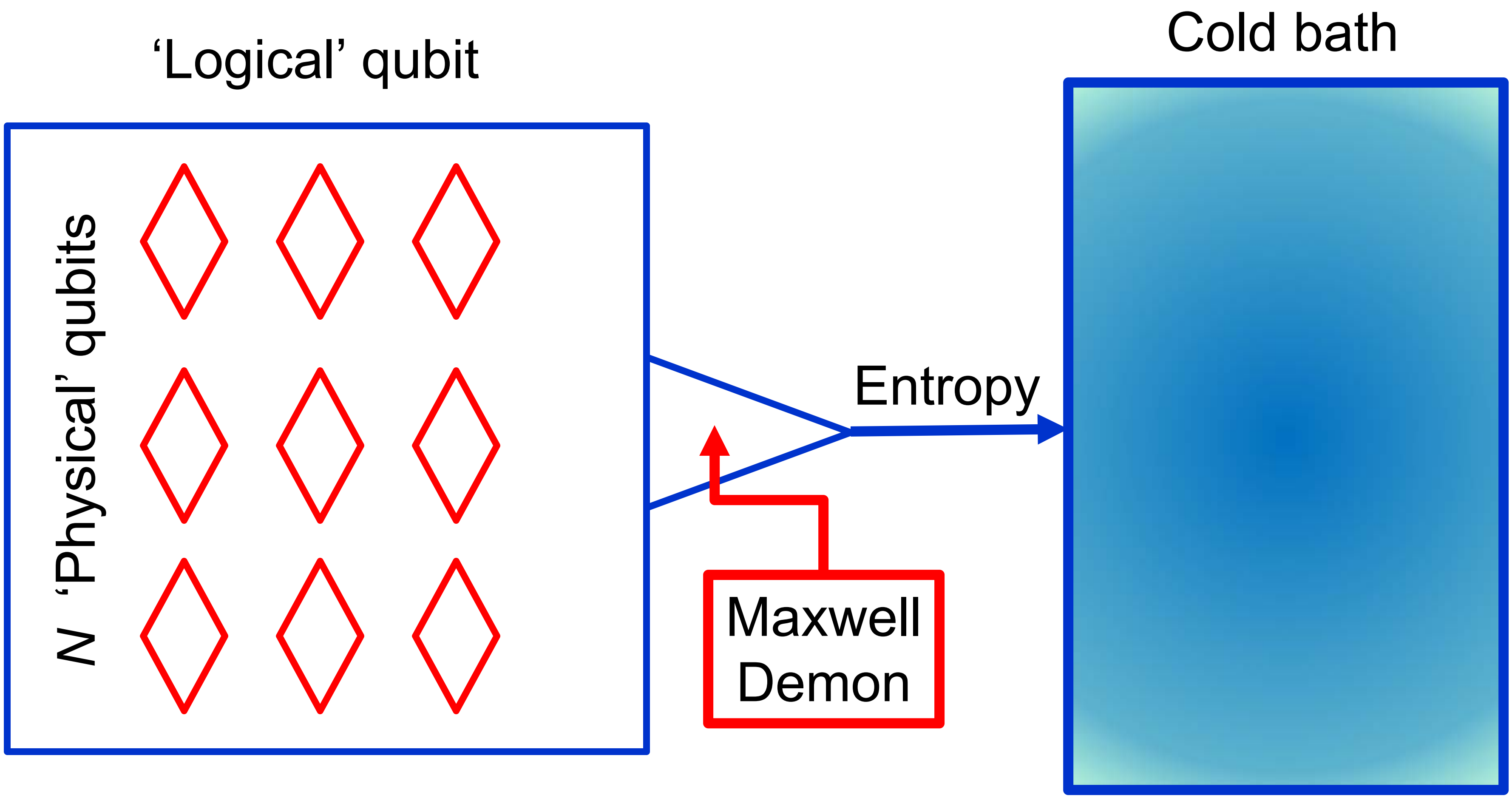


Girvin's Law:

$$\boxed{\text{There is no such thing as too much coherence.}}$$

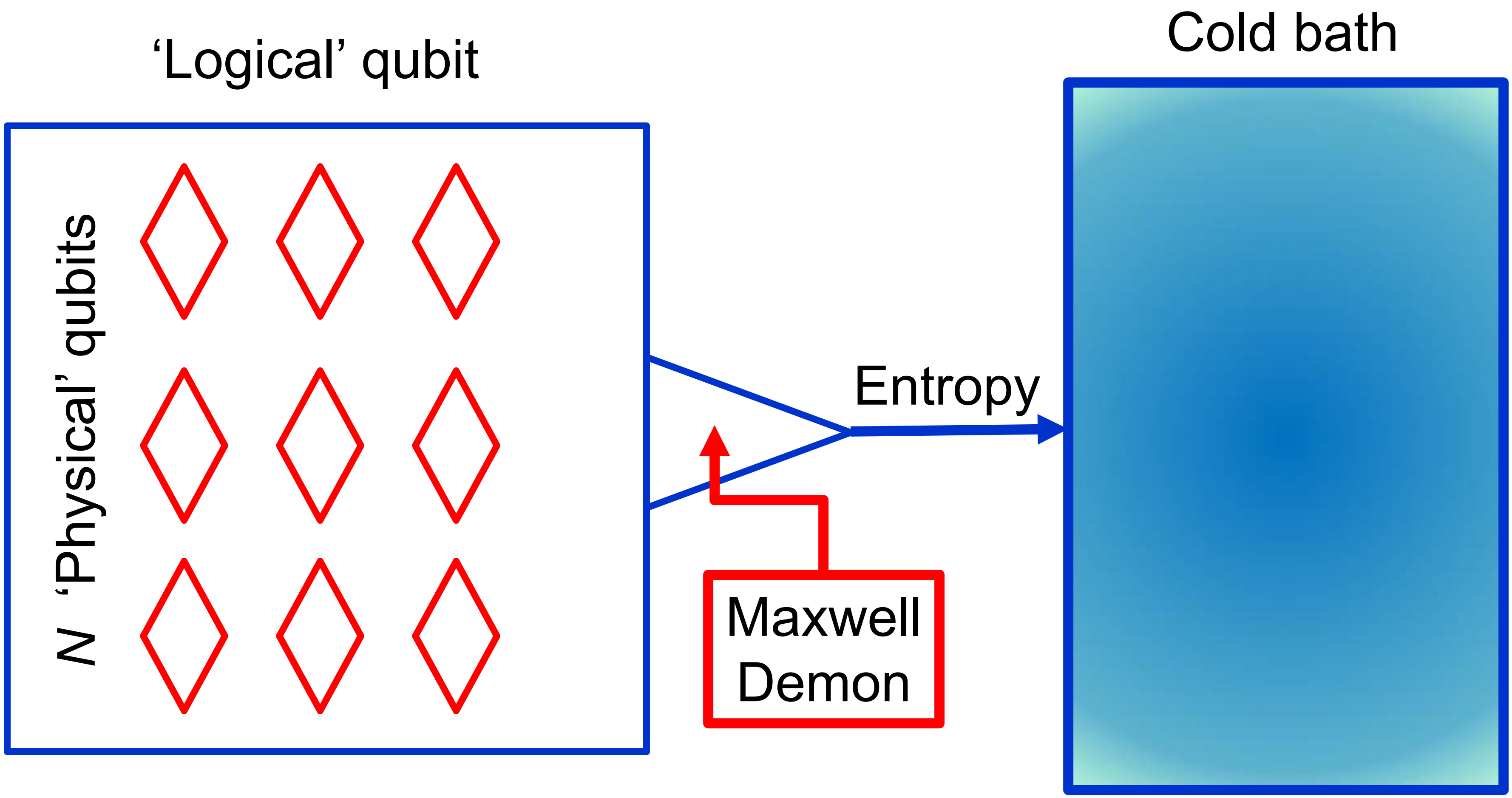
We need quantum error correction!

Quantum Error Correction



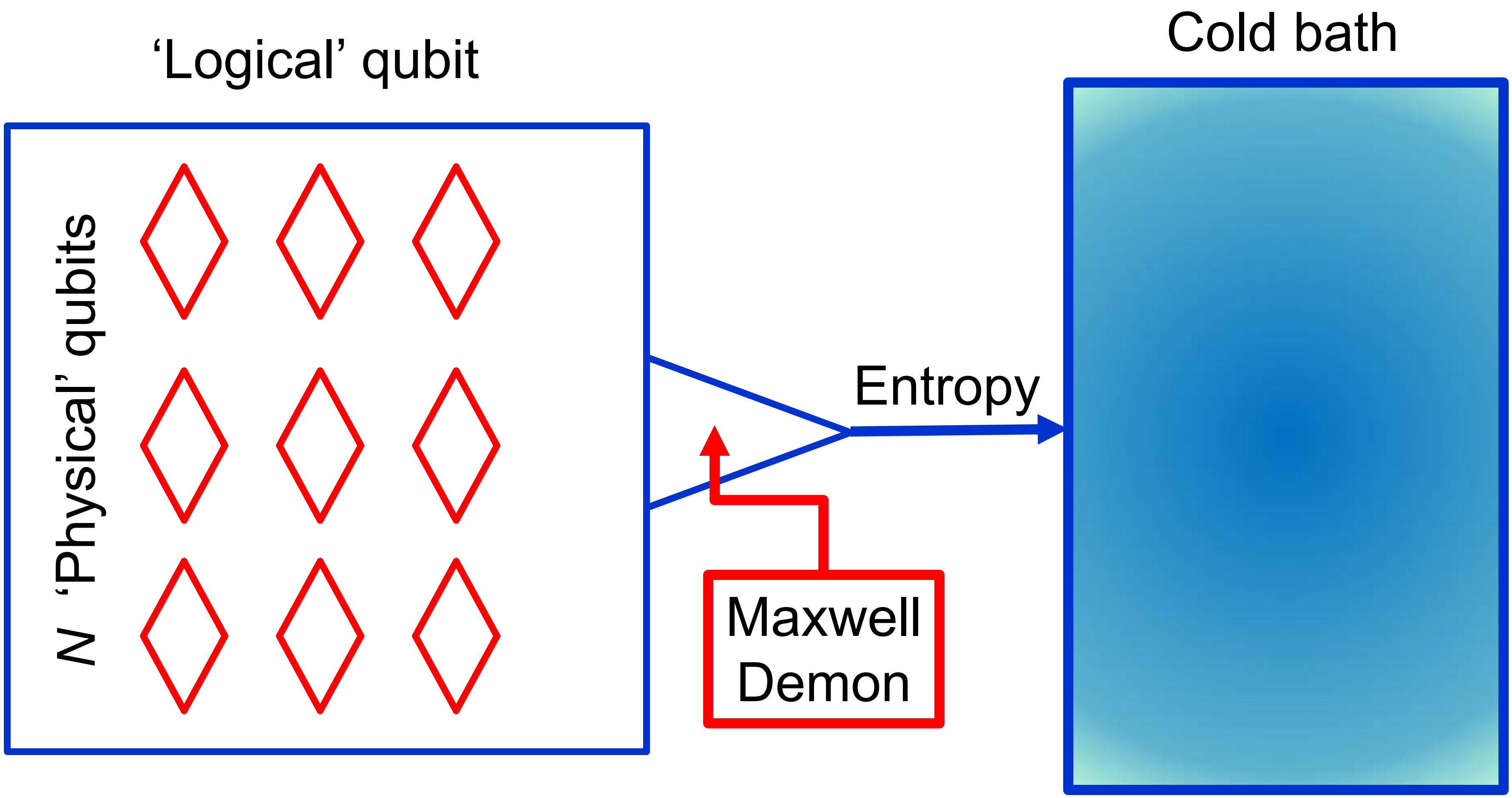
N qubits have errors *N* times faster. Maxwell demon must overcome this factor of *N* – *and not introduce errors of its own!*

Quantum Error Correction



The logical qubit should have <u>longer</u> coherence time than the <u>best</u> of its physical physical components.

Quantum Error Correction



QEC is an emergent collective phenomenon:
adding N-1 worse qubits to the 1 best qubit gives an improvement!

Are We There Yet?

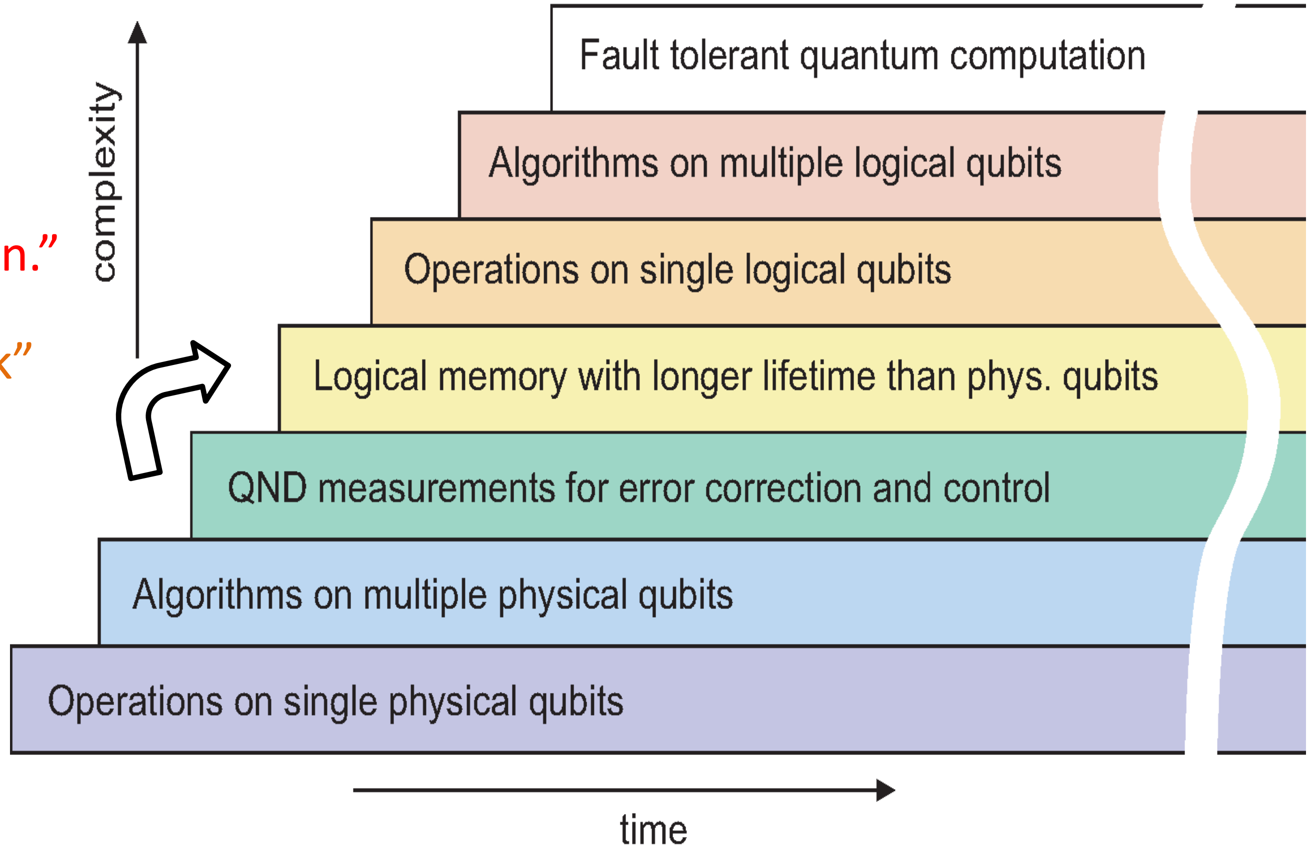
“Age of Qu. Error Correction.”

“Age of Quantum Feedback”

“Age of Measurement”

“Age of Entanglement”

“Age of Coherence”



Goal of next stage: reaching “break-even” point for error correction

“We” are ~ here (also ions, Rydbergs, q-dots, …)

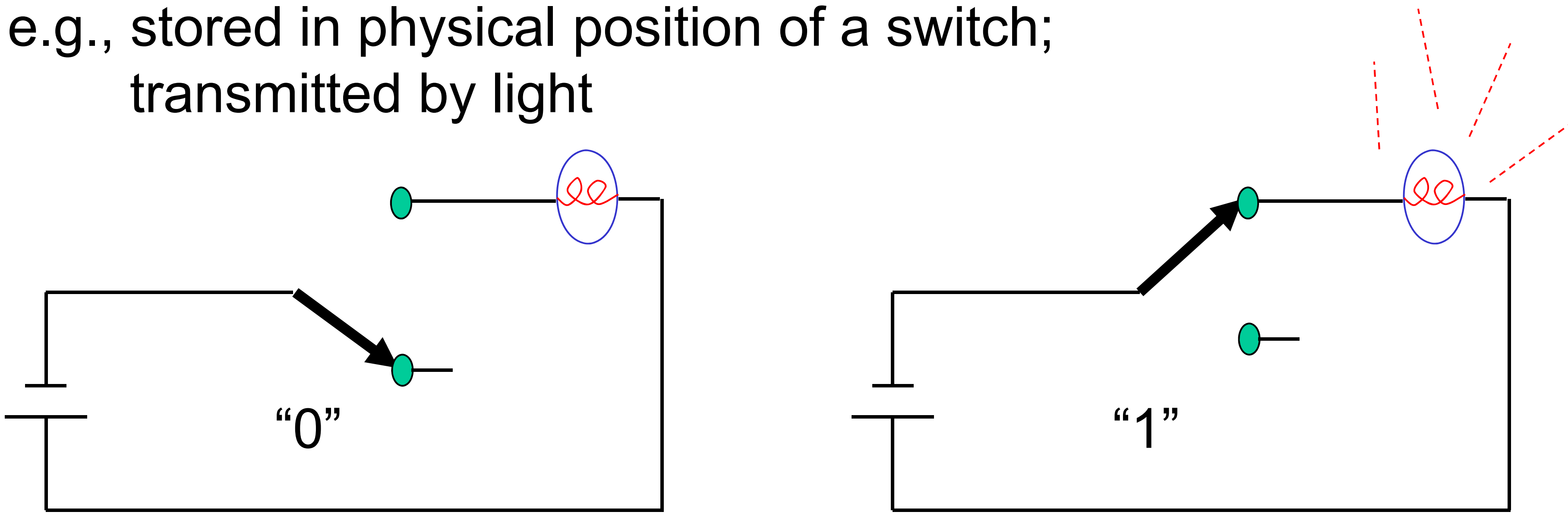
M. Devoret and RS, Science (2013)

Classical bits

Information is physical.

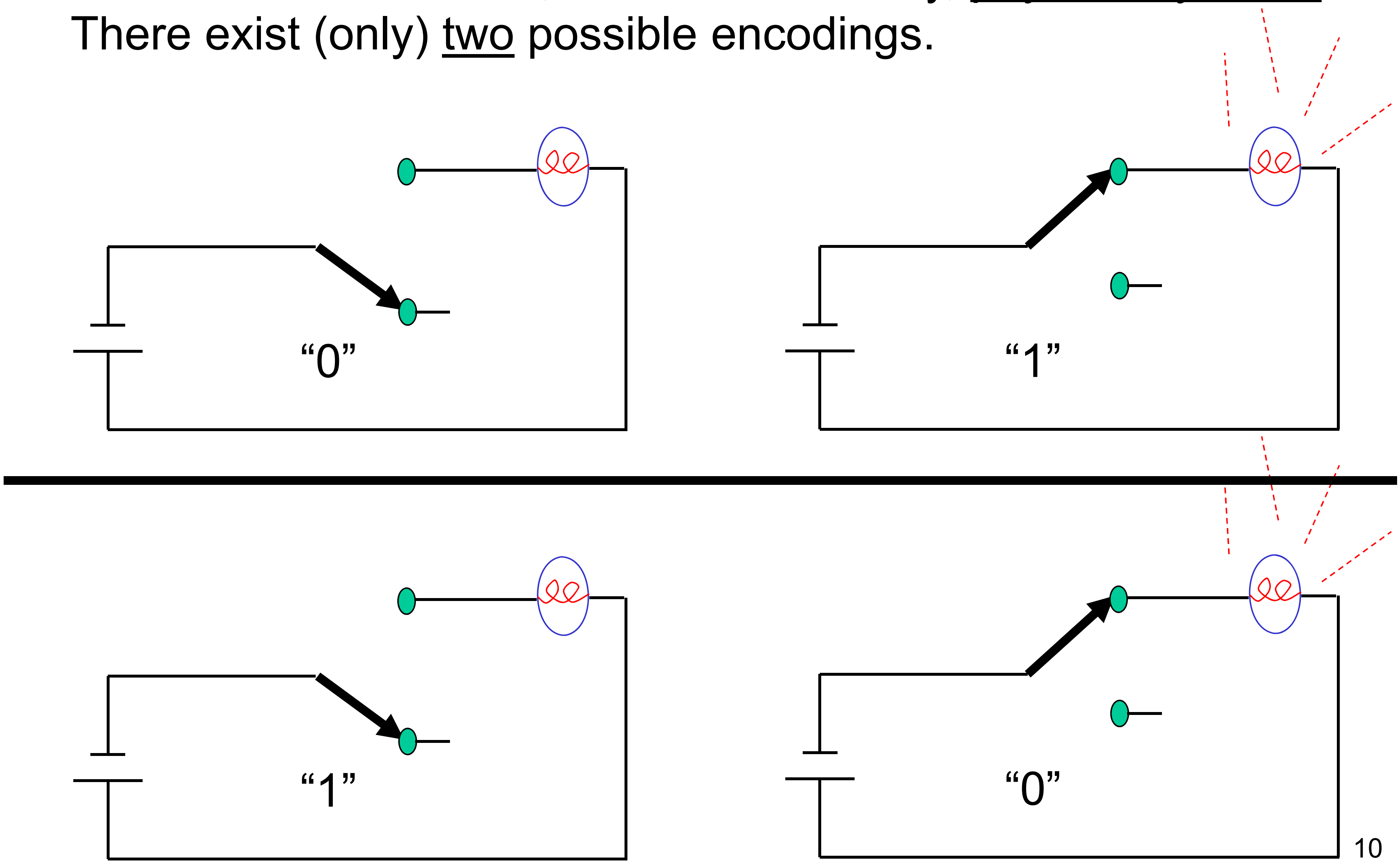
Information is stored in, and transmitted by, physical systems

e.g., stored in physical position of a switch;
transmitted by light



Classical bits

Information is stored in, and transmitted by, physical systems
There exist (only) two possible encodings.



Classical bits

Information is stored in, and transmitted by, <u>physical systems</u>
There exist (only) <u>two</u> possible encodings.

Alice 0110111010	photons →	Bob Wright 0110111010
Alice 0110111010	photons →	Bob Wrong 1001000101

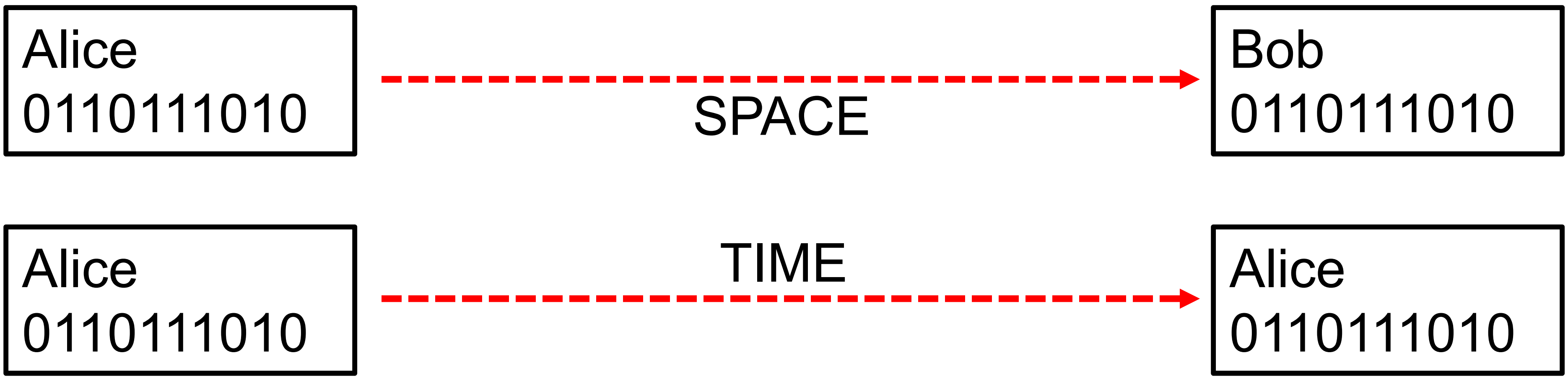
Transformation law between the two possible encodings:

<u>NOT</u>

N.B. The information content (Shannon entropy) Bob receives is not affected by which decoding he uses. Bob can apply <u>NOT</u> to the decoded message *post facto*.

Communication and Memory are Essentially the Same Problem

Information is stored in, and transmitted by, physical systems

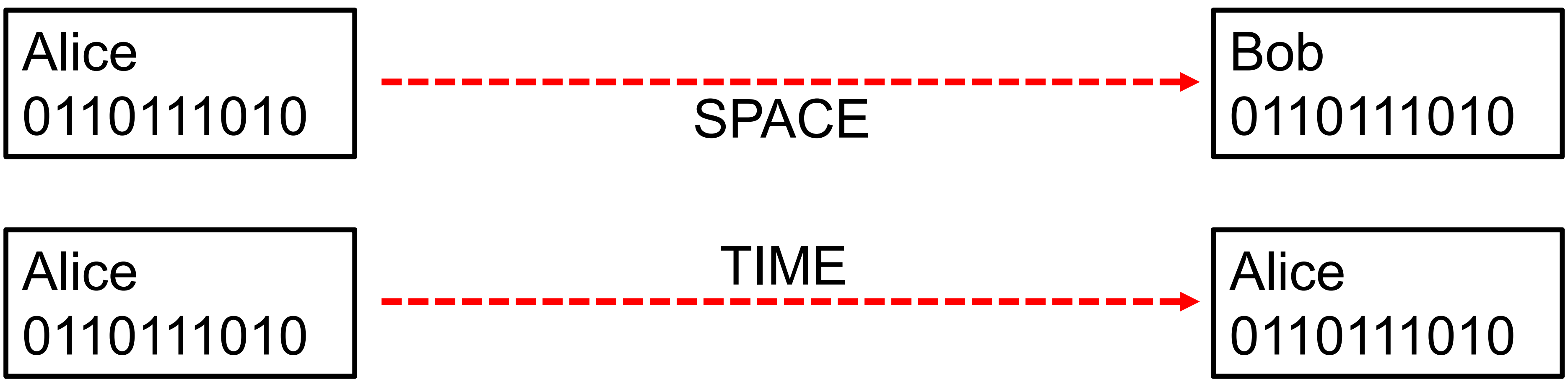


Goal is faithful transmission of information across space and/or time.

Because there are only two encodings there is only one possible error: bit flip

Communication and Memory are Essentially the Same Problem

Information is stored in, and transmitted by, <u>physical systems</u>

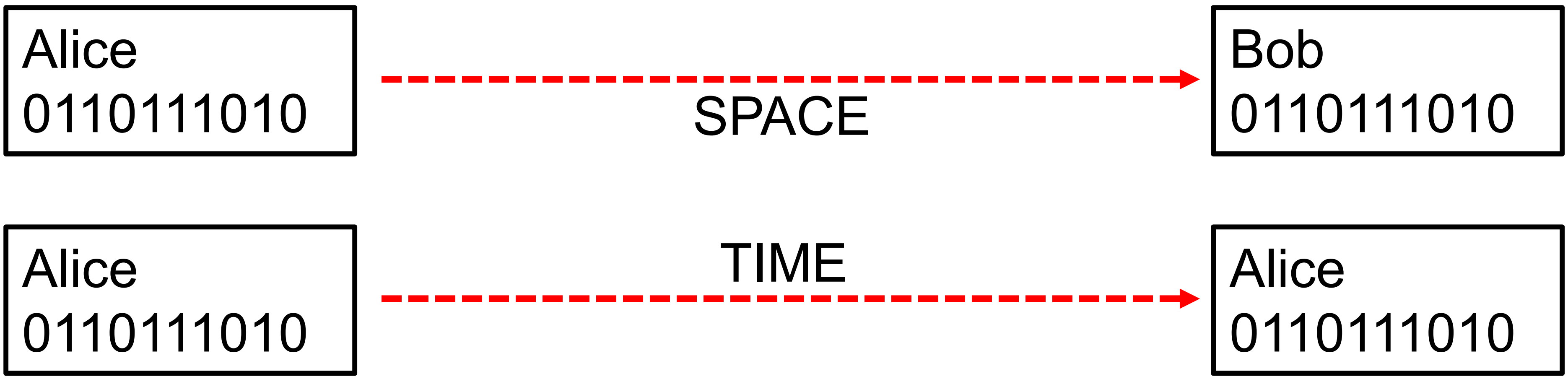


Goal is faithful transmission of information across space and/or time.

Essential tools:
Error correcting codes using ancilla bits

Communication and Memory are Essentially the Same Problem

Information is stored in, and transmitted by, <u>physical systems</u>



Essential tools:
Error correcting codes using ancilla bits

Example: majority rule replication codes

$$0 \rightarrow 000$$

$$1 \rightarrow 111$$

Quantum bits (‘qubits’)

Quantum information is stored in the physical (superposition) states of a quantum system:

- atoms, molecules, ions, superconducting circuits, photons, mechanical oscillators, …

Quantum Information is Paradoxical

Is quantum information carried by waves or by particles?

YES!

Is quantum information analog or digital?

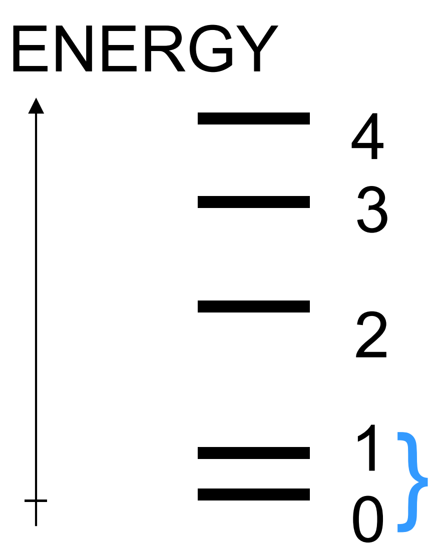
YES!

Quantum information is digital:

Energy levels of a quantum system are discrete.

We use only the lowest two.

Measurement of the state of a qubit yields (only) 1 classical bit of information.



$$\text{excited state } 1 = |e\rangle = |\uparrow\rangle$$

$$\text{ground state } 0 = |g\rangle = |\downarrow\rangle$$

Quantum information is <u>analog</u>:

A quantum system with two distinct states can exist in an Infinite number of physical states ('superpositions') *intermediate* between $|\downarrow\rangle$ and $|\uparrow\rangle$.

Bug:
We are <u>uncertain</u> which state the bit is in. Measurement results are truly random.

Feature:
Qubit is in <u>both</u> states at once, so we can do parallel processing. Potential exponential speed up.

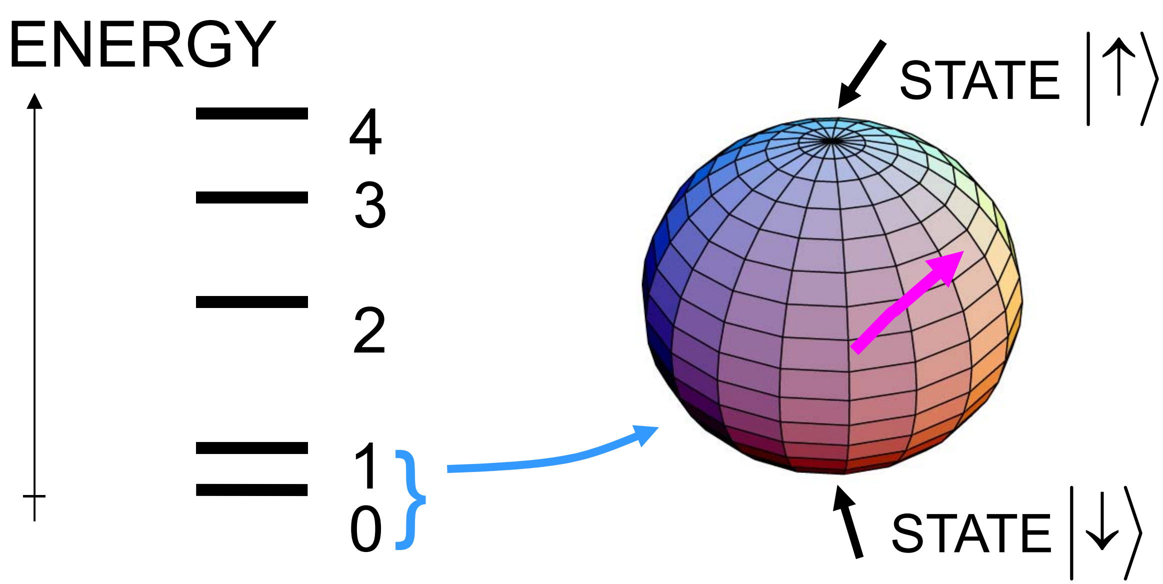
Quantum information is <u>analog</u>:

A quantum system with two distinct states can exist in an Infinite number of physical states ('superpositions') *intermediate* between $|\downarrow\rangle$ and $|\uparrow\rangle$.

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|\uparrow\rangle$$

$\theta =$ latitude

$\varphi =$ longitude

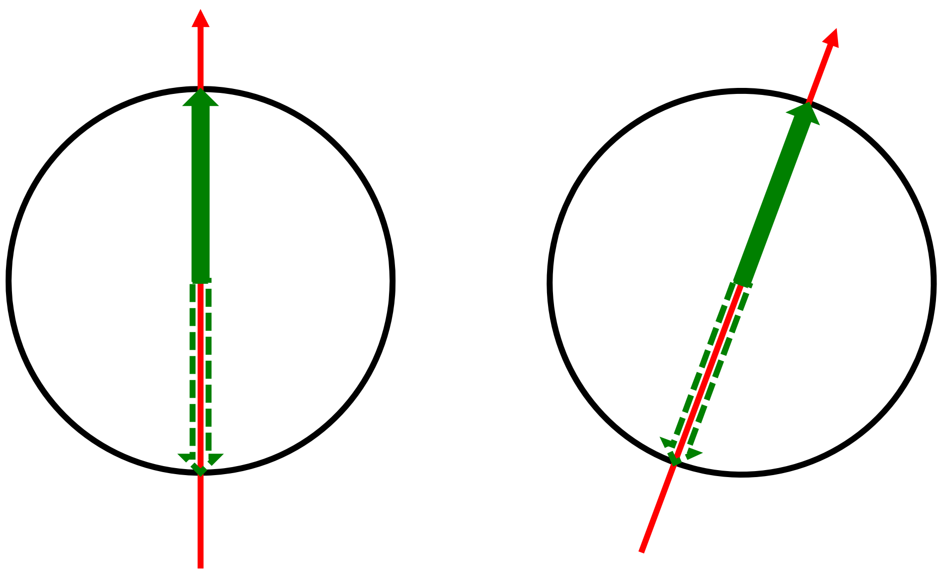


State defined by 'spin polarization vector' on Bloch sphere

Quantum information is analog/digital:

Equivalently: a quantum bit is like a classical bit except there are an infinite number of encodings (aka 'quantization axes').

$Z = \pm 1$ $\qquad$ $Z' = \pm 1$



Alice $\qquad$ Bob

If Alice gives Bob a $Z = +1$,

Bob measures:

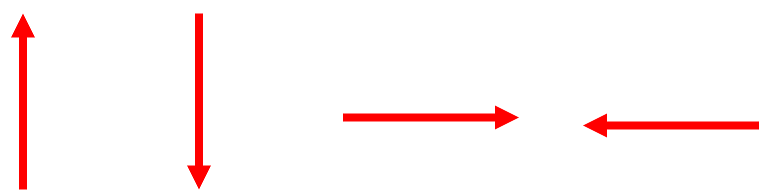
$Z' = +1$ with probability $P_+ = \cos^2 \dfrac{\theta}{2}$

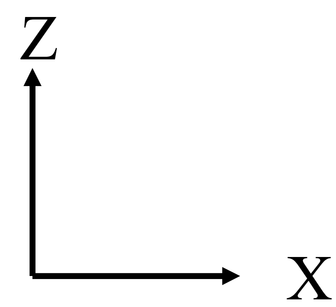
$Z' = -1$ with probability $P_- = \sin^2 \dfrac{\theta}{2}$

'Back action' of Bob's measurement changes the state, but it is invisible to Bob.

What is knowable?

Consider just 4 states:





We are allowed to ask only one of two possible questions:

Does the spin lie along the Z axis? Answer is always yes! (± 1)
Does the spin lie along the X axis? Answer is always yes! (± 1)

BUT WE CANNOT ASK BOTH!
Z and X are ***INCOMPATIBLE*** OBSERVABLES

What is knowable?

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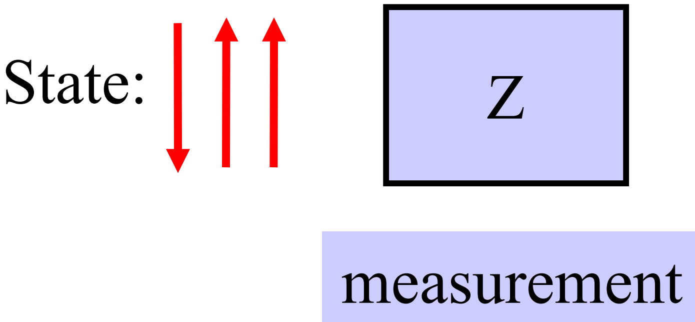
Heisenberg Uncertainty Principle

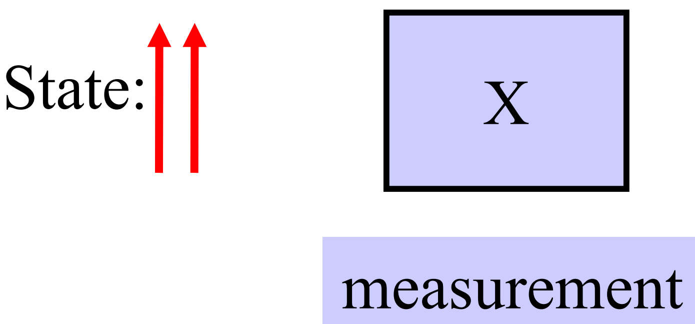
If you know the answer to the Z question
you cannot know the answer to the X question
and vice versa.

(If you know position you cannot know momentum.)

Measurements 1.



Result: quantum state is unaffected.

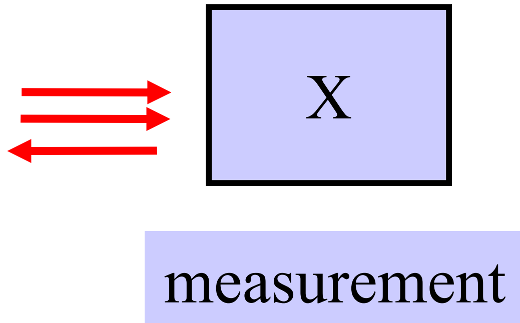


Result: ± 1 randomly!
State is *changed by measurement* to lie along X axis.

Unpredictable result

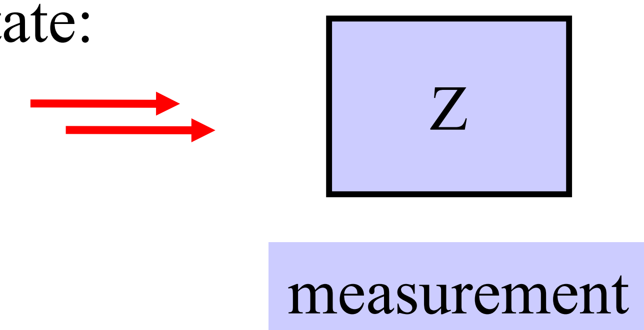
Measurements 2.

State:



Result: quantum state is unaffected.

State:



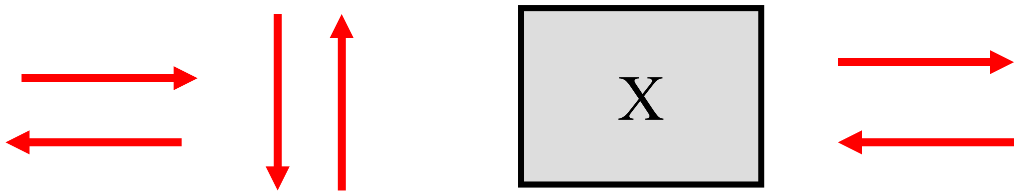
Result: ± 1 randomly! State is *changed by measurement* to lie along Z axis.

Unpredictable result

No Cloning Theorem

Given an unknown quantum state, it is ***impossible*** to make multiple copies

Unknown state:



Guess which measurement to make
---if you guess wrong you change the state
and you have no way of knowing if you did….

No Cloning Theorem

Given an unknown quantum state, it is ***impossible*** to make multiple copies

Big Problem:

Classical error correction is based on cloning!
(or at least on measuring)

Replication code:
$$0 \rightarrow 000$$
$$1 \rightarrow 111$$

Majority Rule voting corrects single bit flip errors.

The Quantum Error Correction Problem

I am going to give you an unknown quantum state.

If you measure it, it will change randomly.

If it develops an error, please fix it.

***Mirable dictu*: It can be done!**

Let's start with classical error <u>heralding</u>

Classical duplication code: $0 \rightarrow 00 \quad 1 \rightarrow 11$

Herald error if bits do not match.

In	Out	# of Errors	Probability	Herald?
00	00	0	$(1-p)^2$	Yes
00	01	1	$(1-p)p$	Yes
00	10	1	$(1-p)p$	Yes
00	11	2	p^2	Fail

And similarly for 11 input.

Using duplicate bits:

-lowers channel bandwidth by factor of 2 (bad)

-lowers the fidelity from $(1-p)$ to $(1-p)^2$ (bad)

-improves unheralded error rate from p to p^2 (good)

In	Out	# of Errors	Probability	Herald?
00	00	0	$(1-p)^2$	Yes
00	01	1	$(1-p)p$	Yes
00	10	1	$(1-p)p$	Yes
00	11	2	p^2	Fail

And similarly for 11 input.

Quantum Duplication Code

No cloning prevents duplication

$$U\underbrace{\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)}_{\text{Unknown quantum state}}\otimes\underbrace{\left|\downarrow\right\rangle}_{\text{Ancilla initialized to ground state}}=\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)\otimes\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)$$

Proof of no-cloning theorem:

α and β are unknown; Hence U cannot depend on them.

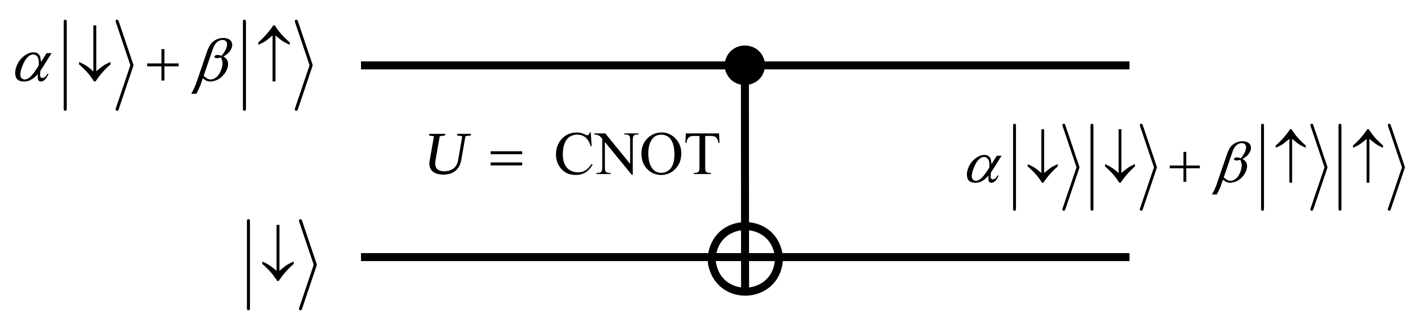
No such unitary can exist if QM is linear.

Q.E.D.

Don’t clone – entangle!

$$U\underbrace{\left(\alpha\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\right)}_{\text{Unknown quantum state}}\otimes\underbrace{\left|\downarrow\right\rangle}_{\text{Ancilla initialized to ground state}}=\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle+\beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle$$

Quantum circuit notation:



Heralding Quantum Errors

$$Z_1, Z_2 = \pm 1$$

Measure the
Joint Parity operator:

$$\Pi_{12} = Z_1 Z_2$$

$$\Pi_{12} \left|\uparrow\right\rangle\left|\uparrow\right\rangle = +\left|\uparrow\right\rangle\left|\uparrow\right\rangle$$

$$\Pi_{12} \left|\downarrow\right\rangle\left|\downarrow\right\rangle = +\left|\downarrow\right\rangle\left|\downarrow\right\rangle$$

$$\Pi_{12} \left|\uparrow\right\rangle\left|\downarrow\right\rangle = -\left|\uparrow\right\rangle\left|\downarrow\right\rangle$$

$$\Pi_{12} \left|\downarrow\right\rangle\left|\uparrow\right\rangle = -\left|\downarrow\right\rangle\left|\uparrow\right\rangle$$

$$\Pi_{12}\left(\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle + \beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle\right) = +\left(\alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle + \beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle\right)$$

$\Pi_{12} = -1$ heralds single bit flip errors

Heralding Quantum Errors

$$\Pi_{12} = Z_1 Z_2$$

<u>Not</u> easy to measure a joint operator while not accidentally measuring individual operators!

(Typical ‘natural’ coupling is $M_Z = Z_1 + Z_2$)

$|\uparrow\rangle|\uparrow\rangle$ and $|\downarrow\rangle|\downarrow\rangle$ are very different,

yet we must make that difference invisible

But it can be done if you know the right people...

Heralding Quantum Errors

Example of error heralding:

$$|\Psi\rangle = \alpha|\downarrow\rangle|\downarrow\rangle + \beta|\uparrow\rangle|\uparrow\rangle$$

Introduce single qubit error on 1 (over rotation, say)

$$e^{i\frac{\theta}{2}X_1}|\Psi\rangle = \cos\frac{\theta}{2}|\Psi\rangle + i\sin\frac{\theta}{2}X_1|\Psi\rangle$$

Relative weight of α, β is untouched.

Probability of error: $\sin^2\frac{\theta}{2}$

If no error is heralded, state collapses to $|\Psi\rangle$

and there is no error!

Heralding Quantum Errors

Example of error heralding:

$$|\Psi\rangle = \alpha|\downarrow\rangle|\downarrow\rangle + \beta|\uparrow\rangle|\uparrow\rangle$$

Introduce single qubit rotation error on 1 (say)

$$\mathrm{e}^{i\frac{\theta}{2}X_1}|\Psi\rangle = \cos\frac{\theta}{2}|\Psi\rangle + i\sin\frac{\theta}{2}X_1|\Psi\rangle$$

Relative weight of α, β is untouched.

Probability of error: $\sin^2\frac{\theta}{2}$

If error is heralded, state collapses to $\mathrm{X}_1|\Psi\rangle$

and there is a full bitflip error. We cannot correct it because we don't know which qubit flipped.

Heralding Quantum Errors

Quantum errors are continuous (analog!).

But the detector result is discrete.

The measurement back action renders the error discrete (digital!)

– either no error or full bit flip.

Correcting Quantum Errors

Extension to 3-qubit code allows full correction of bit flip errors

$$\left|\Psi\right\rangle = \alpha\left|\downarrow\right\rangle\left|\downarrow\right\rangle\left|\downarrow\right\rangle + \beta\left|\uparrow\right\rangle\left|\uparrow\right\rangle\left|\uparrow\right\rangle$$

$$\Pi_{12} = Z_1 Z_2 \text{ and } \Pi_{32} = Z_3 Z_2$$

Provide two classical bits of information to diagnose and correct all 4 possible bitflip errors:

$$I, X_1, X_2, X_3$$

Correcting Quantum Errors

Extension to 5,7,or 9-qubit code allows full correction of ALL single qubit errors

I (no error)

$X_1,\ldots,X_N$ (single bit flip)

$Z_1,\ldots,Z_N$ (single phase flip; no classical analog)

$Y_1,\ldots,Y_N$ (single bit AND phase flip; no classical analog)

For *N=5,* there are 16 errors and 32 states

32= 16 x 2

Just enough room to encode which error occurred and still have one qubit left to hold the quantum information.

The cat code and hardware-efficiency

‘Extending the lifetime of a quantum bit with error correction in superconducting circuits,’ Ofek et al., (*Nature*, 20 July, 2016)

(Slides courtesy R. Schoelkopf)

Cat-codes Can Be "Hardware-Efficient"

Leghtas, Mirrahimi, et al., PRL **111**, 120501(2013).

- Advantages include:
  - Long lifetimes*:

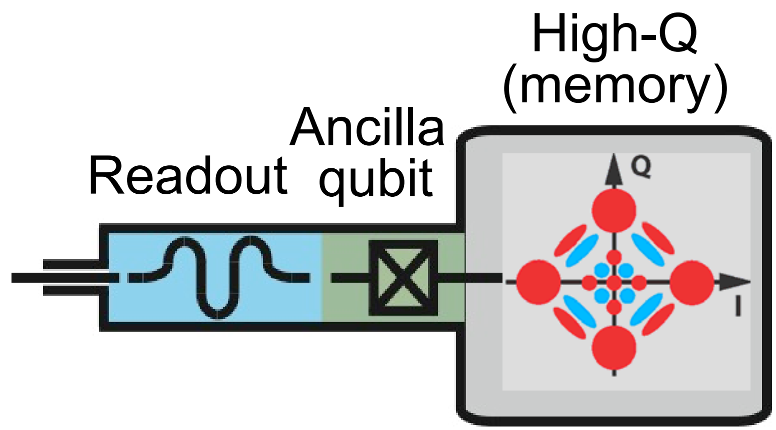
    $Q \sim 10^8,\ \tau_c \sim 0.005$ sec

  - Large Hilbert space
  - One dominant error channel:

    single photon loss

$$\kappa = \frac{1}{\tau_C} = \frac{\omega}{Q}$$

Quality Factor



Our approach :

- Cavity is the memory
- One error syndrome

1. 1 qubit!
2. Single readout channel!

What is the best encoding scheme?

*Reagor et al., arXiv 1508.05882

earlier ideas: Gottesman, Kitaev & Preskill, PRA 64 , 012310 (2001)

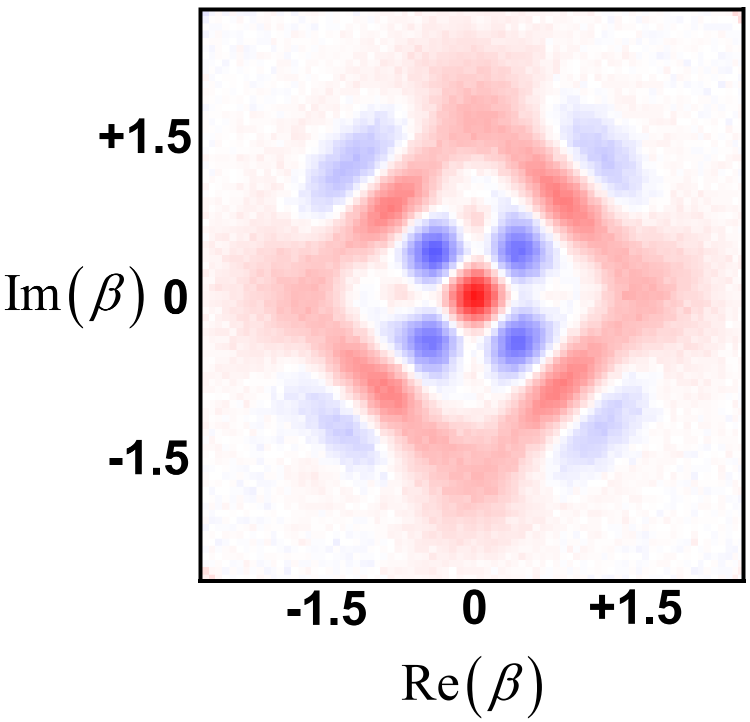
Redundant Encoding in Cat States

Hardware-Efficient Code:
Corrects for single photon loss

Logical code words in cavity:

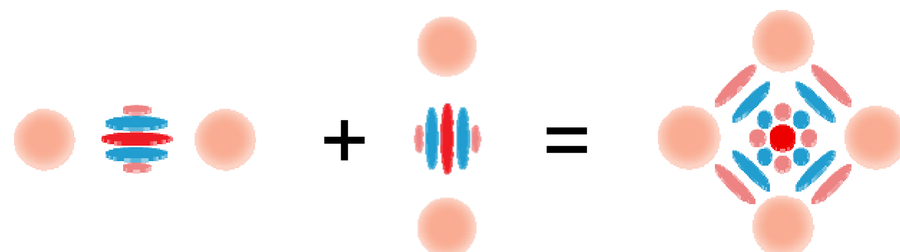
$$|W_1\rangle = |\mathcal{C}_\alpha^+\rangle = \mathcal{N}(|\alpha\rangle + |-\alpha\rangle)$$

$$|W_2\rangle = |\mathcal{C}_{i\alpha}^+\rangle = \mathcal{N}(|i\alpha\rangle + |-i\alpha\rangle)$$



Map qubit state to cavity:

$$c_g|g\rangle + c_e|e\rangle \Rightarrow c_g|W_1\rangle + c_e|W_2\rangle$$



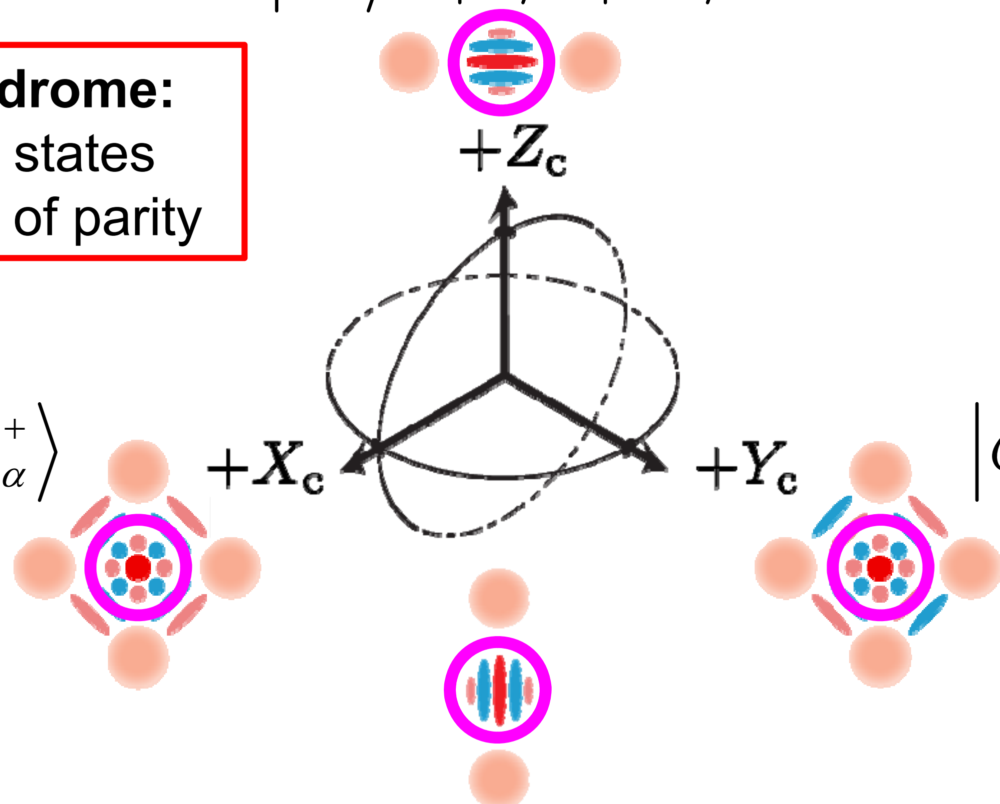
Store a ***qubit*** as a ***superposition*** of two cats of same ***parity****

*Leghtas, Mirrahimi, et al., PRL 111, 120501, (2013)

Redundant Encoding in Cat States

Error Syndrome:
All logical states eigenstates of parity

$$\left|\mathcal{C}_{\alpha}^{+}\right\rangle = \left|\alpha\right\rangle + \left|-\alpha\right\rangle$$



$$\left|\mathcal{C}_{\alpha}^{+}\right\rangle + \left|\mathcal{C}_{i\alpha}^{+}\right\rangle$$

$$\left|\mathcal{C}_{\alpha}^{+}\right\rangle + i\left|\mathcal{C}_{i\alpha}^{+}\right\rangle$$

$$\left|\mathcal{C}_{i\alpha}^{+}\right\rangle = \left|i\alpha\right\rangle + \left|-i\alpha\right\rangle$$

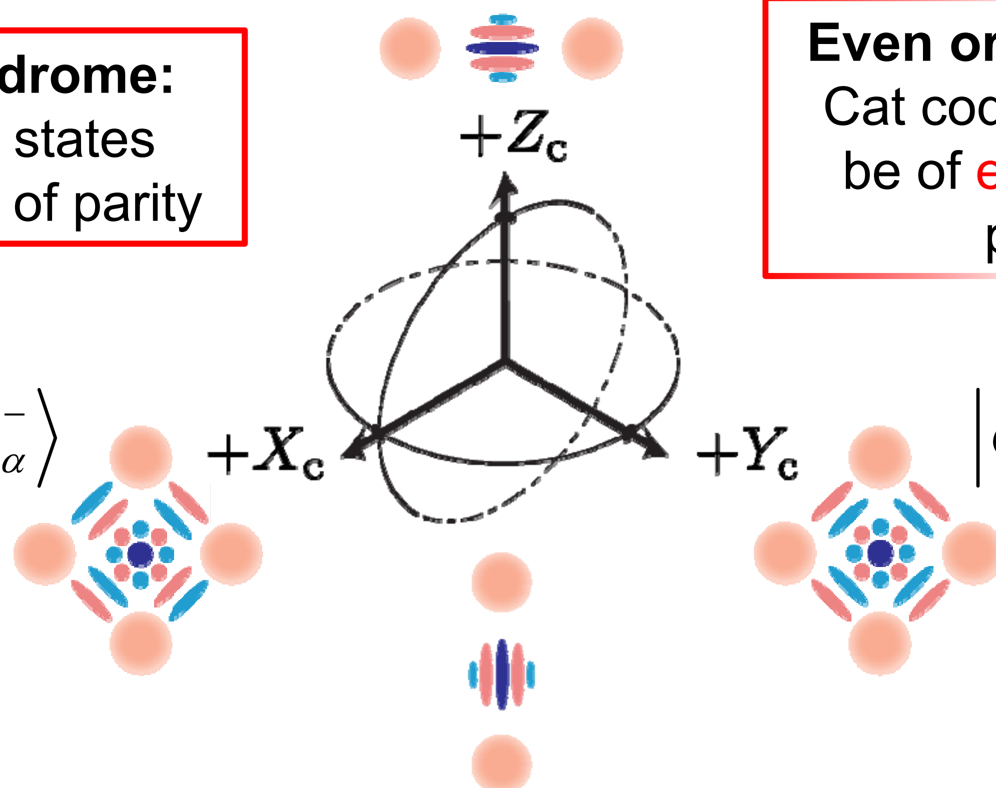
Redundant Encoding in Cat States

Error Syndrome:
All logical states eigenstates of parity

$$\left|\mathcal{C}_{\alpha}^{-}\right\rangle = \left|\alpha\right\rangle - \left|-\alpha\right\rangle$$

Even or Odd basis:
Cat code basis can be of even or odd parity

$$\left|\mathcal{C}_{\alpha}^{-}\right\rangle + \left|\mathcal{C}_{i\alpha}^{-}\right\rangle$$



$$\left|\mathcal{C}_{\alpha}^{-}\right\rangle + i\left|\mathcal{C}_{i\alpha}^{-}\right\rangle$$

$$\left|\mathcal{C}_{i\alpha}^{-}\right\rangle = \left|i\alpha\right\rangle - \left|-i\alpha\right\rangle$$

Error model for cavity: Damping into a cold bath

(Exact) Kraus representation (POVM) organized by number of photon losses (detector 'clicks'). SHO is a special case: time of clicks unimportant, only total number of clicks counts:

$$\rho(t) = \sum_{\ell=0}^{\infty} E_\ell \rho(0) E_\ell^\dagger$$

$$E_\ell \equiv \sqrt{\gamma_\ell}\, e^{-\frac{\kappa}{2}\hat{n}t} a^\ell, \qquad \gamma_\ell \equiv \sqrt{\frac{(1-e^{-\kappa t})^\ell}{\ell!}}$$

$$\sum_{\ell=0}^{\infty} E_\ell^\dagger E_\ell = \hat{I}$$

M. Michael et al., Phys. Rev. X **6**, 031006 (2016)
'New class of error correction codes for a bosonic mode'

Coherent states have special properties under the Kraus operations:

$$E_\ell |\alpha\rangle = \sqrt{\gamma_\ell}\,\alpha^\ell \left| e^{-\frac{\kappa}{2}t}\alpha \right\rangle$$

$$\boxed{|\alpha\rangle \to \frac{E_\ell|\alpha\rangle}{\langle\alpha|E_\ell{}^\dagger E_\ell|\alpha\rangle^{1/2}} = \left(\frac{\alpha}{|\alpha|}\right)^\ell \left| e^{-\frac{\kappa}{2}t}\alpha \right\rangle}$$

State is (pseudo) invariant! (up to an important phase).

Effect of photon loss on code words

$$a|W_1\rangle = a\left(|\alpha\rangle + |-\alpha\rangle\right) \to \left(|\alpha\rangle - |-\alpha\rangle\right) \quad (\text{if } \alpha \text{ real})$$

$$a^2|W_1\rangle = a^2\left(|\alpha\rangle + |-\alpha\rangle\right) \to \left(|\alpha\rangle + |-\alpha\rangle\right) = |W_1\rangle$$

$$a|W_2\rangle = a\left(|i\alpha\rangle + |-i\alpha\rangle\right) \to i\left(|i\alpha\rangle - |-i\alpha\rangle\right)$$

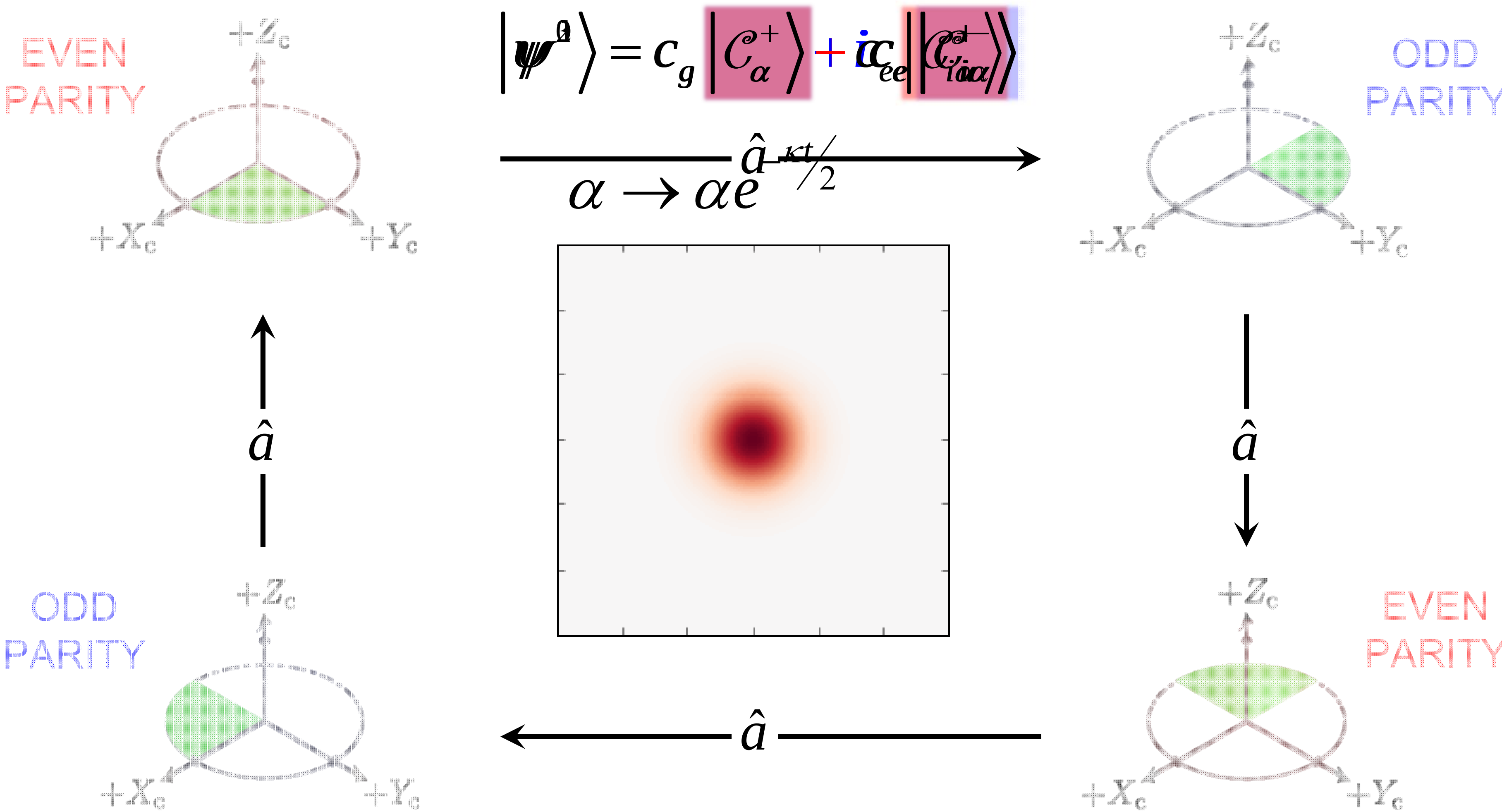
$$a^2|W_2\rangle = a^2\left(|i\alpha\rangle + |-i\alpha\rangle\right) = (i)^2\left(|i\alpha\rangle + |-i\alpha\rangle\right) = -|W_2\rangle$$

After loss of 4 photons cycle repeats:

$$a^4\left(\xi_1|W_1\rangle + \xi_2|W_2\rangle\right) \to \left(\xi_1|W_1\rangle + \xi_2|W_2\rangle\right)$$

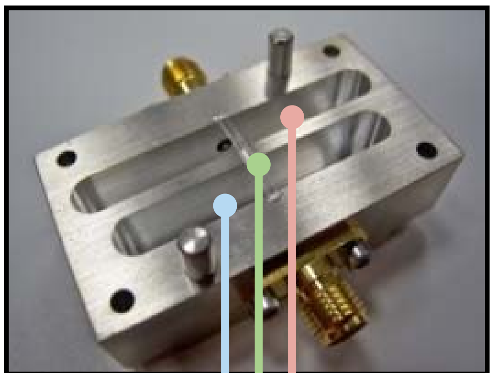
We can recover the state if we know: $\boxed{N_{\text{Loss}} \mod 4}$

To Live & Die in a Cavity: What κ Does

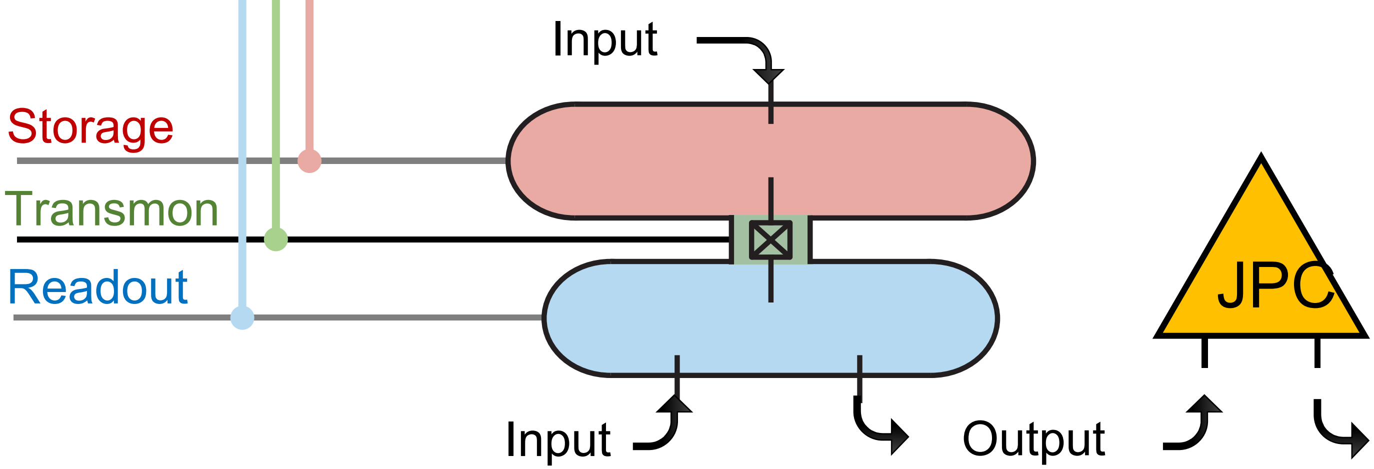


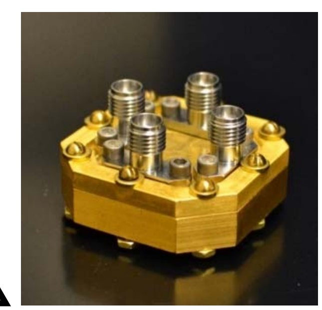
The Experiment: Implementing the cat code with superconducting circuits

QEC Setup: 2 Cavities + 1 Transmon Ancilla



$$H = \overbrace{\omega_s a^\dagger a}^{8.2\,GHz} + \overbrace{\omega_q |e\rangle\langle e|}^{6.5\,GHz} - \chi_{sa} a^\dagger a |e\rangle\langle e| - K_{ss} (a^\dagger a)^2$$





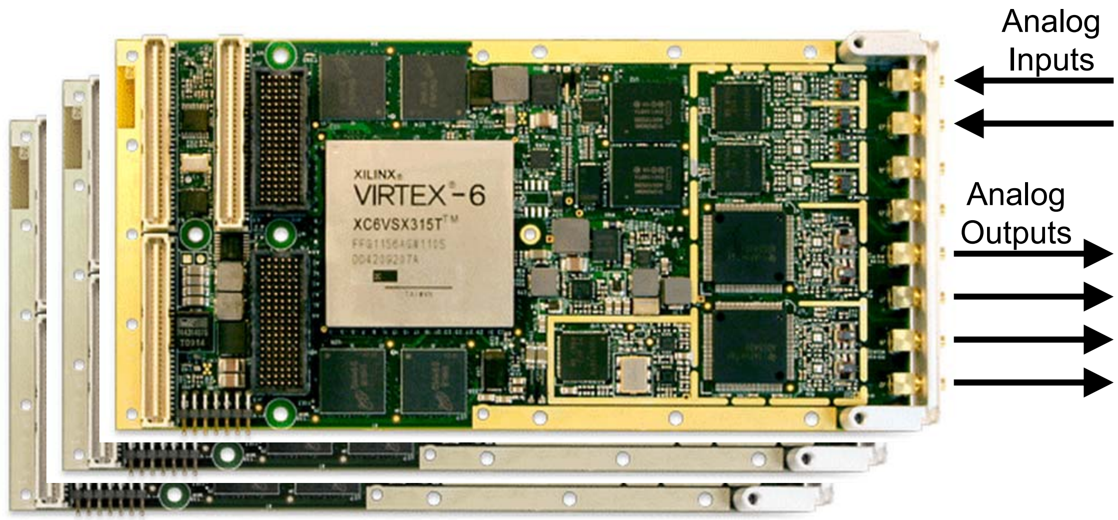
Ancilla (transmon) readout fidelity ~ 99.5%
in 400 ns

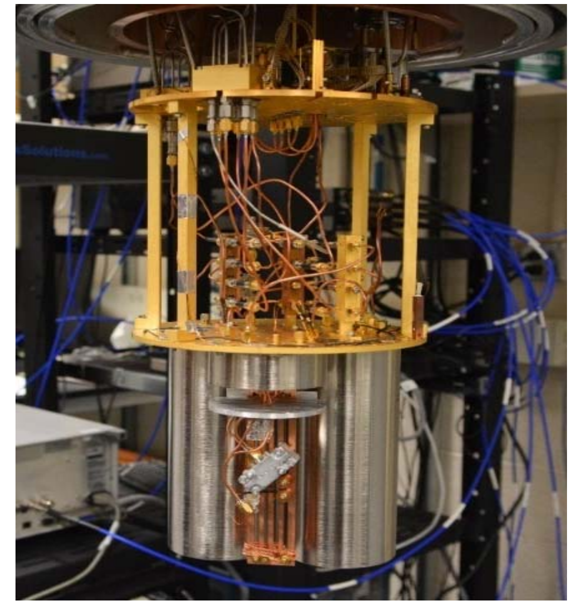
Thank you QLab

The Experiment: Implementing the cat code with superconducting circuits

1. Map ancilla qubit state into photonic cat code words
2. Monitor number parity jumps M
3. Conditioned on M, choose unitary U_M to map cavity state back to ancilla qubit
4. Perform process tomography to determine fidelity
5. Compare error-corrected logical qubit performance to simple (0,1) photon encoding (best uncorrectable code)

2016: First true Error Correction Engine that works

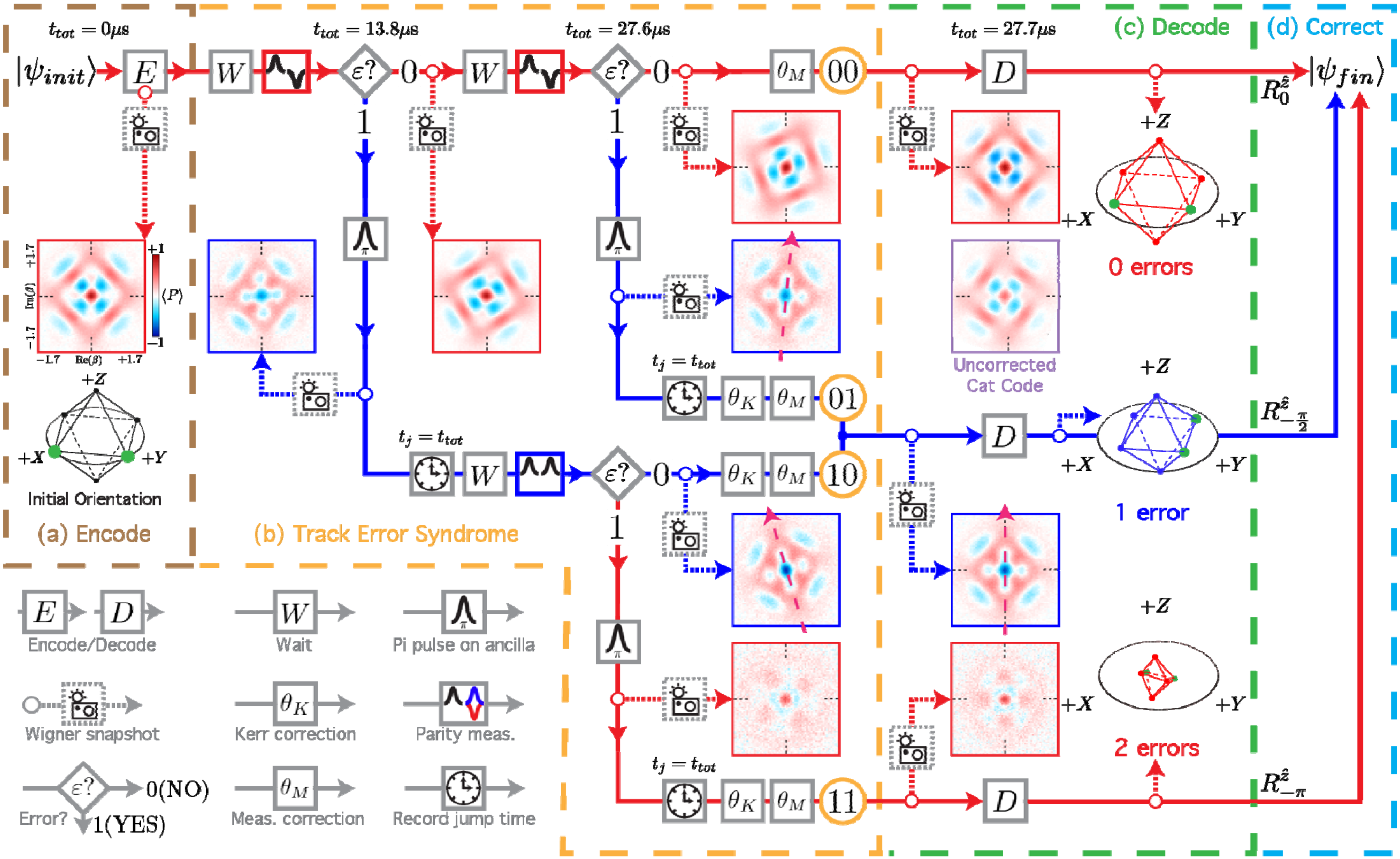




A prototype quantum computer being prepared for cooling close to absolute zero.

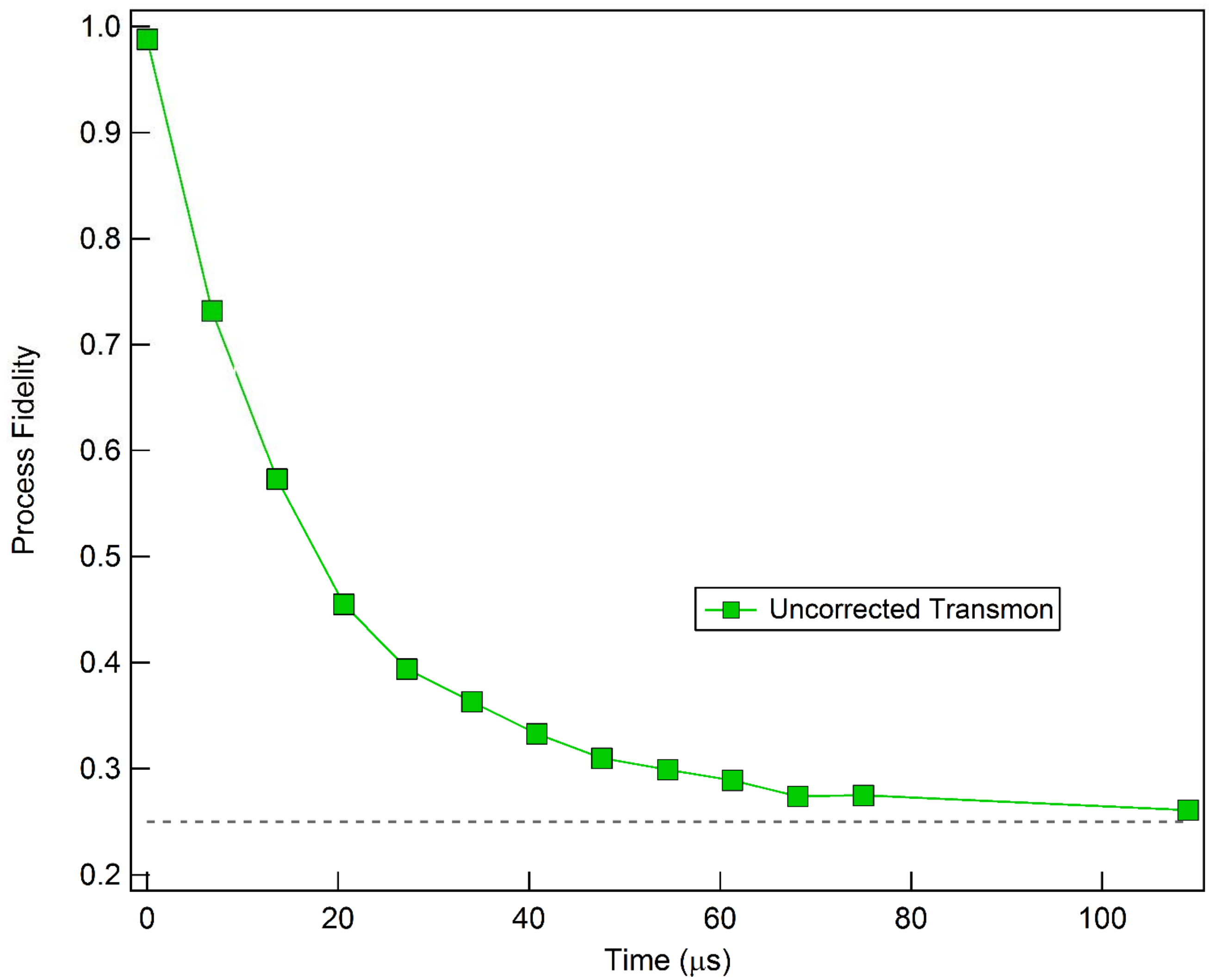
- Commercial FPGA with custom software developed at Yale
- Single system performs all measurement, control, & feedback
  (latency ~200 nanoseconds)
- 15% of the latency is the time it takes signals to move at the speed of light from the quantum computer to the controller and back!

Implementing a Full QEC System: Debugger View



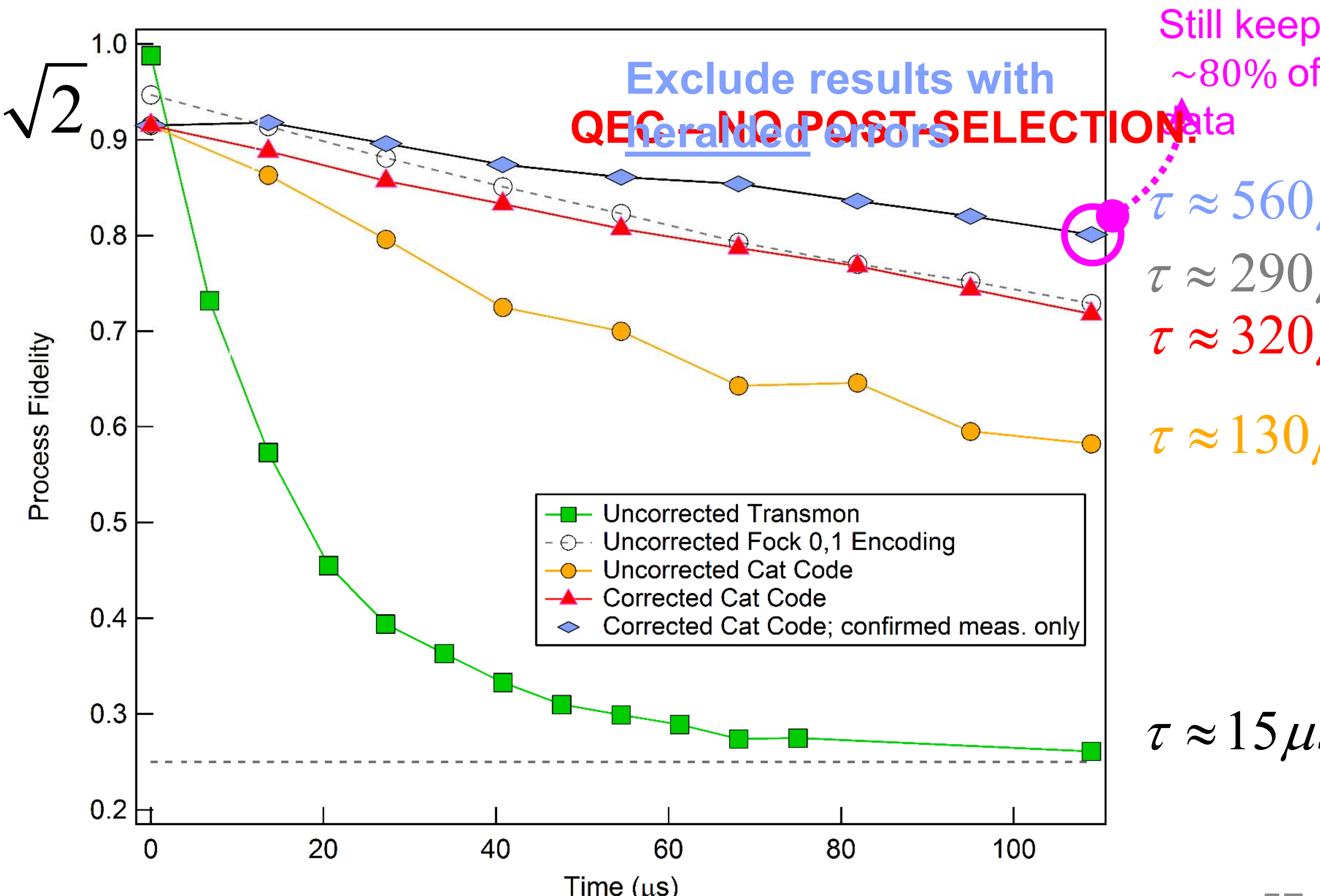
(This is all real, raw data.)

Process Fidelity: Uncorrected Transmon



$\tau \approx 15 \mu s$

$\alpha = \sqrt{2}$



$\tau \approx 560\mu s$

$\tau \approx 290\mu s$

$\tau \approx 320\mu s$

$\tau \approx 130\mu s$

$\tau \approx 15\mu s$

Future directions:

-still higher Q cavities;

-cat codes on two cavities;
logical operations for cat states in two cavities

-scaling to many cavities coupled by qubits

-’kitten codes’ [PRX **6**, 031006 (2016)]

-’cat pumping’ to overcome amplitude decay
Leghtas et al. *Science* **347**, 853 (2015)

We are on the way!

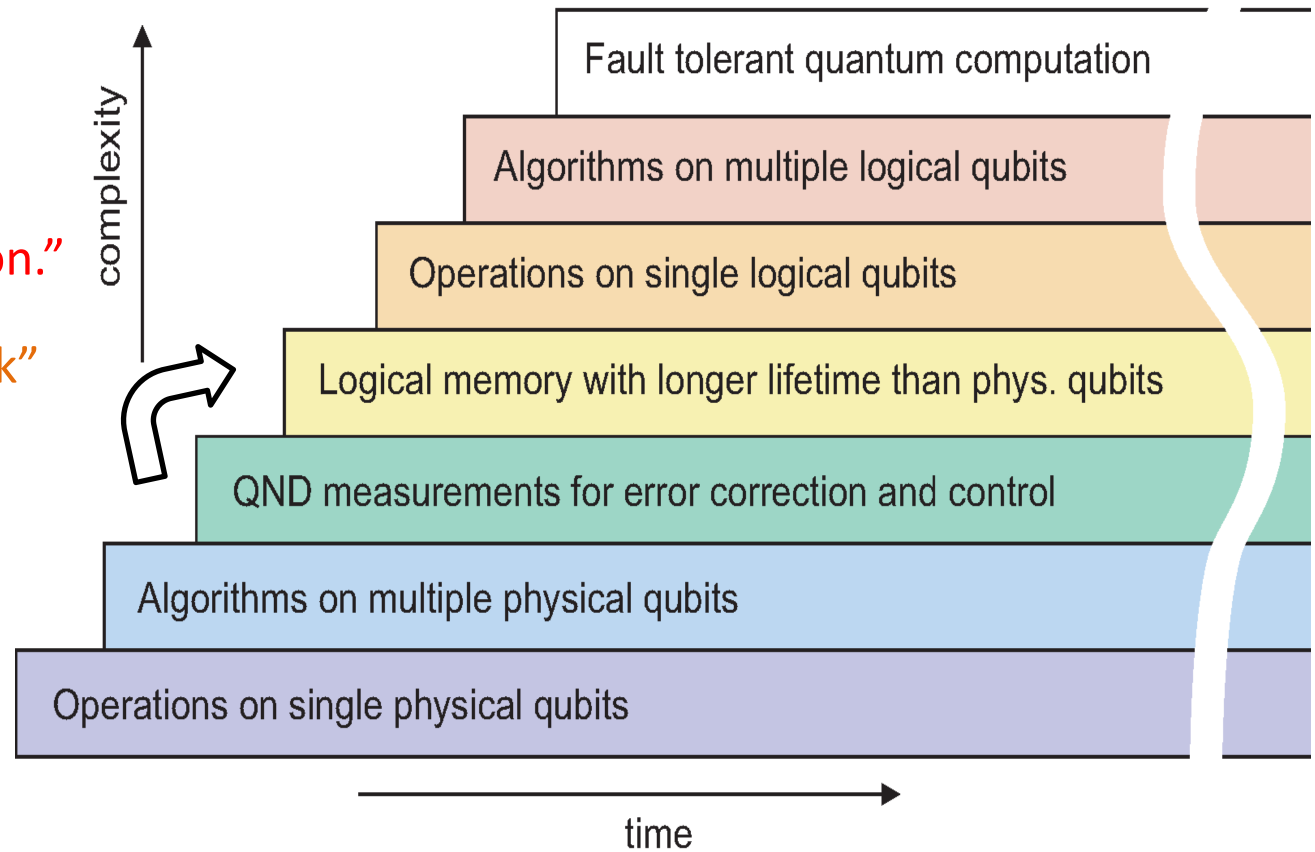
"Age of Qu. Error Correction."

"Age of Quantum Feedback"

"Age of Measurement"

"Age of Entanglement"

"Age of Coherence"



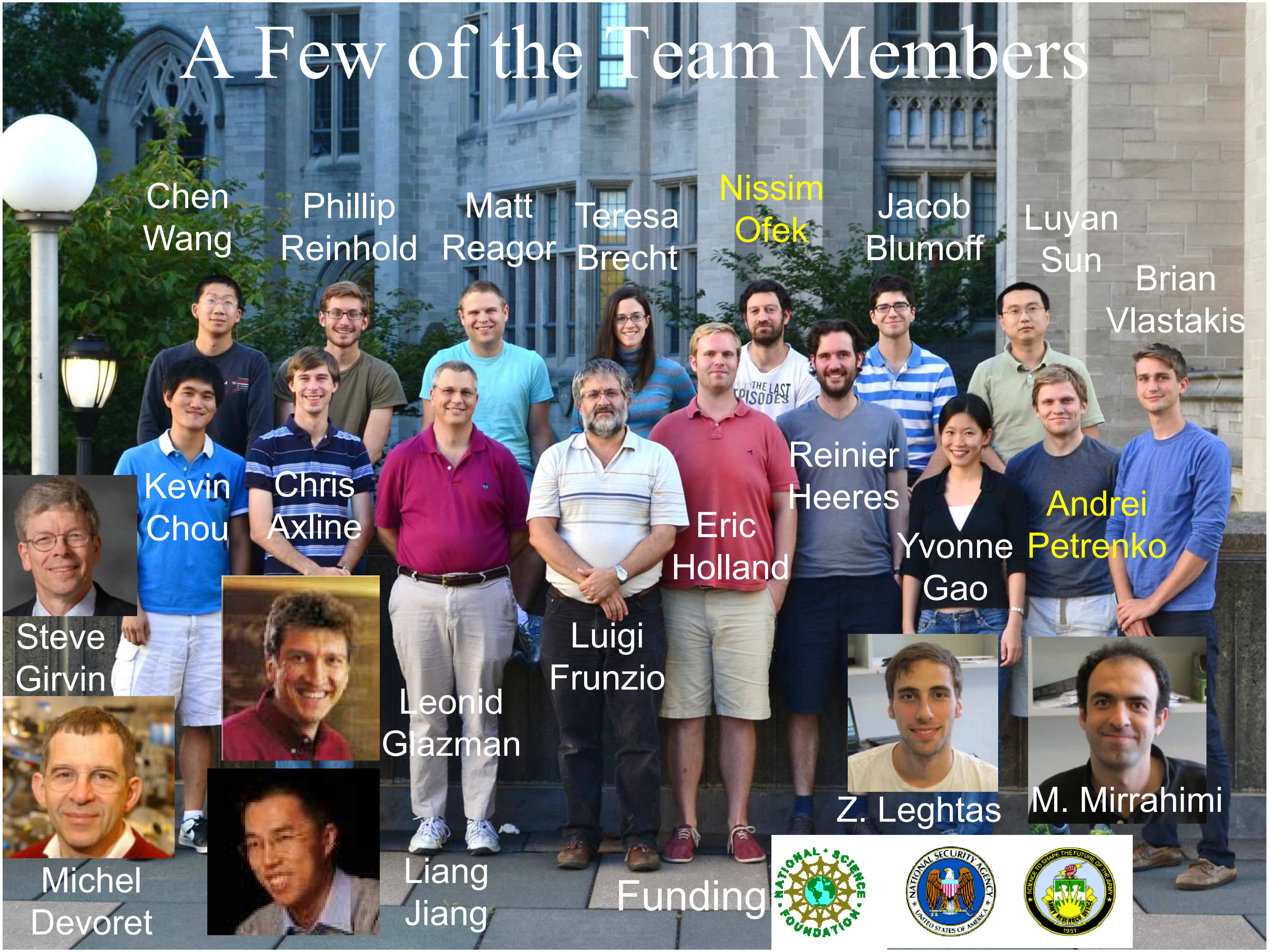
Have achieved goal of reaching "break-even" point for error correction

➡ "We" are ~ here (also ions, Rydbergs, q-dots, …)

M. Devoret and RS, Science (2013)

When it comes to quantum mechanics you have to think different

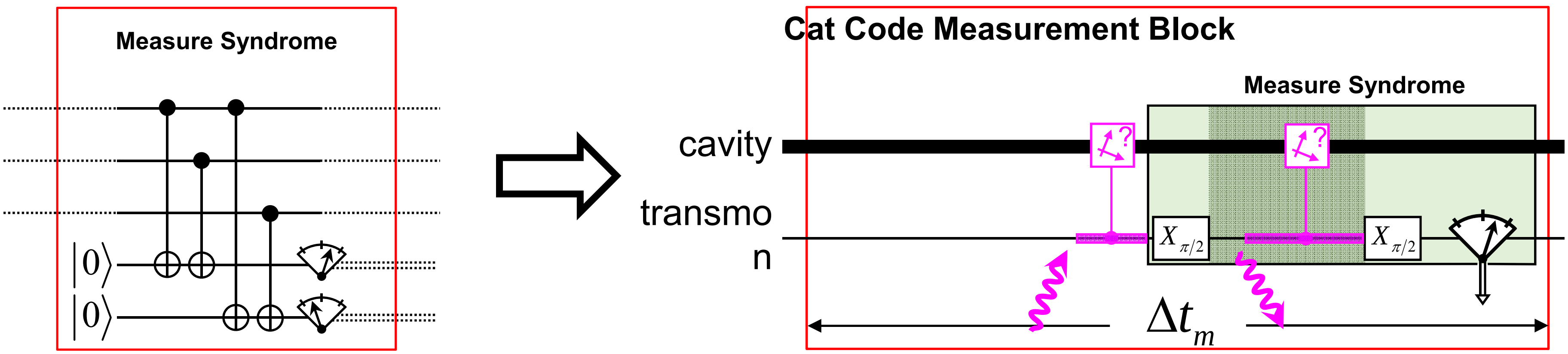
A Few of the Team Members



A few more….

Extra Slides

The Cat Code – Failure Modes



Failures	Sources	Expected Lifetime Gain: Max. Meas. Rate (1/1μs)	Expected Lifetime Gain: Opt. Meas. Rate (1/25μs)
1. Double Errors	cavity a^2	x 125	x 5
2. Uncorrectable Errors	cavity $a^\dagger$	x 20	x 20
3. Readout Errors	transmon T_φ	x 25	x 8
4. Ancilla Preparation	transmon $\Gamma_\uparrow$	x 1000	x 3
5. Undesired Couplings	cavity $a^{\dagger 2} a^2$	x 2000	x 7
6. Forward Propagation	transmon T_1	**x 0.7**	x 2