

# DOE/NSF Quantum Science Summer School 3

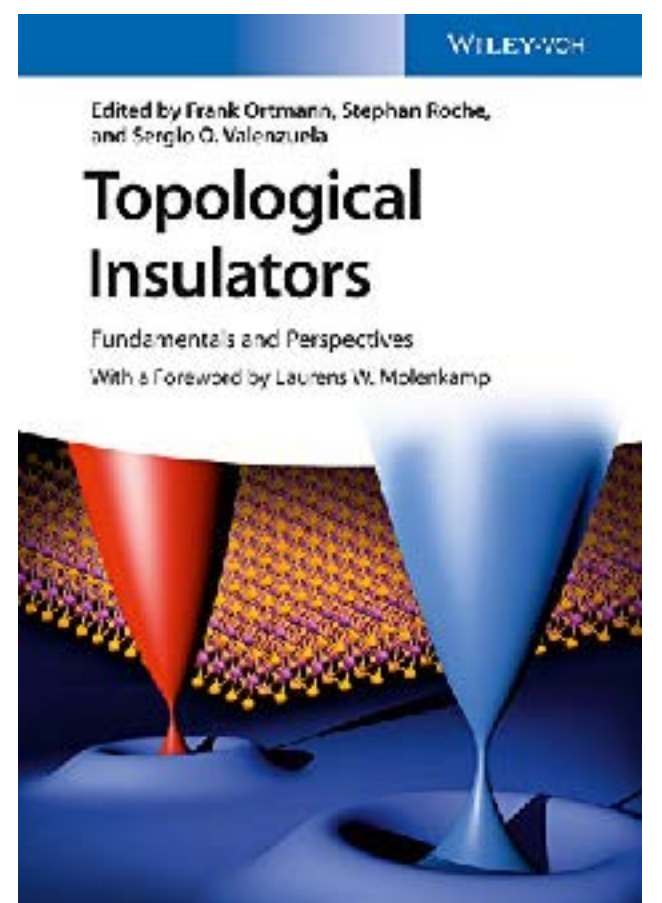
Lecture 1:

An introduction to spintronics  
Topological insulators

Lecture 2:

Topological spintronics

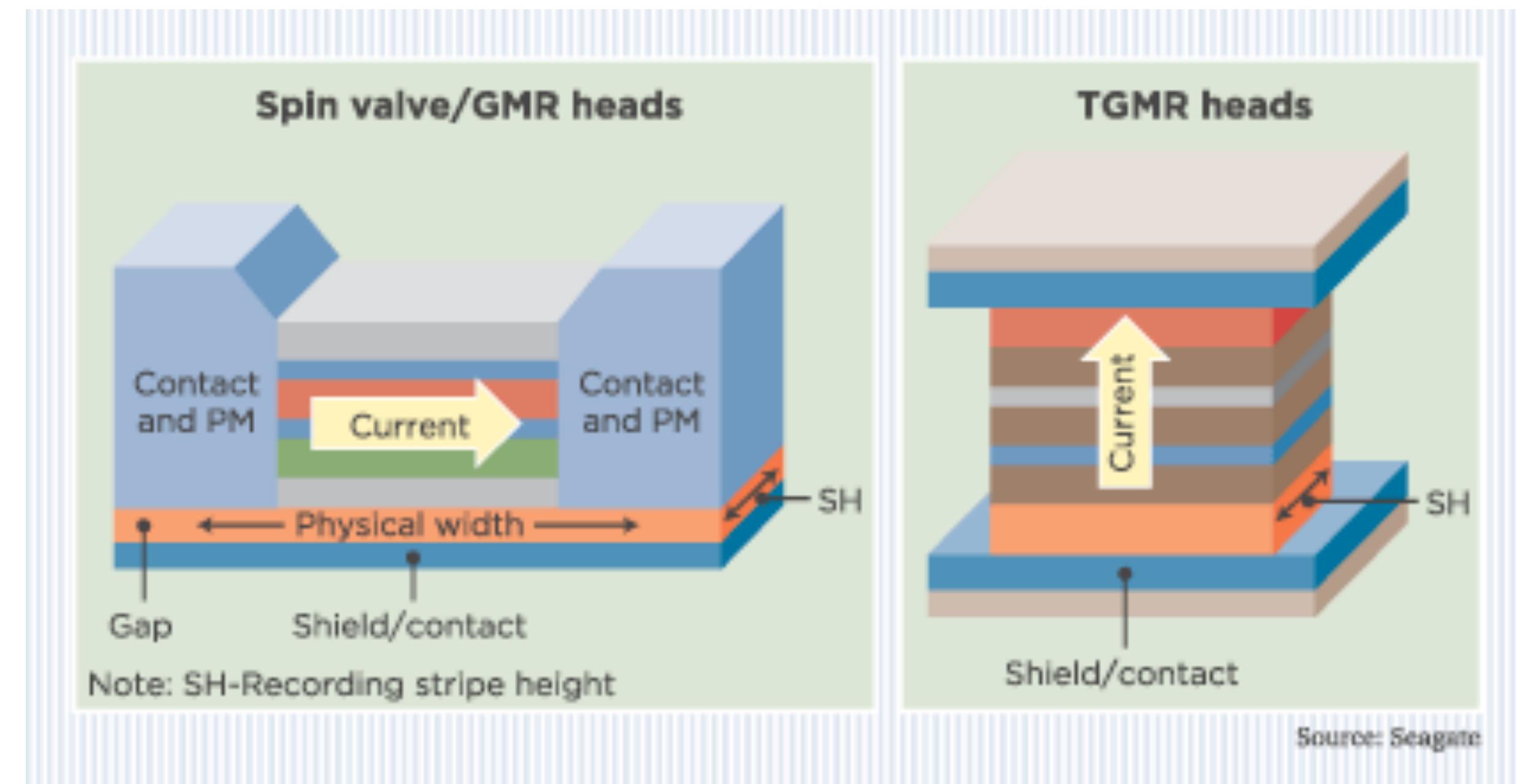
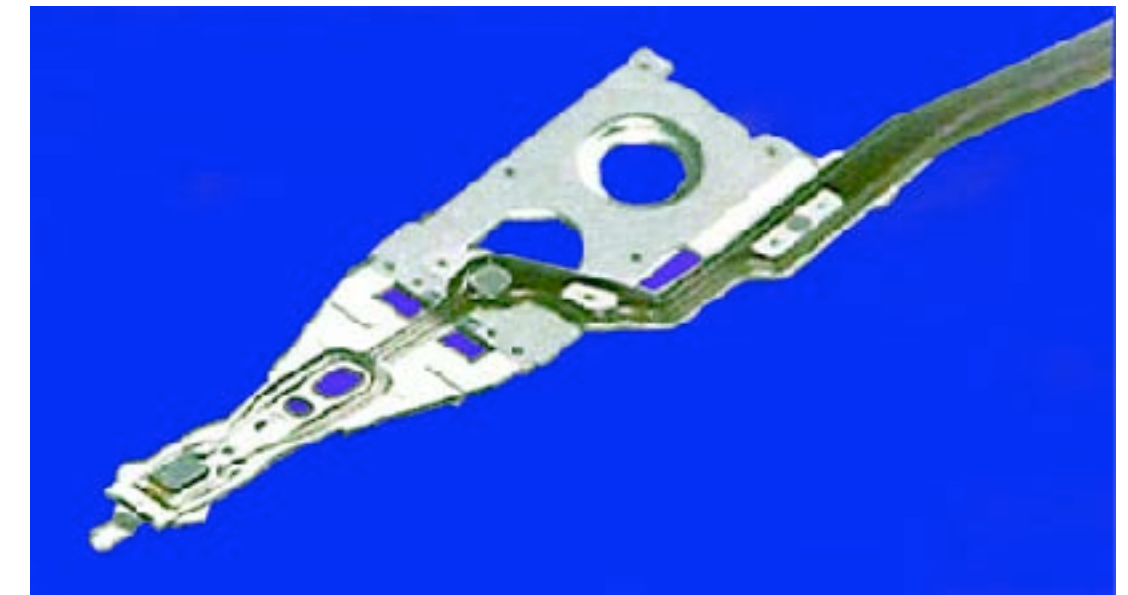
- Spintronics: overview of concepts & devices
- Topological insulators: concepts, materials, phenomena
- Topological spintronics: concepts, materials, phenomena, devices





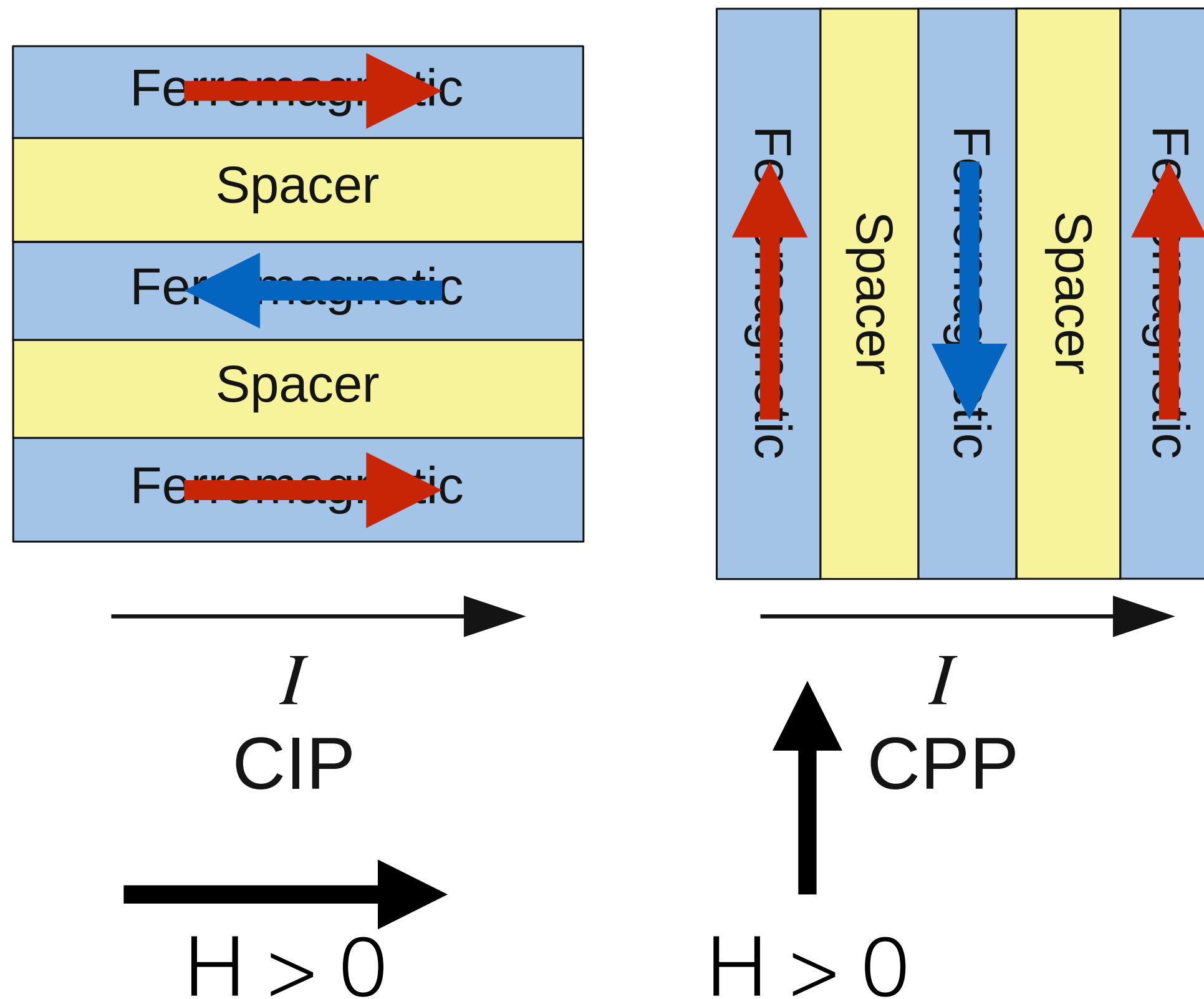
# The origins of “spintronics”

- Spintronics: “spin transport electronics.”
- First used as a misnomer for magnetic field sensors that “read” magnetic memory via spin-dependent scattering of charge currents in ferromagnetic multilayers
- Giant magnetoresistance (GMR)
- Tunnel magnetoresistance (TMR)
- Direct impact: magnetic hard drives



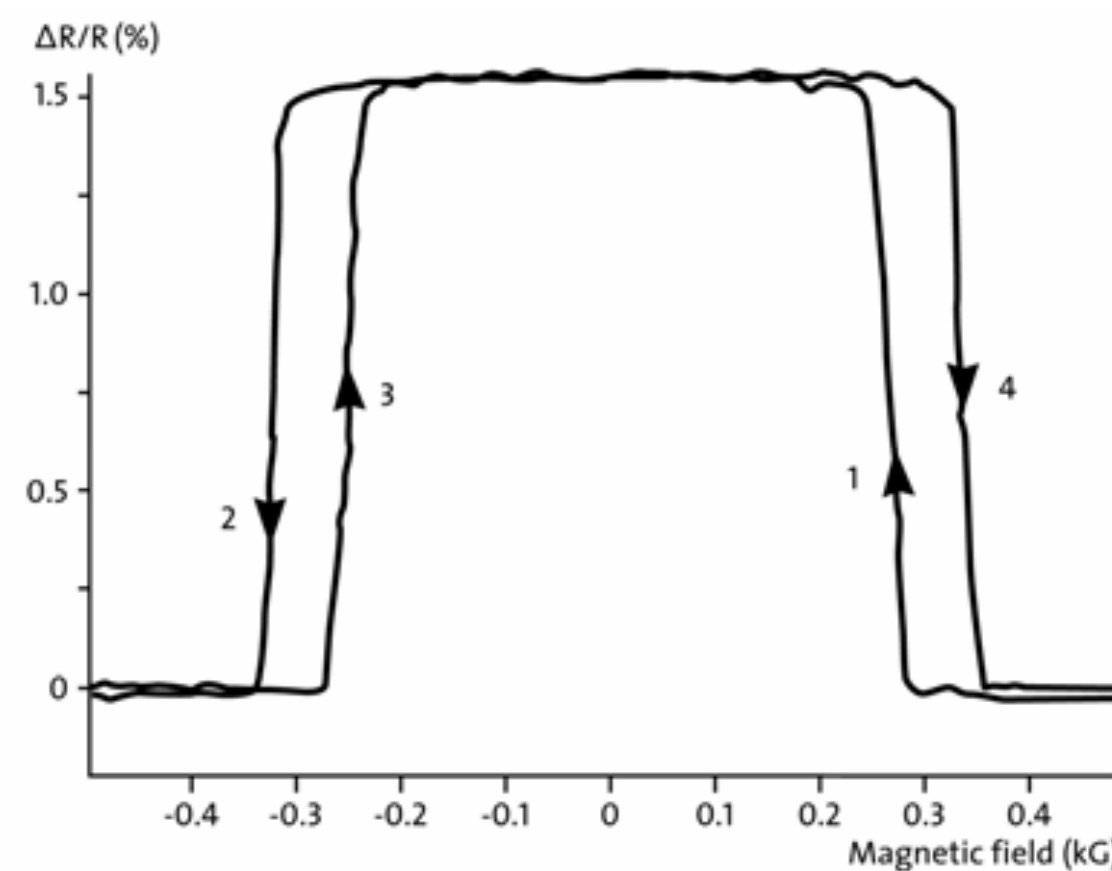
(Graphics: Seagate, [nobel.org](http://nobel.org))

# Giant magnetoresistance (GMR)



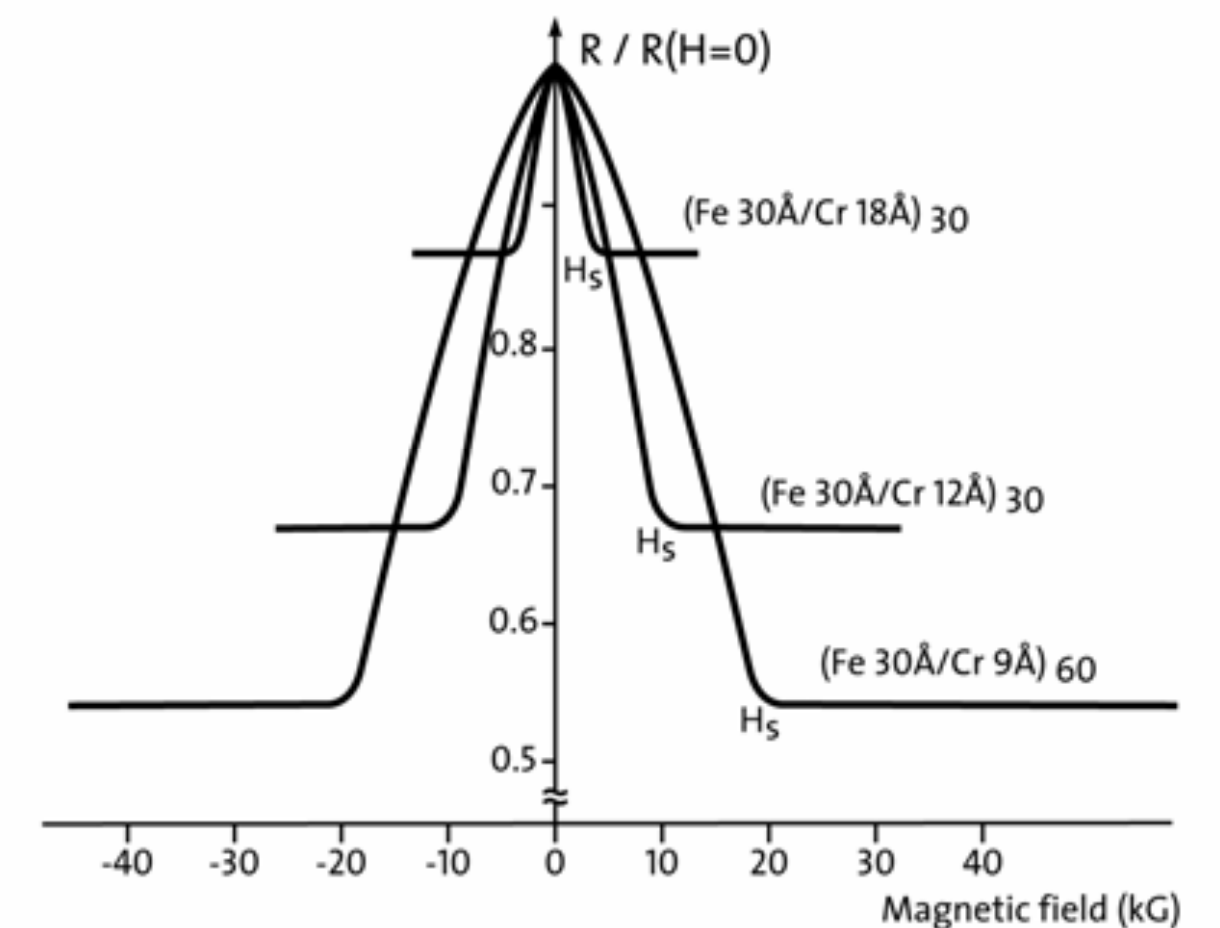
Two geometries for GMR:  
Current-in-plane (CIP)  
Current-perpendicular-to-plane (CPP)

Fe/Cr/Fe trilayer



Binach *et al.*, PRB **39**,  
4828 (1989)

(Fe/Cr)<sub>n</sub> multilayer



Baibich *et al.*, PRL **61**,  
2472 (1988)

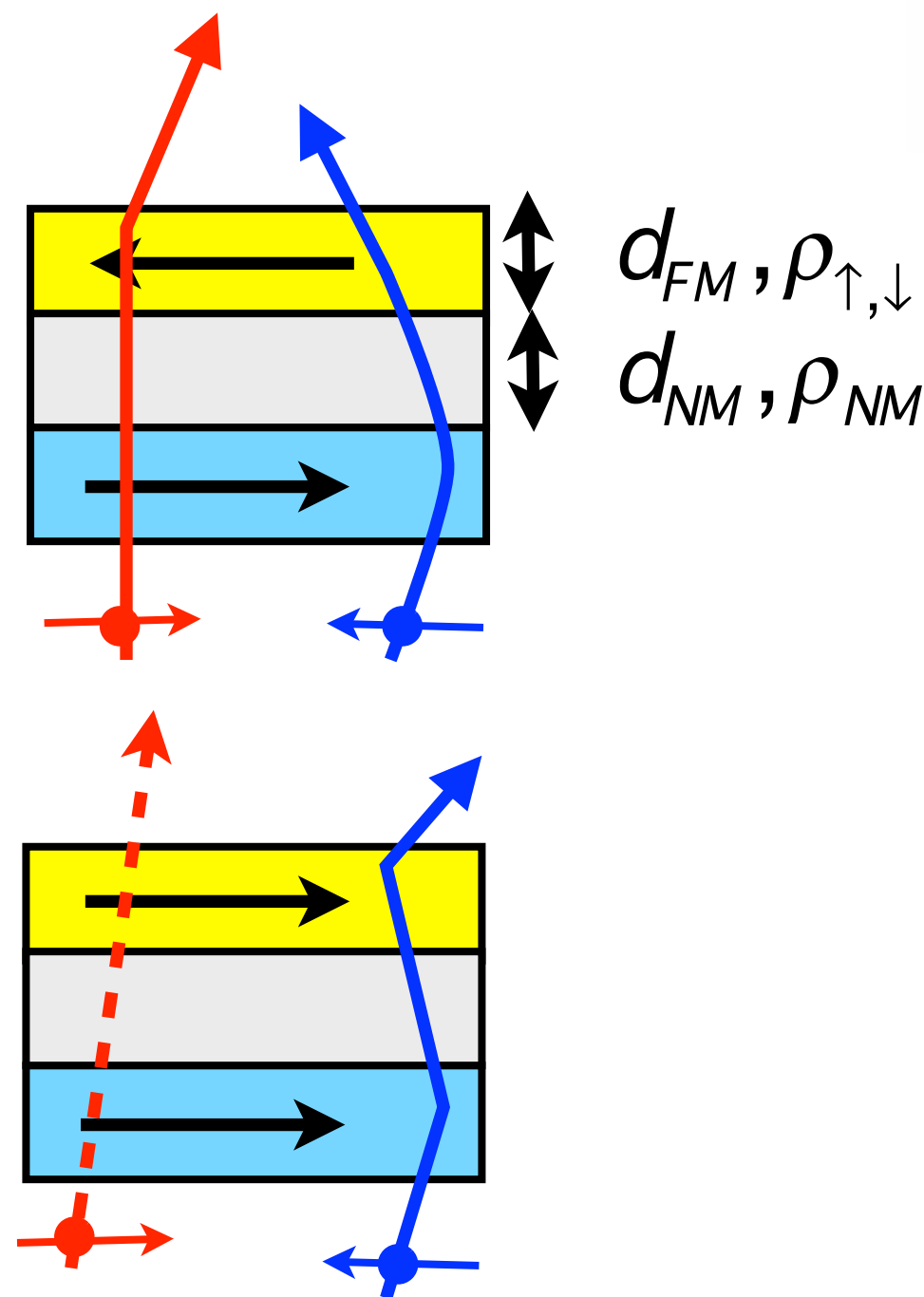
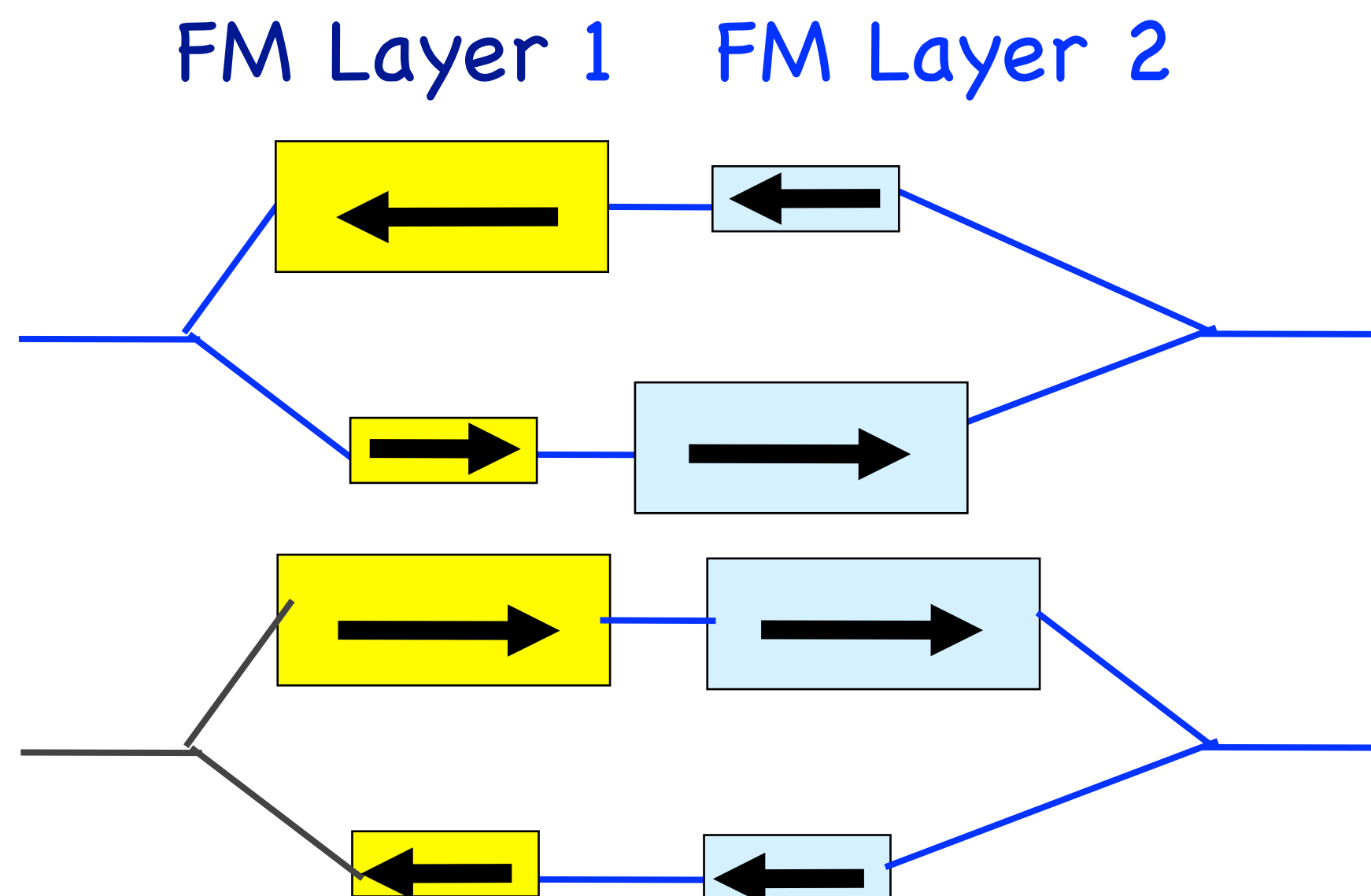
Which of the magnetization configurations in the animation has a higher resistance?  
Why?



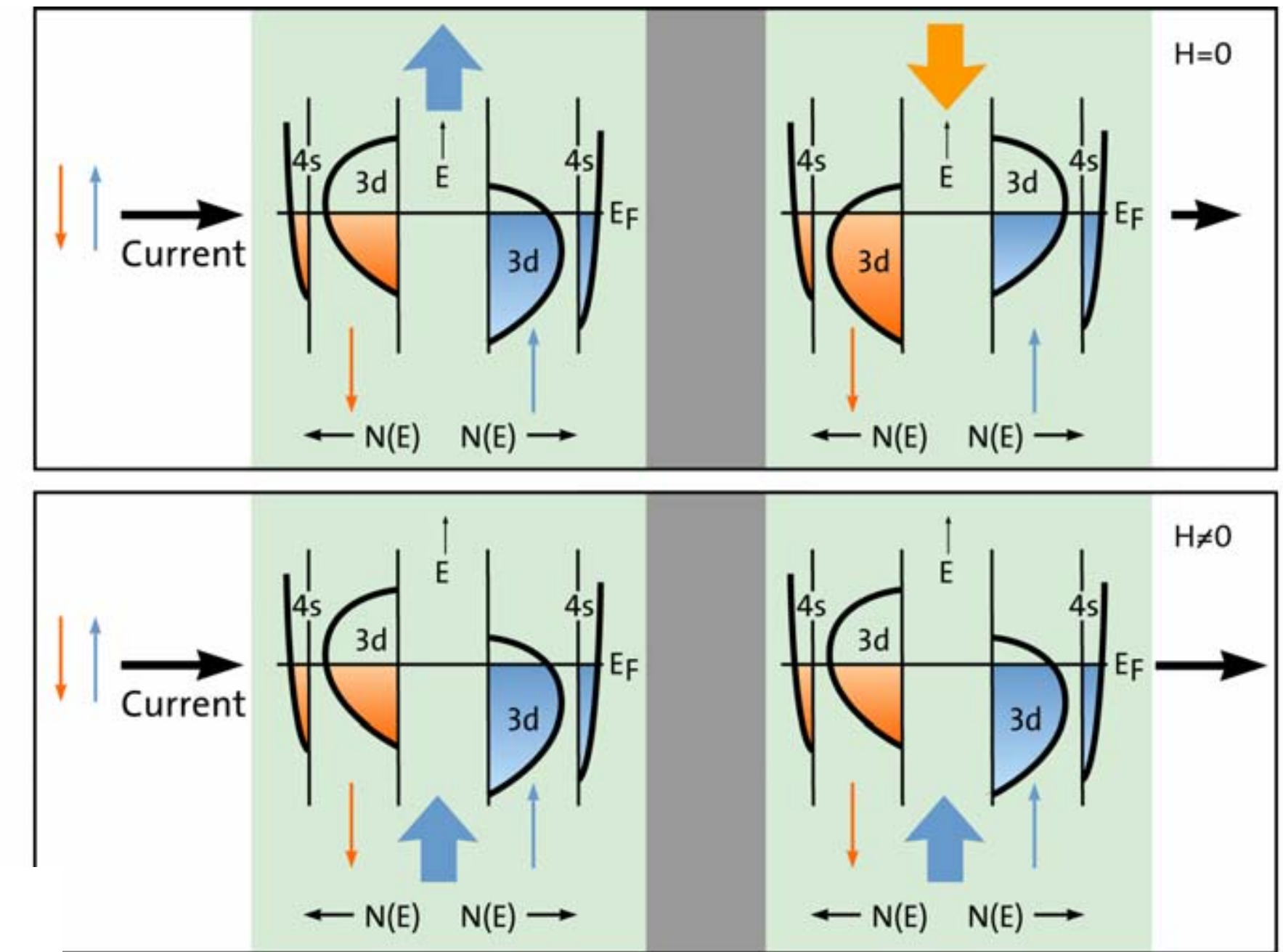
# Giant magnetoresistance (GMR)

Intuitive explanation of GMR: Mott's two channel model of transport in ferromagnets; majority & minority electrons have different conductivity; spin diffusion length  $\gg$  mean free path.

More sophisticated approaches: Boltzmann, first principles



(graphic: [nobel.org](http://nobel.org))



$$\frac{\Delta R}{R} = \frac{(\alpha - 1)^2}{4(\alpha + p \frac{d_{NM}}{d_{FM}})(1 + p \frac{d_{NM}}{d_{FM}})}$$

$$\alpha = \frac{\rho_{\downarrow}}{\rho_{\uparrow}} \quad p = \frac{\rho_{NM}}{\rho_{\uparrow}}$$

# Tunnel magnetoresistance (TMR)

Tunnel magnetoresistance (TMR) arises from spin-dependent tunneling: transmission probability higher when magnetization is parallel, lower when anti-parallel.

Julliere model (Phys. Lett. A **54**, 225 (1975):

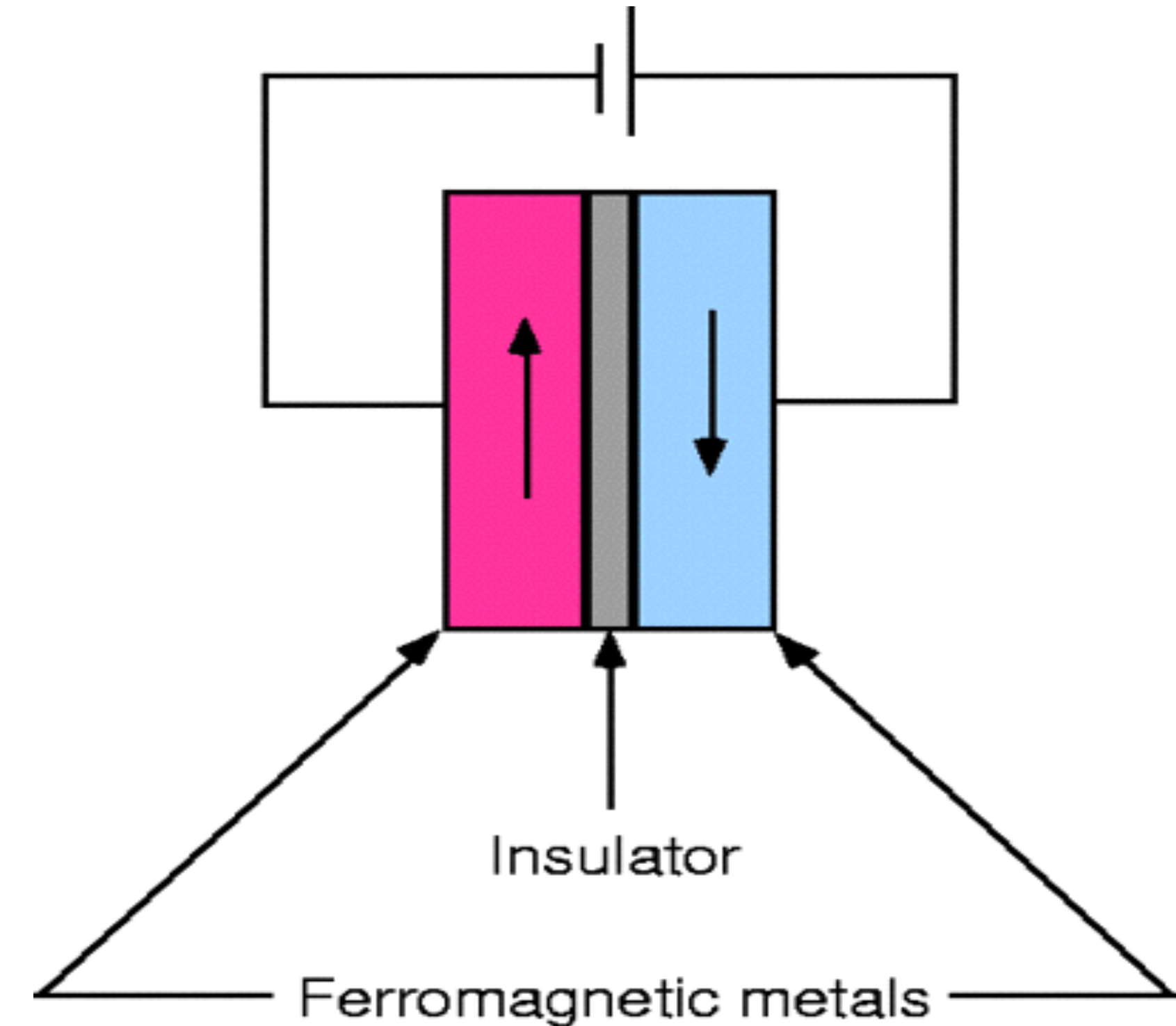
A simple exercise

Define spin polarization of FM:  $P = \frac{n^{\uparrow} - n^{\downarrow}}{n^{\uparrow} + n^{\downarrow}}$   $\frac{n^{\uparrow}}{n^{\downarrow}} = \frac{1 + P}{1 - P}$

Relate the tunneling conductance to the product of the density of states

$$\text{TMR} = \frac{R_{\uparrow\downarrow} - R_{\uparrow\uparrow}}{R_{\uparrow\uparrow}} = \frac{2P_1P_2}{1 - P_1P_2}$$

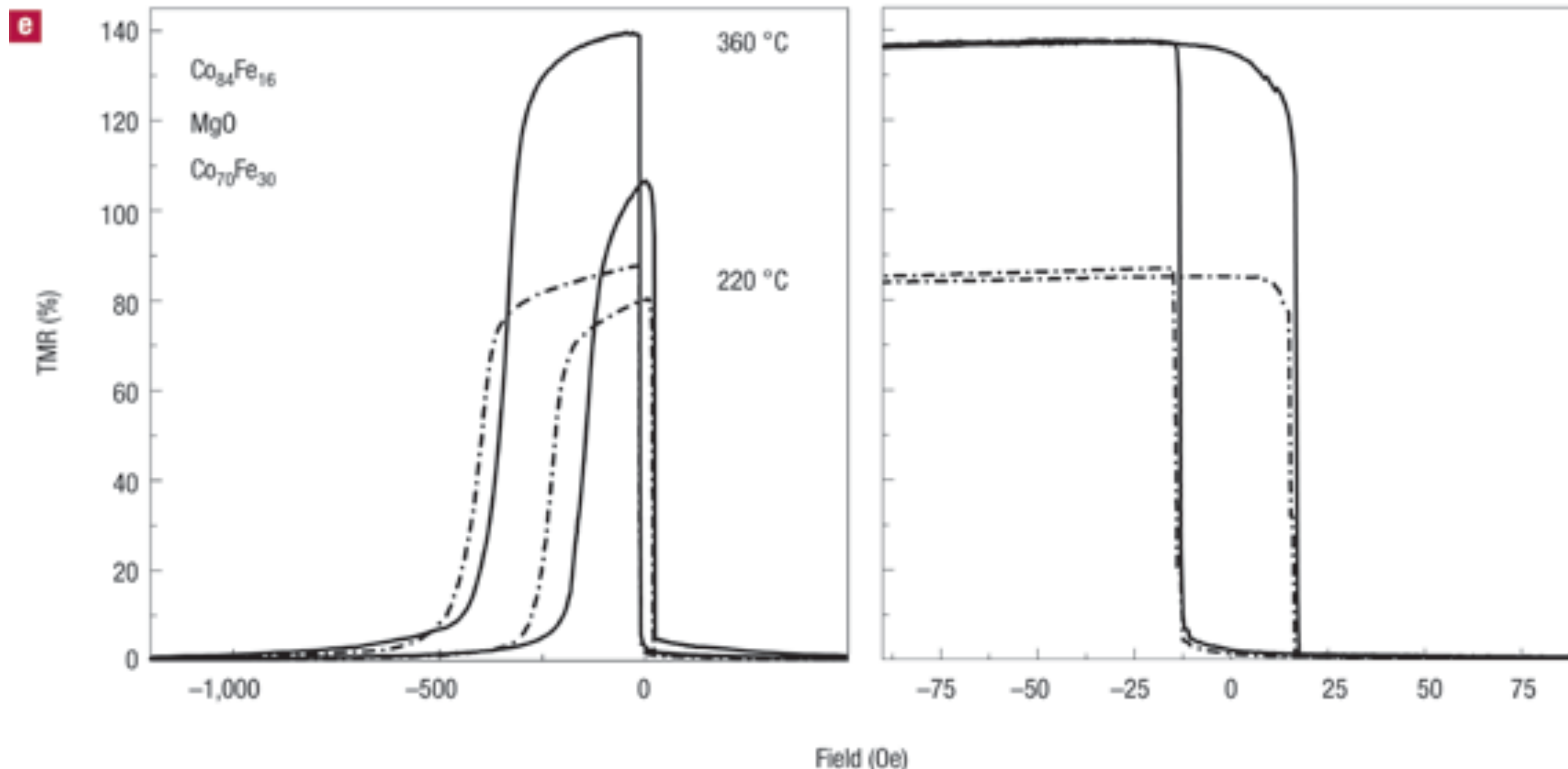
Work out the difference in conductance between parallel and antiparallel magnetization configurations



Popular model used for interpreting early experiments but this neglects many important factors. What are they?

# More sophisticated modeling of TMR

Full understanding of TMR requires calculation of the quantum mechanical matrix elements from first principles, using Bloch wave functions of the specific materials involved, interfaces, spin non-conserving processes, etc. Predictions based on first principles led to the experimental configurations with very large TMR currently in use in actual devices (e.g. CoFe/MgO/CoFe)



See Zhang & Butler,  
Spintronics Handbook  
(Springer 2016)

Parkin *et al*, *Nature Mater.* **3**, 862 (2004).

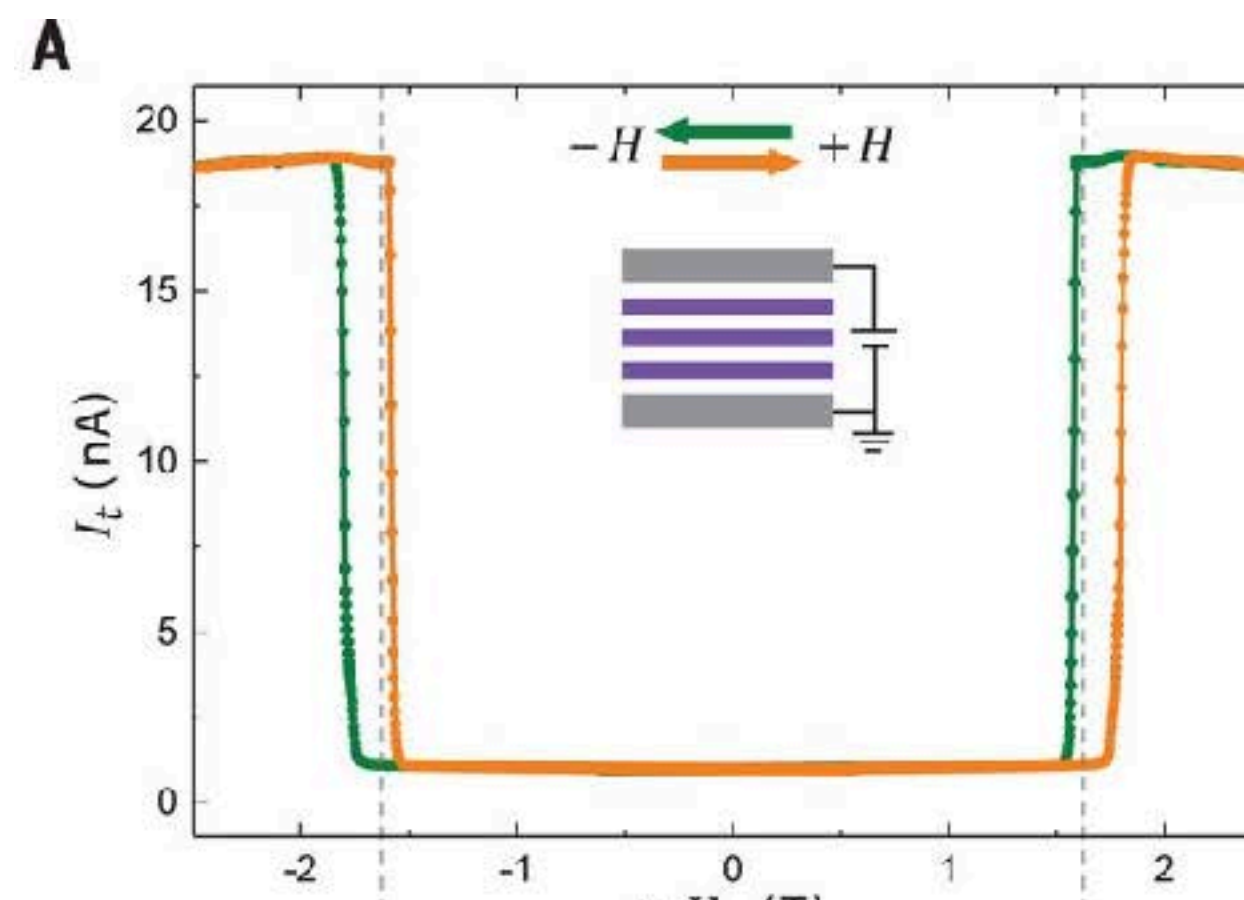
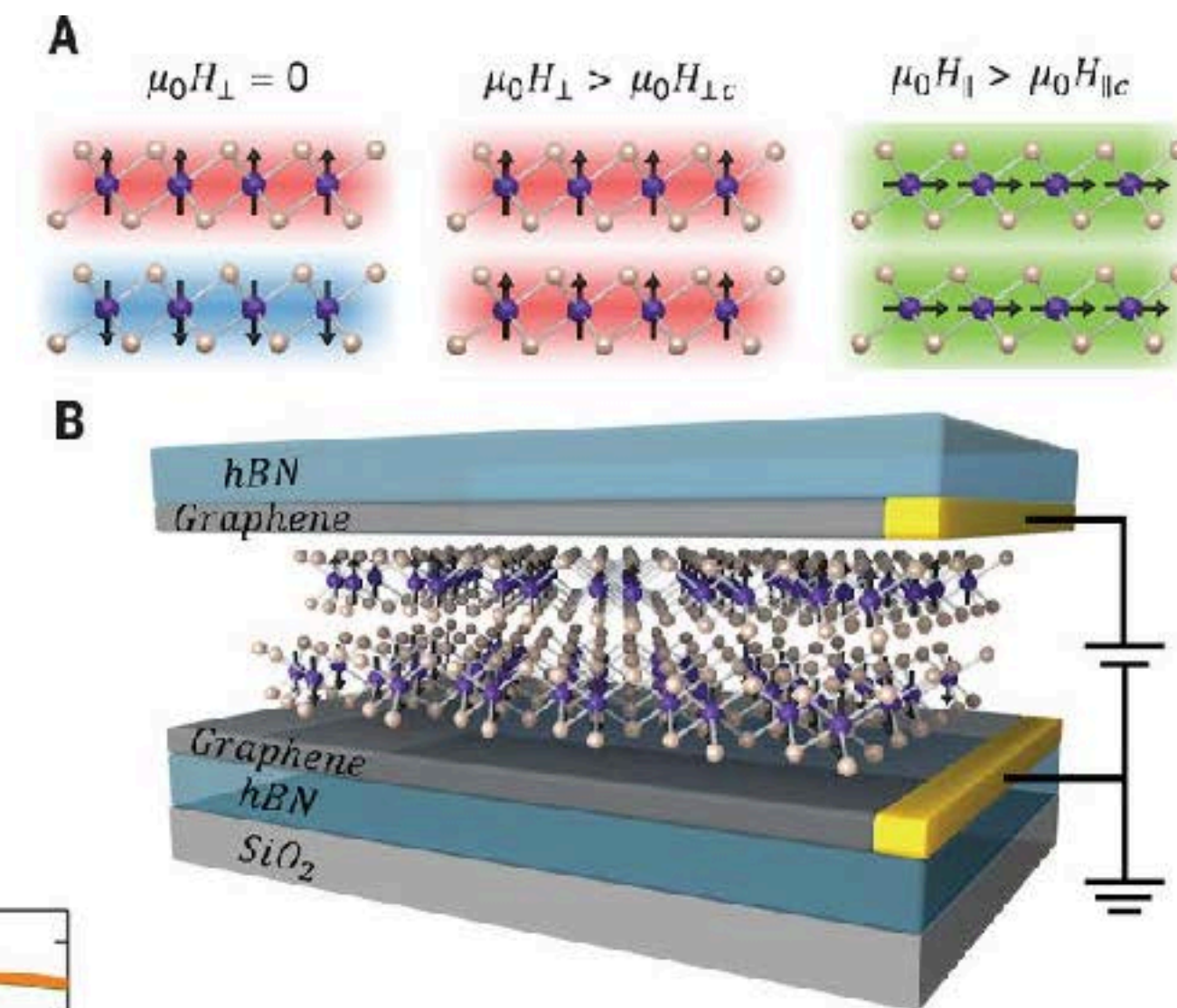


# Spintronics beyond GMR & TMR in metallic multilayers and tunnel junctions: a vast field

- Spin-based magnetic random access memory (MRAM).
- Spin as a state variable in “beyond CMOS” logic switches & architecture.
- Spin as a ‘qubit’ for quantum computing or sensing.
- THz spintronics with antiferromagnets.

## Materials for spintronics beyond FM metals

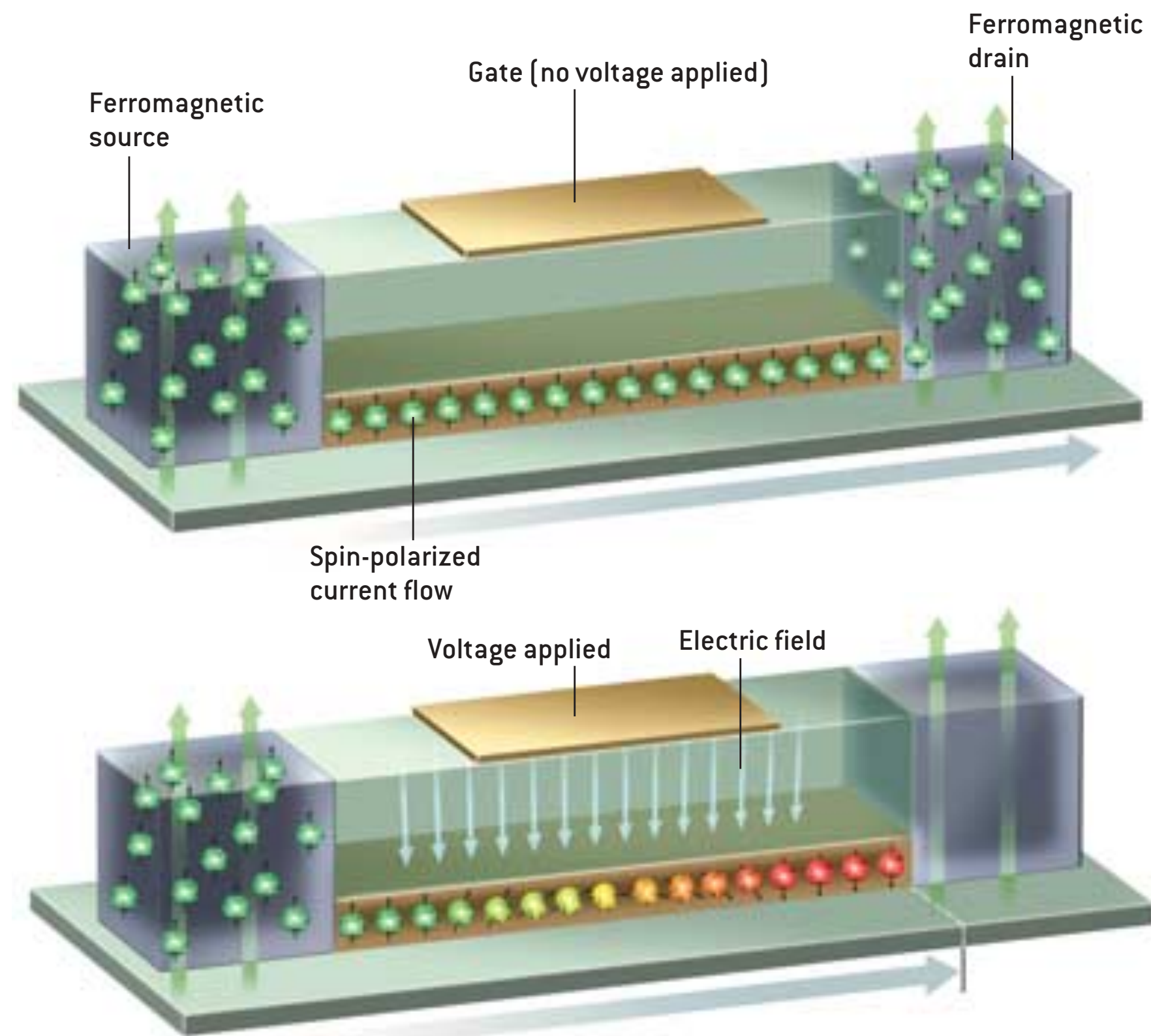
- Semiconductors & magnetic semiconductors
- Antiferromagnetic/ferrimagnetic/ferromagnetic insulators
- Topological quantum materials
- Organic materials
- 2D materials
- Oxides...



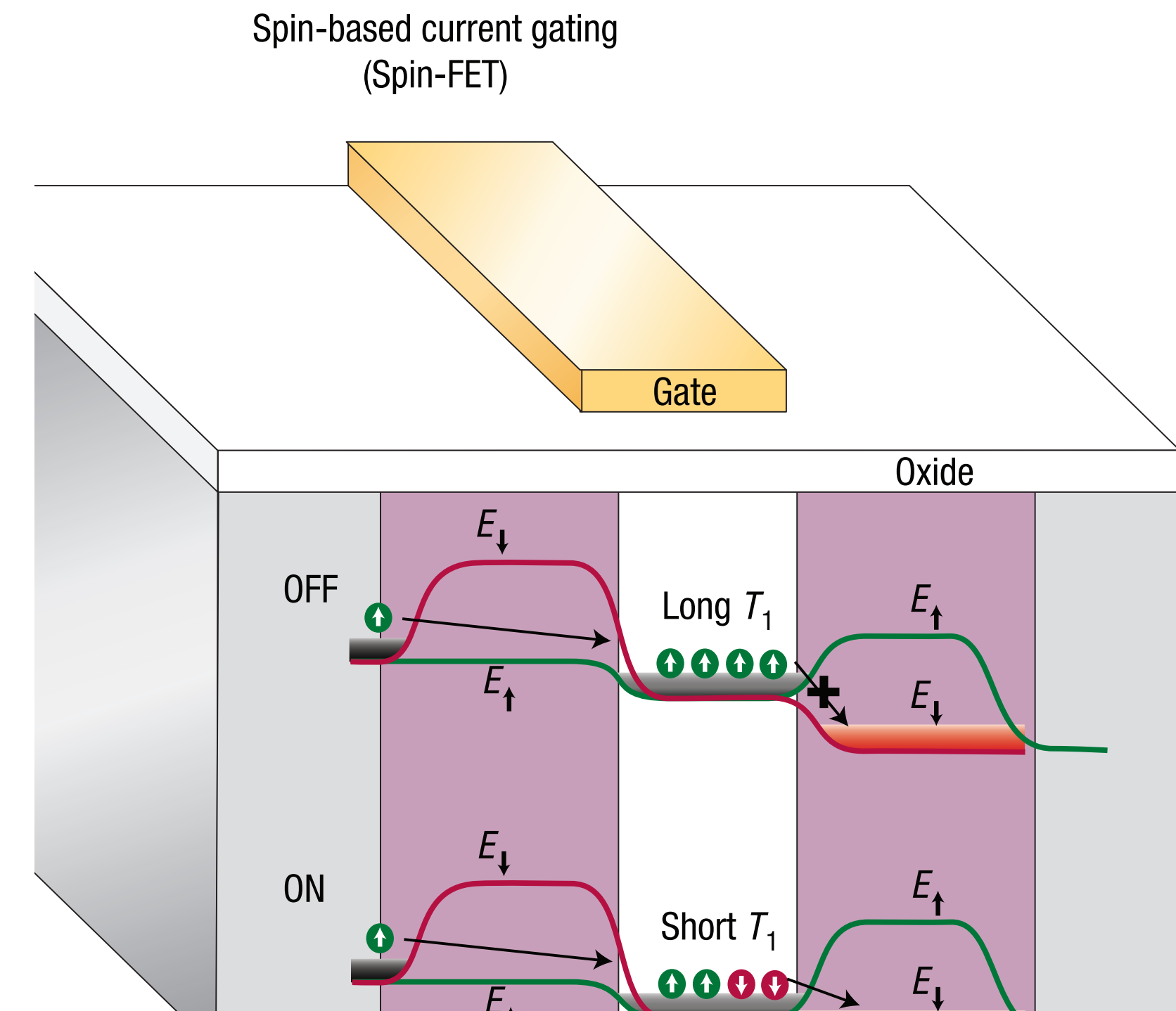
Song *et al.*,  
*Science* **360**,  
1214 (2018)



# Semiconductor spintronics: spin FETs



Datta & Das, APL **56**, 665 (1990)

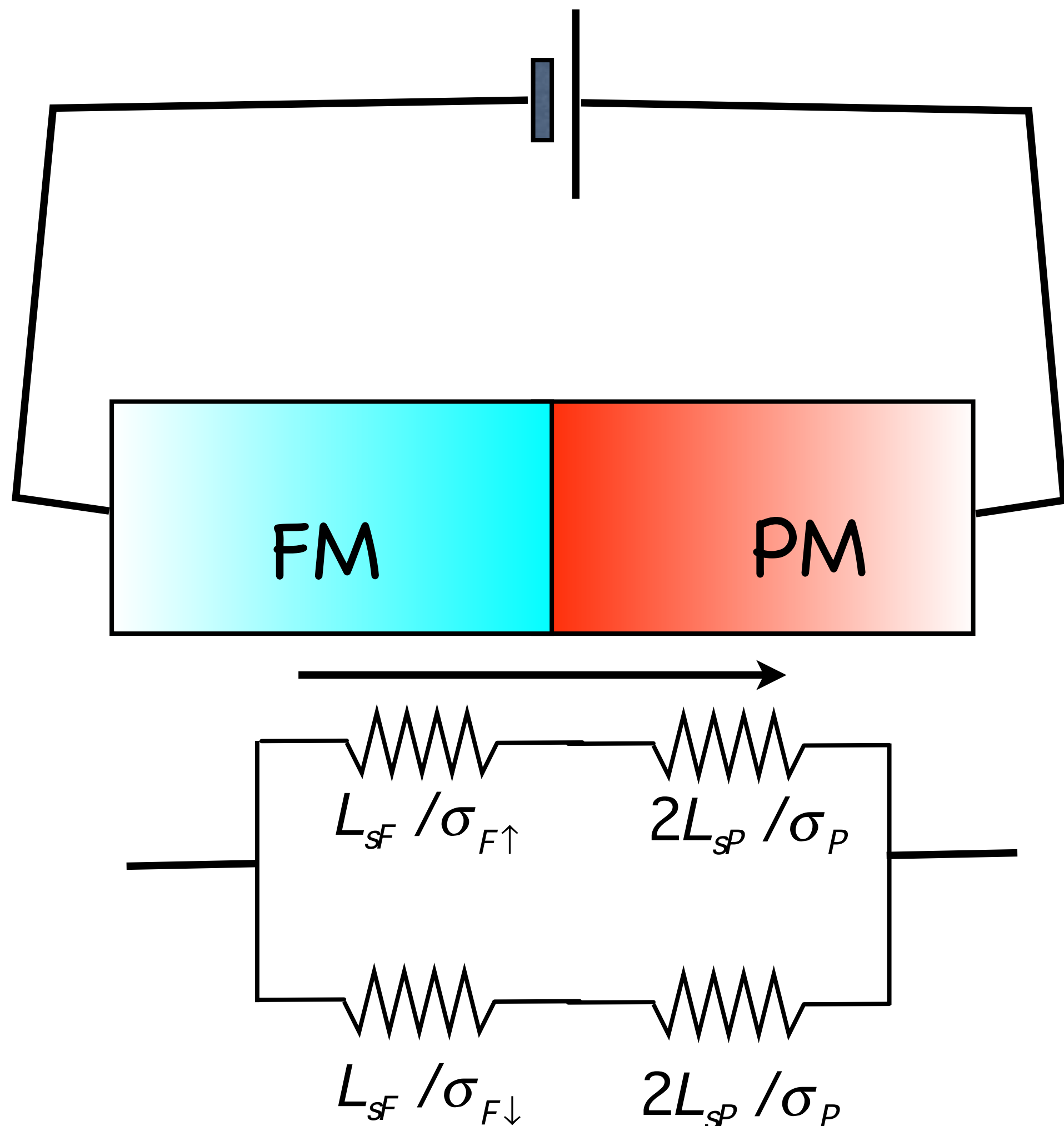


Hall & Flatte, APL 88, 162503 (2006)

Spin-based transistor switches have been of interest since the first concept described by Datta & Das (“Electronic analog of the electro-optic modulator”). Many new ideas since but few realizations that are technologically competitive to CMOS.

# Semiconductor spintronics: spin injection

Spin-dependent injection of electrons from a ferromagnetic conductor (equilibrium spin polarization =  $P_0$ ) into a paramagnetic conductor (equilibrium spin polarization = 0).



The physics turns out to be surprising: using a drift-diffusion analysis, we can show that if the (spin-independent) conductivity of the FM is much larger than that of the PM, spin injection is almost impossible!

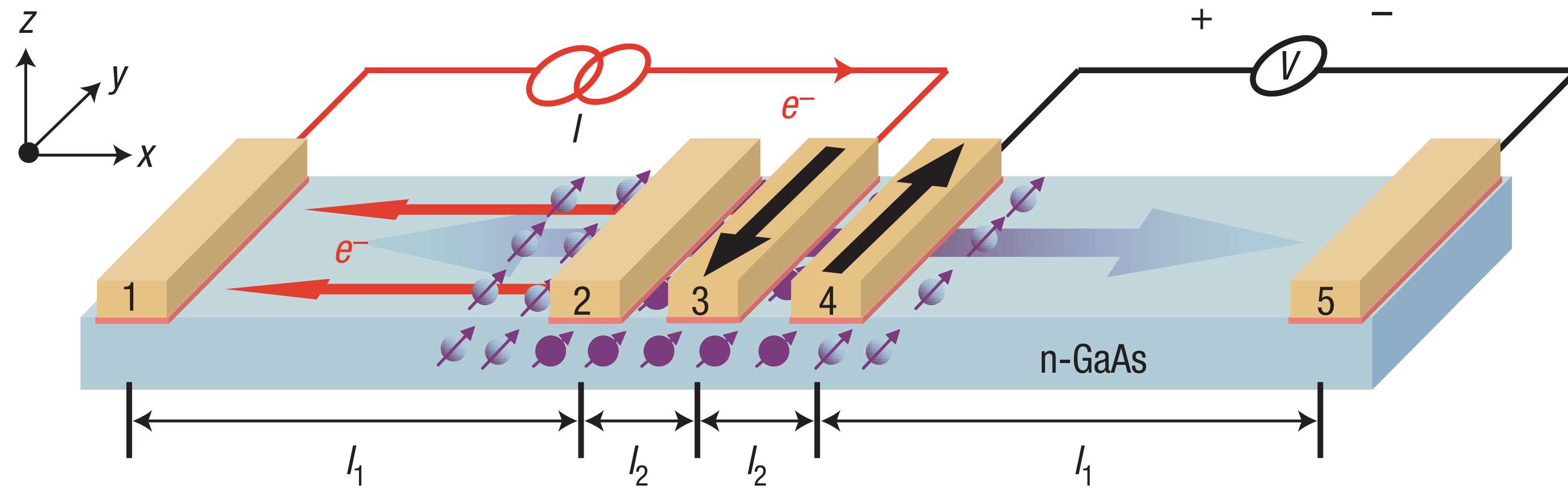
This is called the “conductivity mismatch problem” (Schmidt *et al.*, *Phys. Rev. B* **62** (2000))

$$P_j = \frac{R_F^* \left( \frac{\sigma_{sF}}{\sigma_F} \right) + R_C \left( \frac{\Sigma_s}{\Sigma} \right)}{R_F^* + R_P^* + R_C}$$

How do we solve this problem?



# Semiconductor spintronics: spin transport

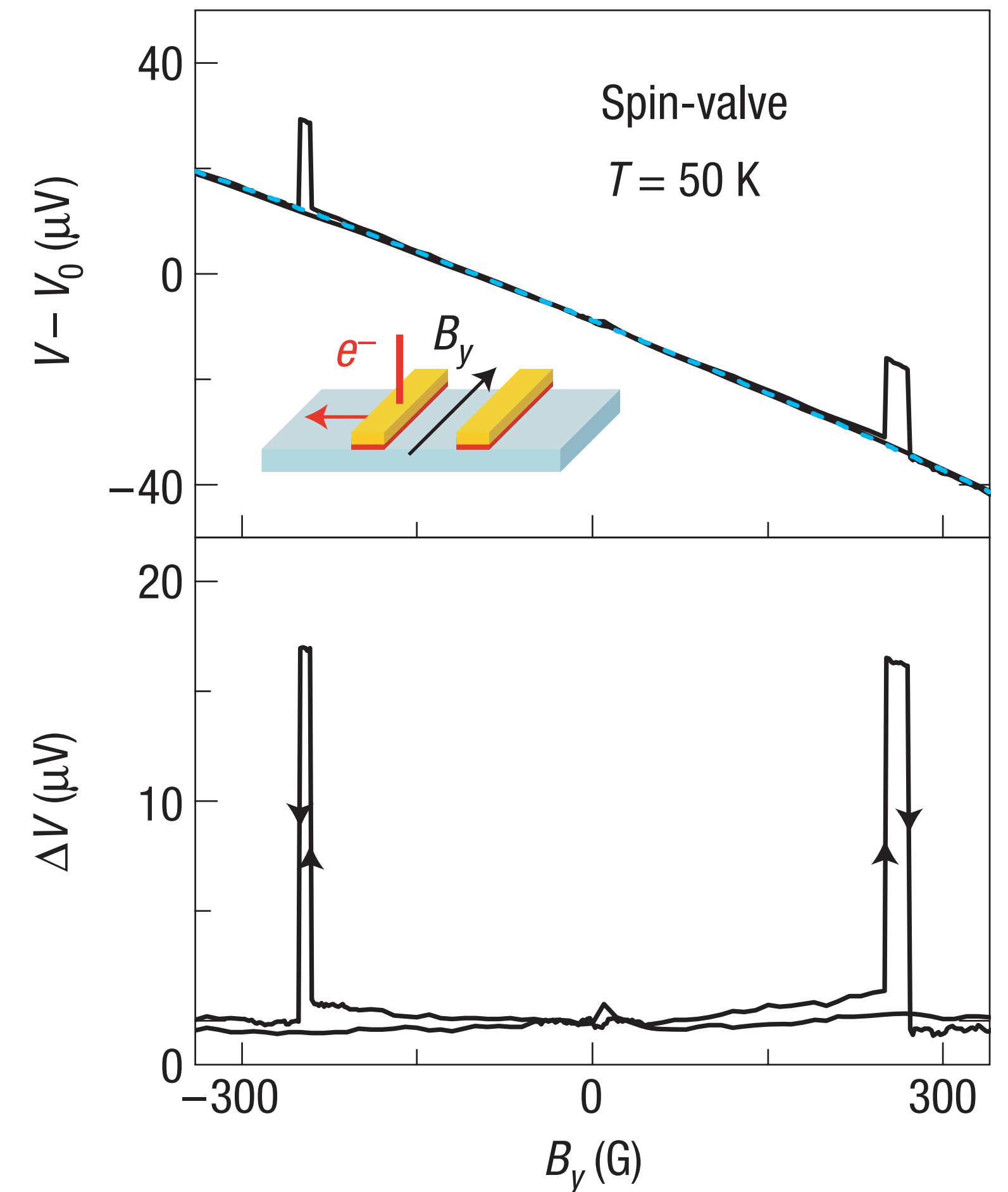


Ferromagnetic contacts can source spin polarized current in a semiconductor and detect it via a 'spin valve' effect.

Spin diffusion lengths can be long ( $\sim 100 \mu\text{m}$ ).

Cautions:

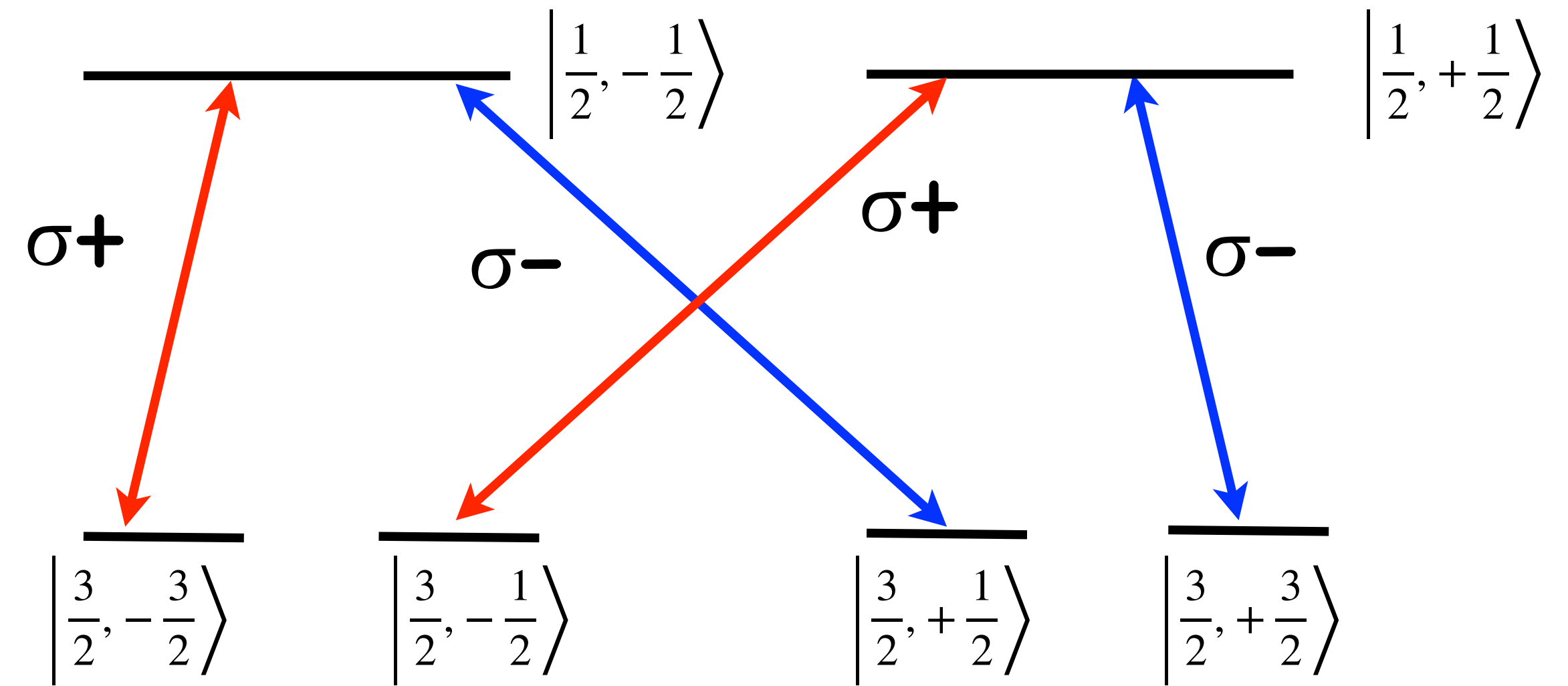
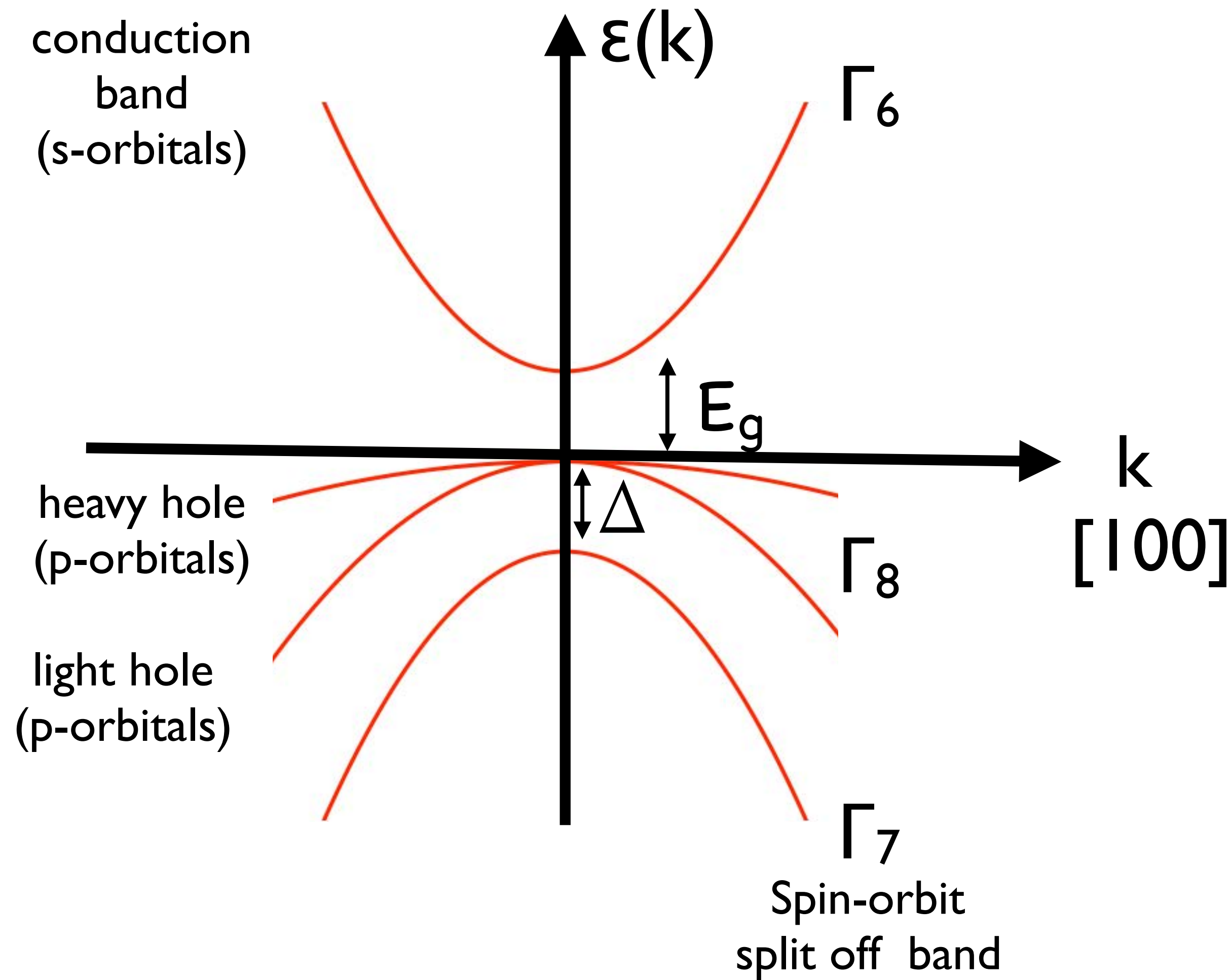
1. Transparent contacts do not work as efficient spin injectors! We need a tunnel barrier!
2. **Non-local detection** scheme needed to avoid artifacts from fringe fields.



Lou *et al.*, *Nature Physics* **3**, 197 (2007)

$$P_j = \frac{j_s}{j} = \frac{\sigma_s}{\sigma} + \frac{4}{j} \nabla \mu_s \frac{\sigma_{\uparrow} \sigma_{\downarrow}}{\sigma}$$

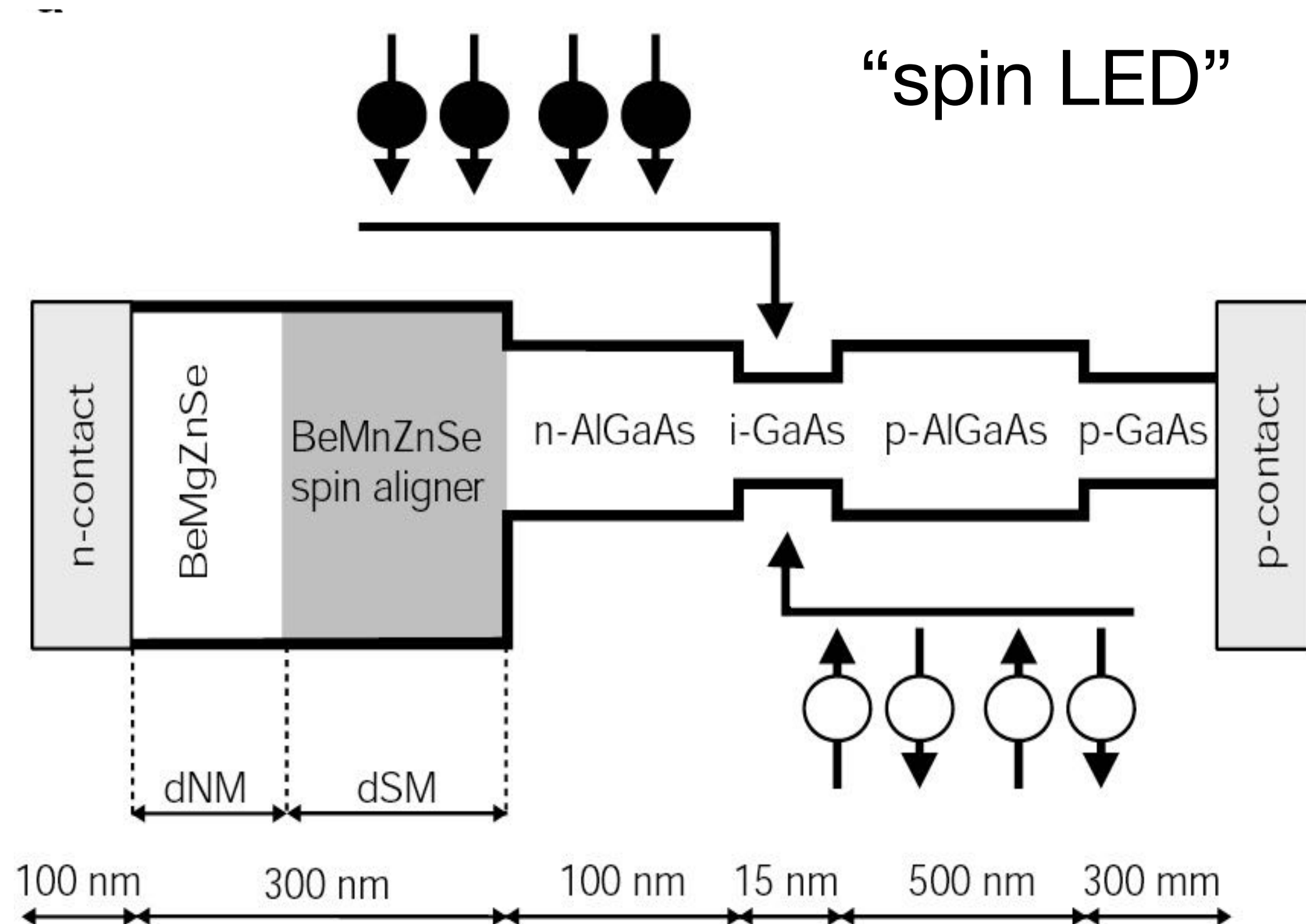
# Semiconductor spintronics: photons & spins



Early experiments in semiconductor spintronics used circularly polarized light to couple to spins in conduction and valence band states of semiconductors.

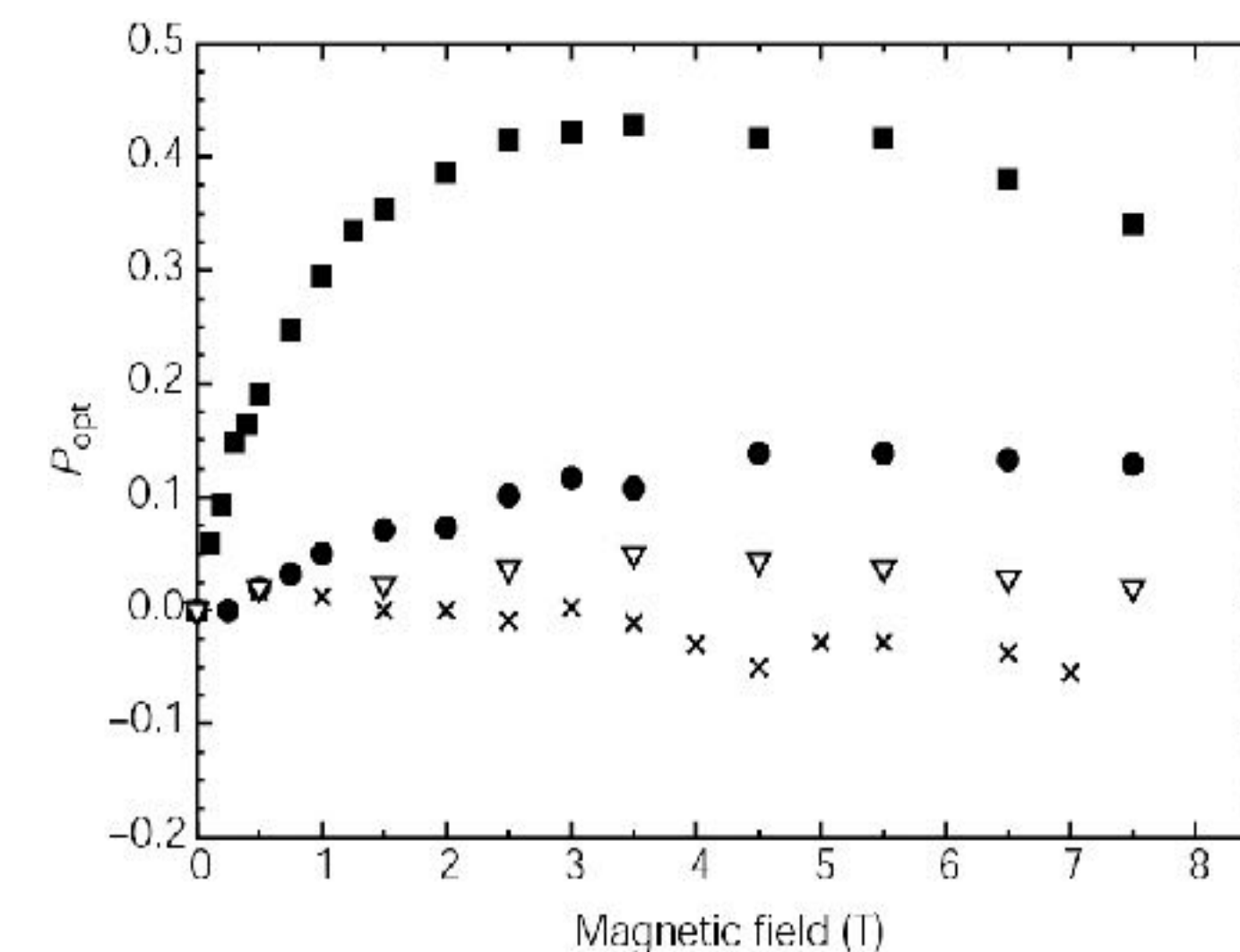
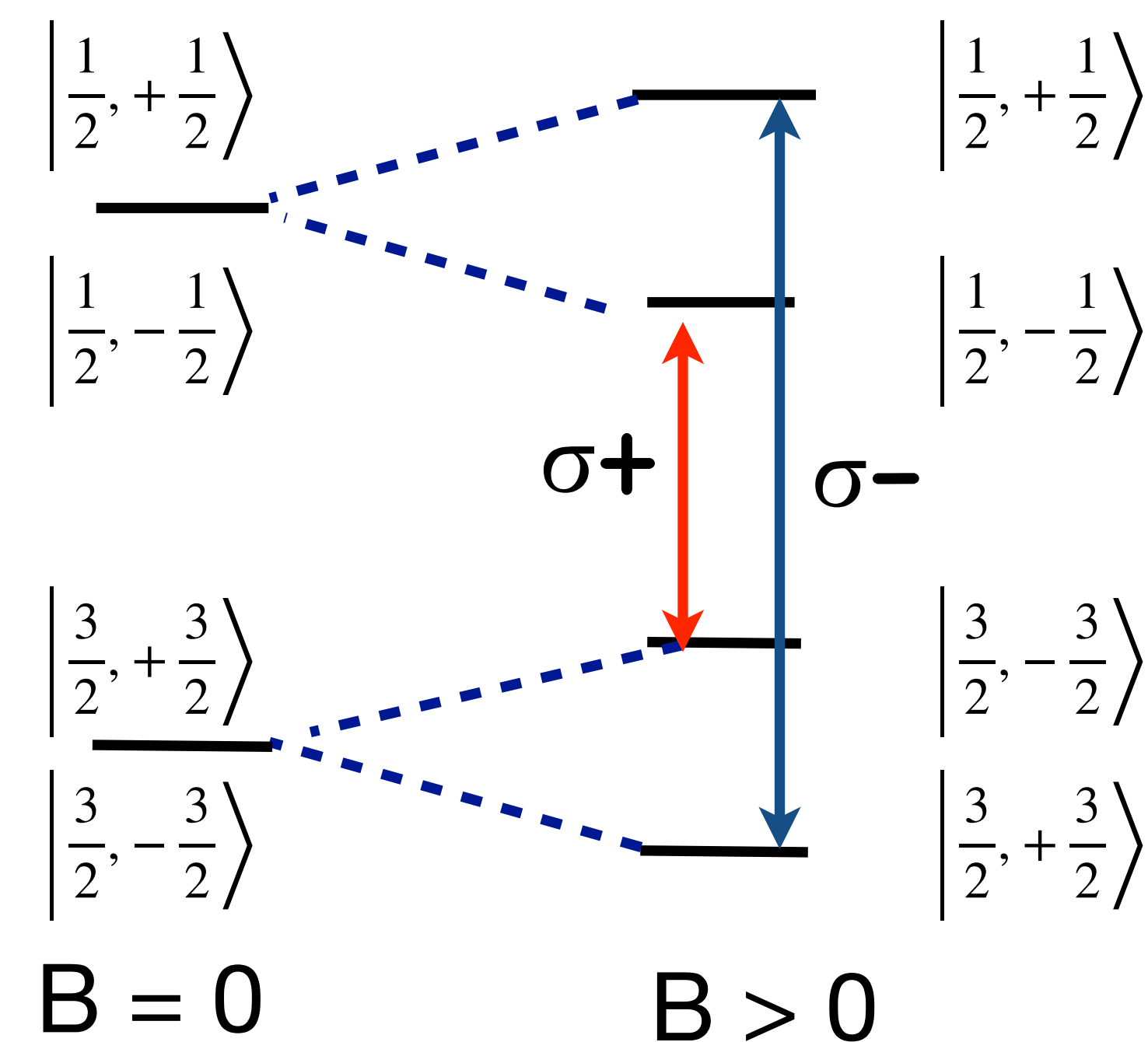


# Semiconductor spintronics: spin-photon coupling

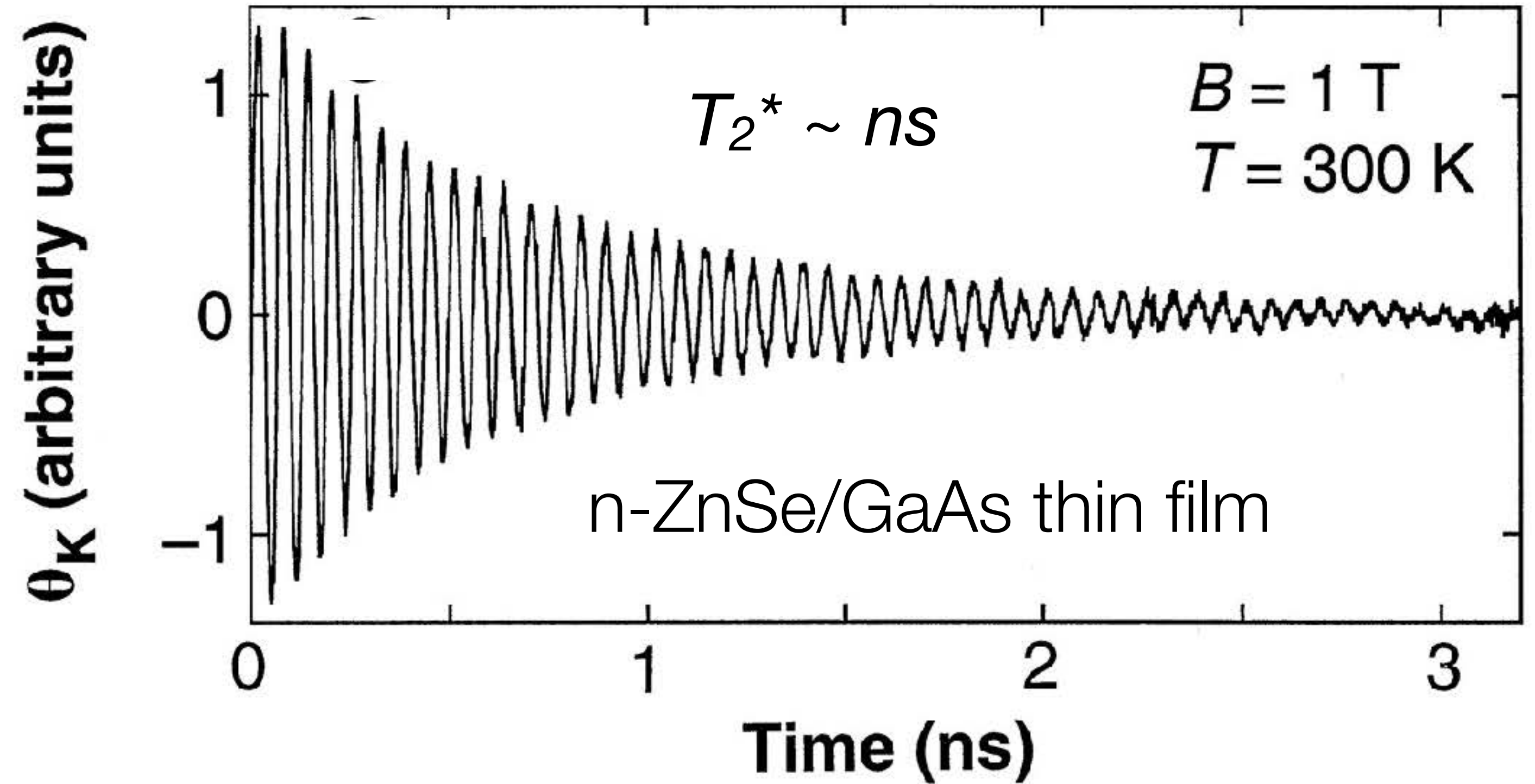
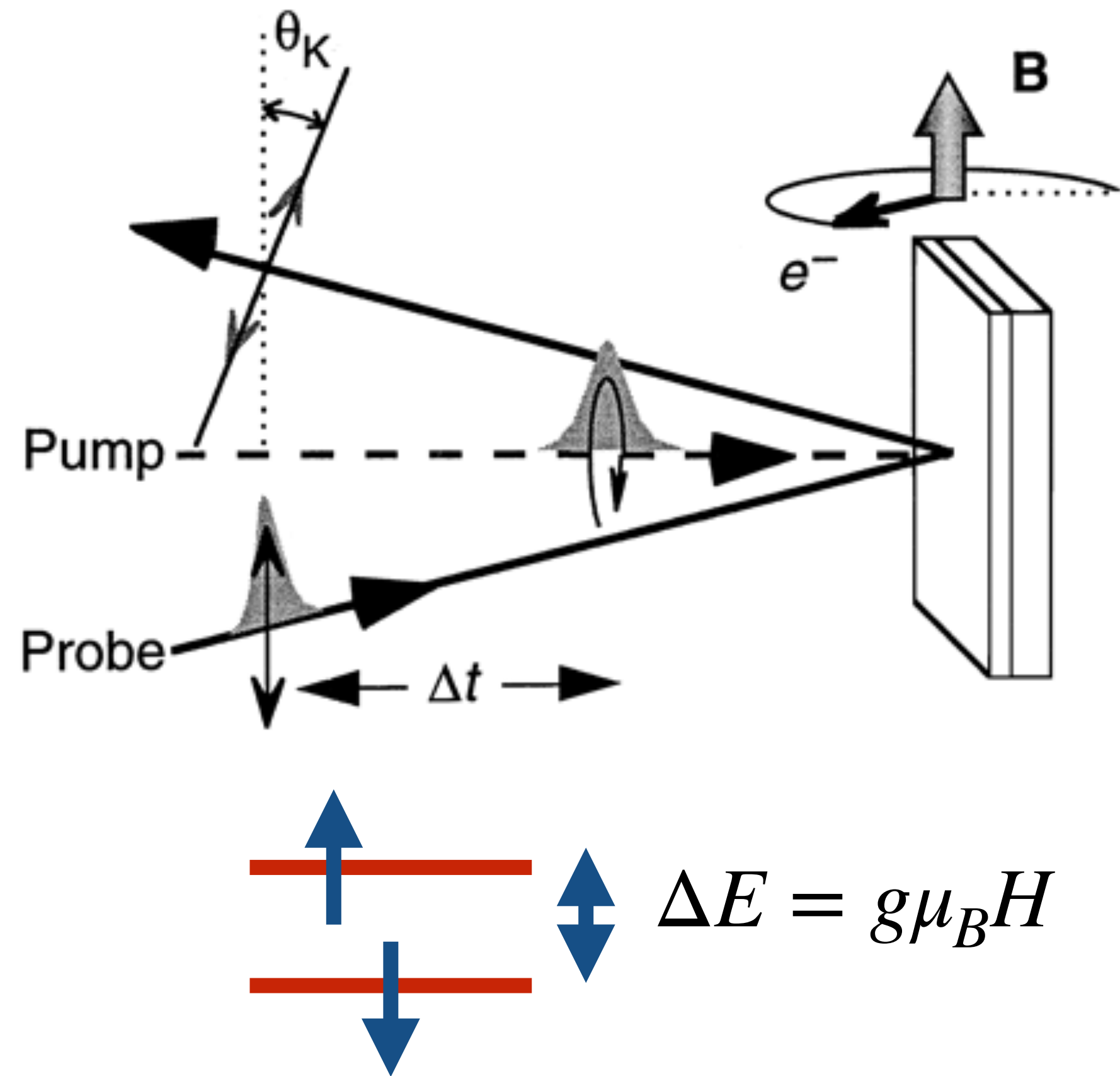


Recombination of spin polarized electrons and unpolarized holes leads to circular polarized photon emission: spin LED

Fiederling *et al.*, *Nature* **402**, 787 (1999); Ohno *et al.*, *ibid* 790 (1999)



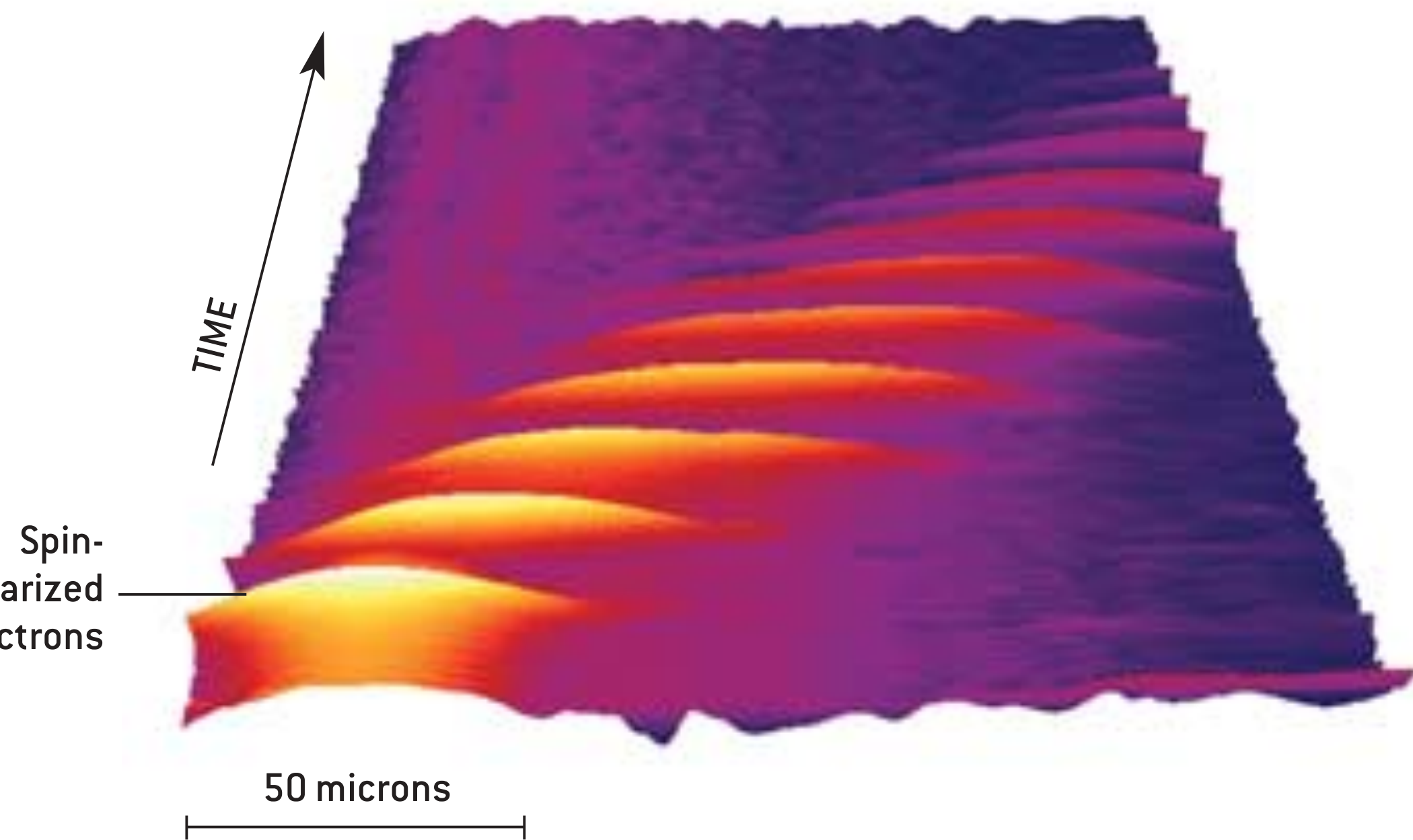
# Semiconductor spintronics: spin coherence



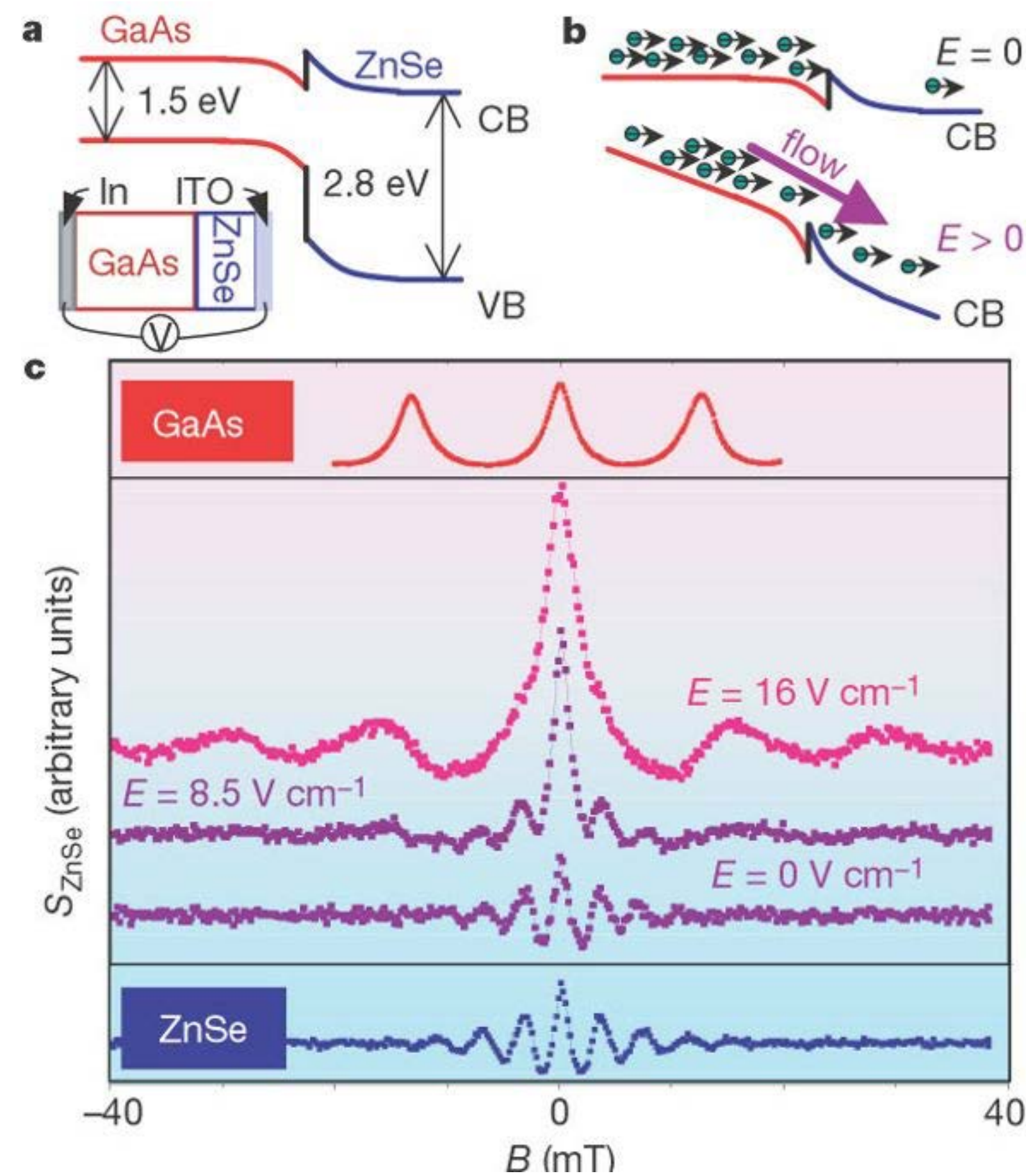
Time-resolved Kerr rotation probed spin coherence in n-doped semiconductors and discovered relatively long spin inhomogeneous dephasing times ( $\sim ns$  at 300 K).



# Semiconductor spintronics: spin coherence



Kikkawa & Awschalom, *Nature* **397**, 139 (1999)



Malajovich, Berry,  
Samarth &  
Awschalom, *Nature*  
**411**, 770 (2001)

Time- and spatially-resolved Kerr rotation probed coherent spin transport in n-doped laterally and across interfaces over macroscopic lengths ( $\sim 100 \mu\text{m}$  at 300 K).



# Semiconductor spintronics: spin qubits

Long spin coherence times in semiconductors: motivation for using spin in semiconductors as a qubit.

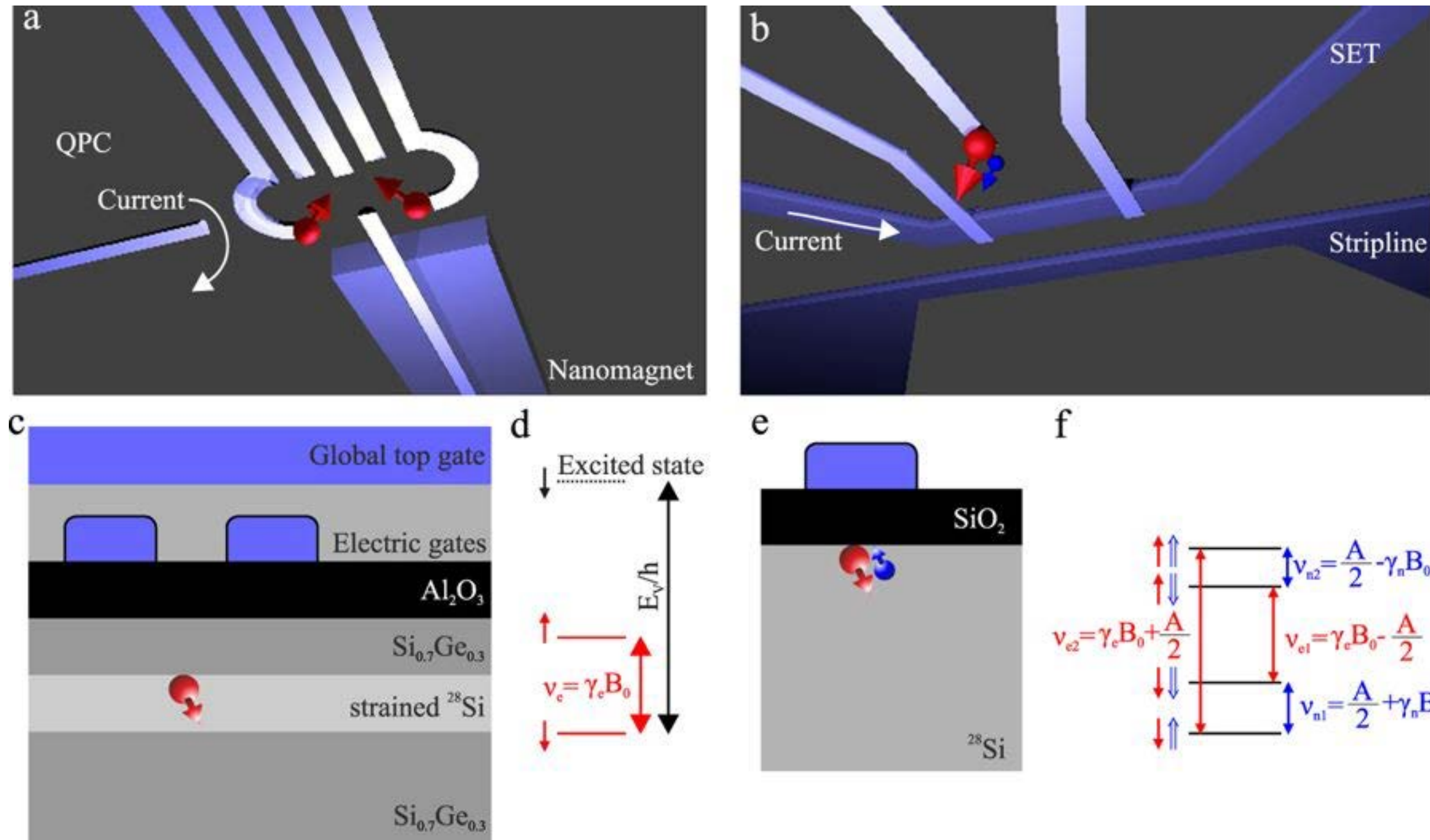
Trap, probe, and manipulate spin state of a single electron in semiconductor nanostructures.

Quantum dot in an electrically gated 2DEG [Si/SiGe, GaAs/GaAlAs].

Single donor atom coupled to an electrostatic gate [P-doped Si].

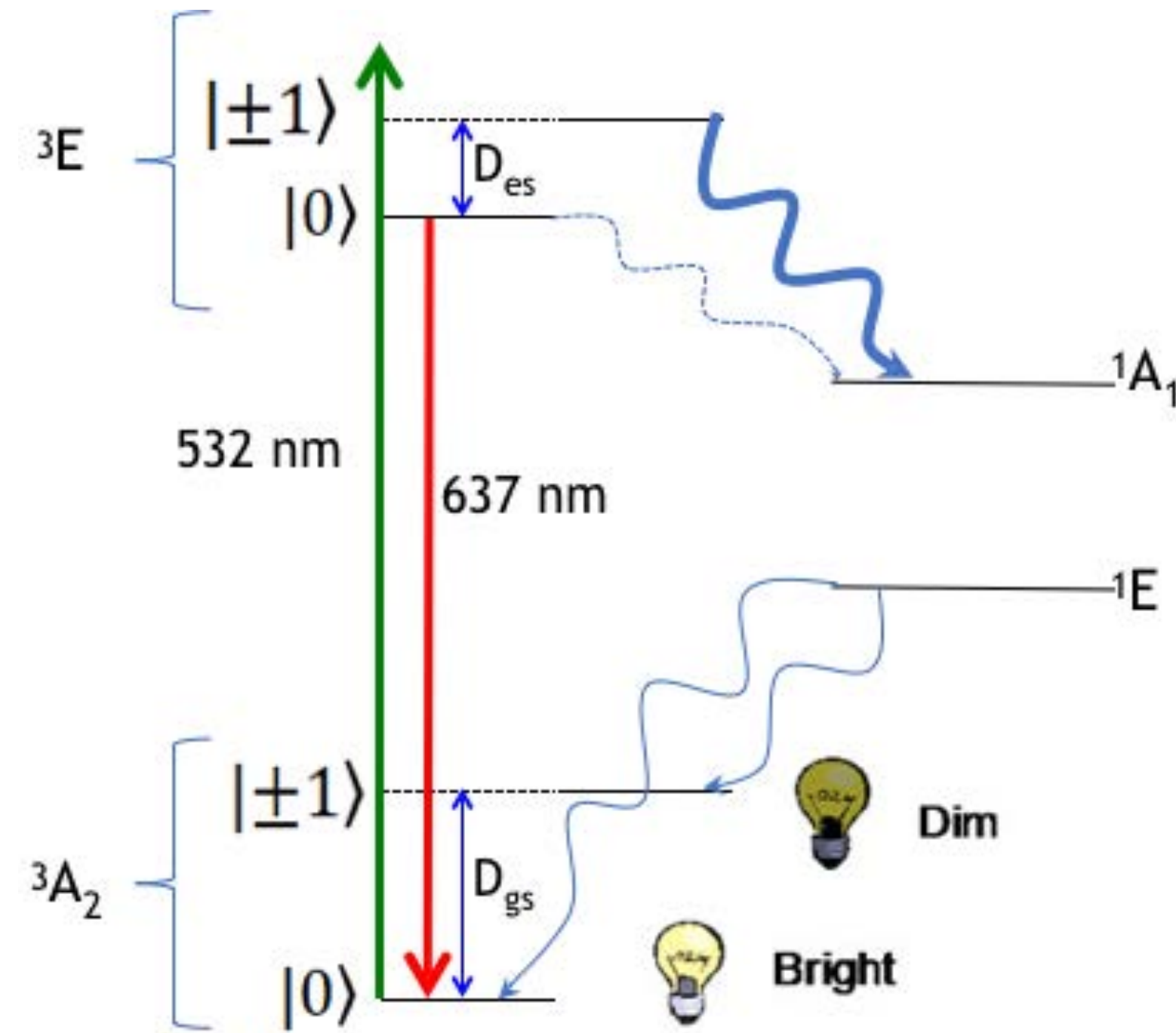
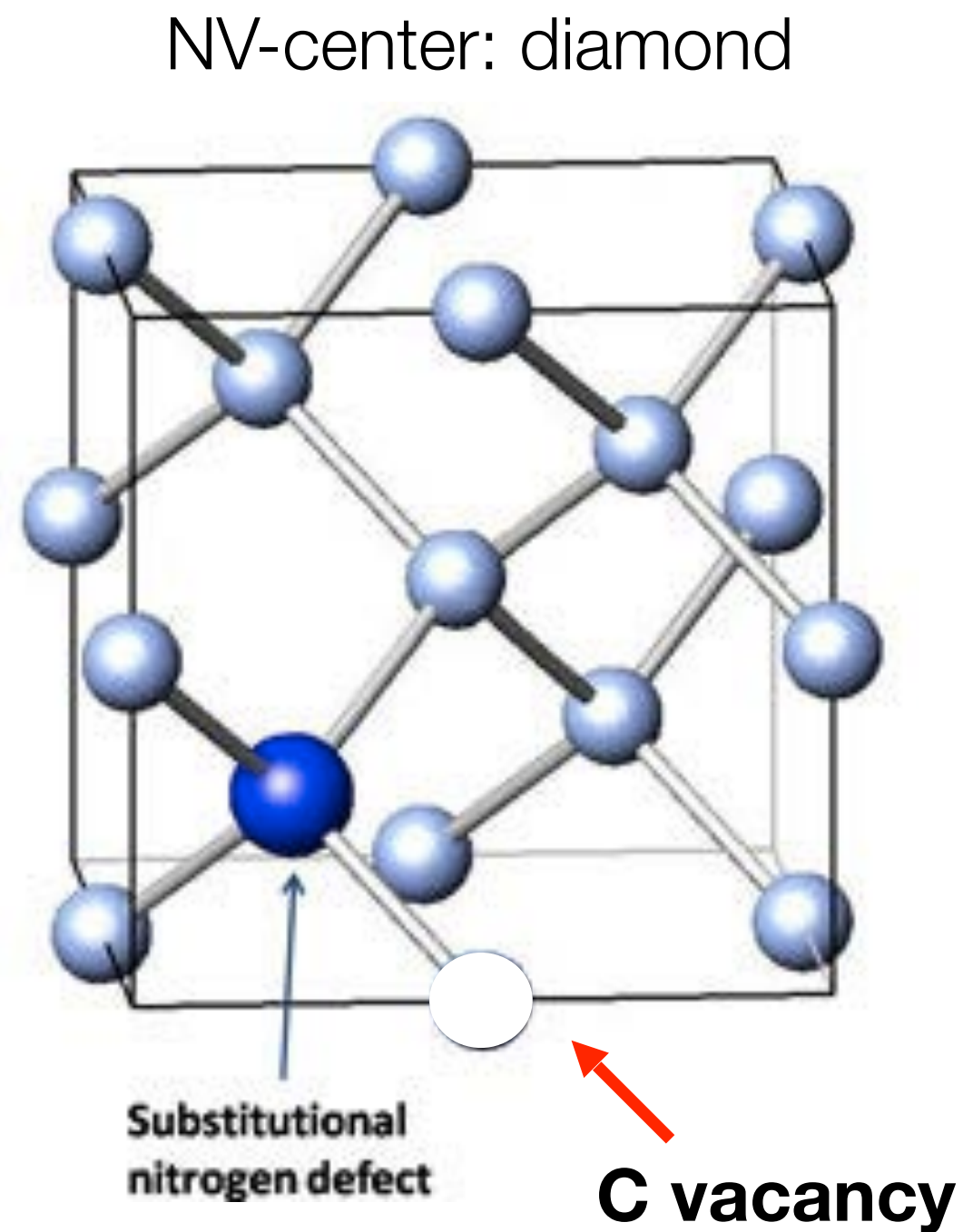
The two level Zeeman-split spin states of these single confined electrons define the qubit.

Spin dephasing times in semiconductor qubits observed up to 200  $\mu\text{s}$ .





# Semiconductor spintronics: defect spin qubits



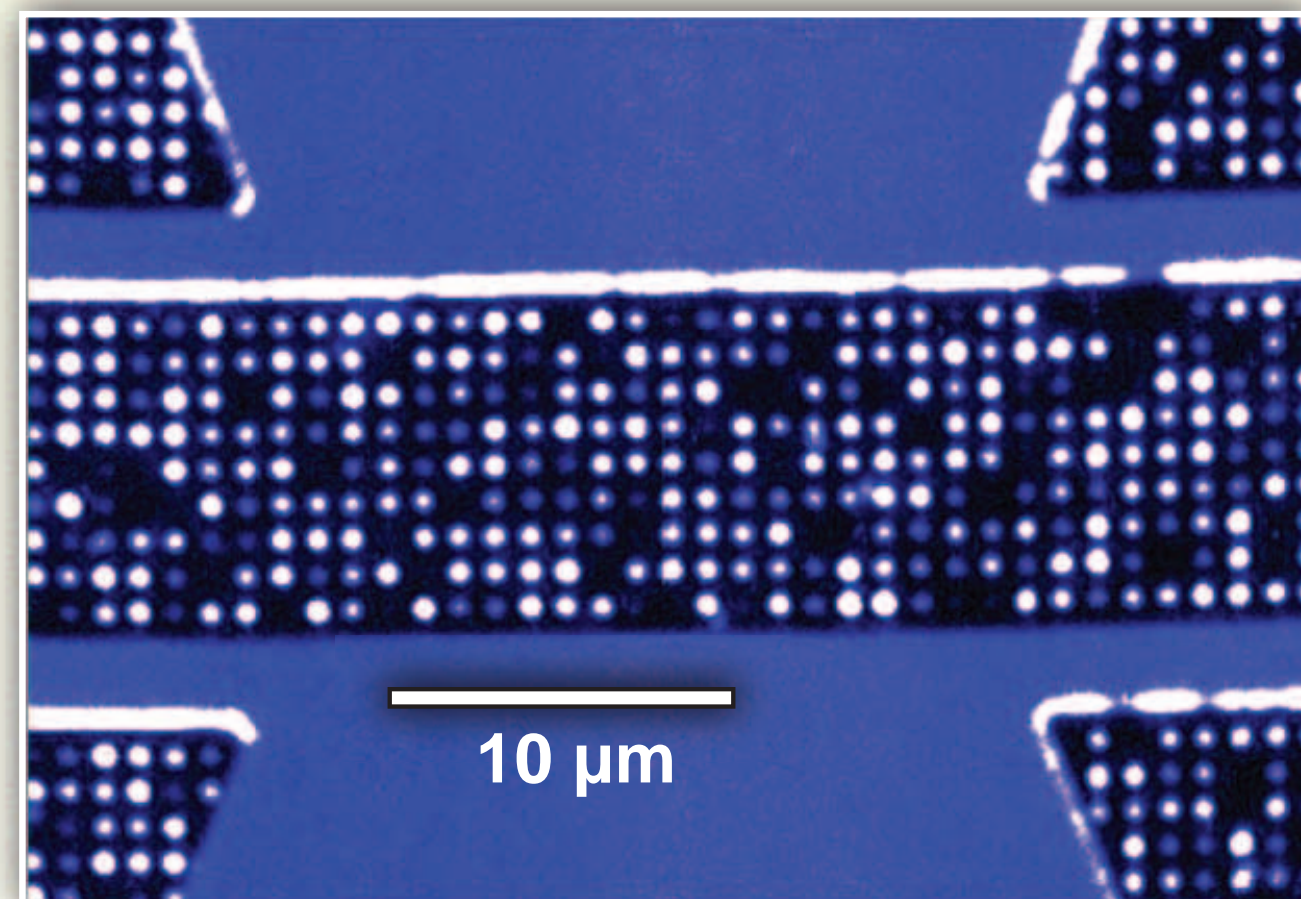
Defect states in some materials act as single spin qubits

Nitrogen-vacancy (NV) center in diamond

Si divacancies in SiC

Spin coherence times can be very long ( $\mu$ s) even at room temperature

Spin states can be coherently manipulated and addressed using microwaves and IR - visible photons.

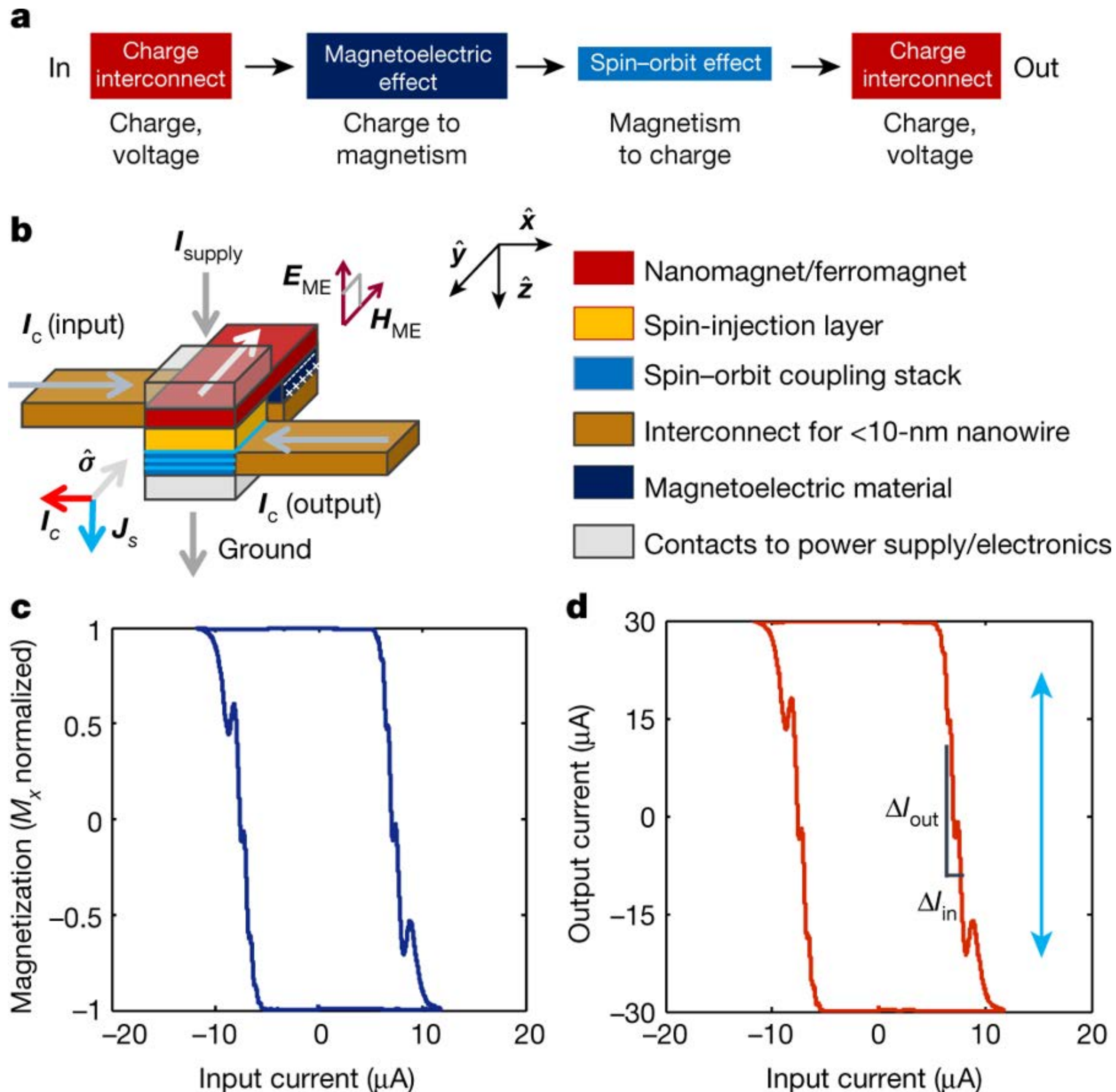


Chip scale nano fabrication of single spin arrays in diamond.  
D. M. Toyli *et al.*, *Nano Lett.* **10**, 3168 (2010).

See lectures by A. Jayich



# Spintronics: new ideas for spin-based logic

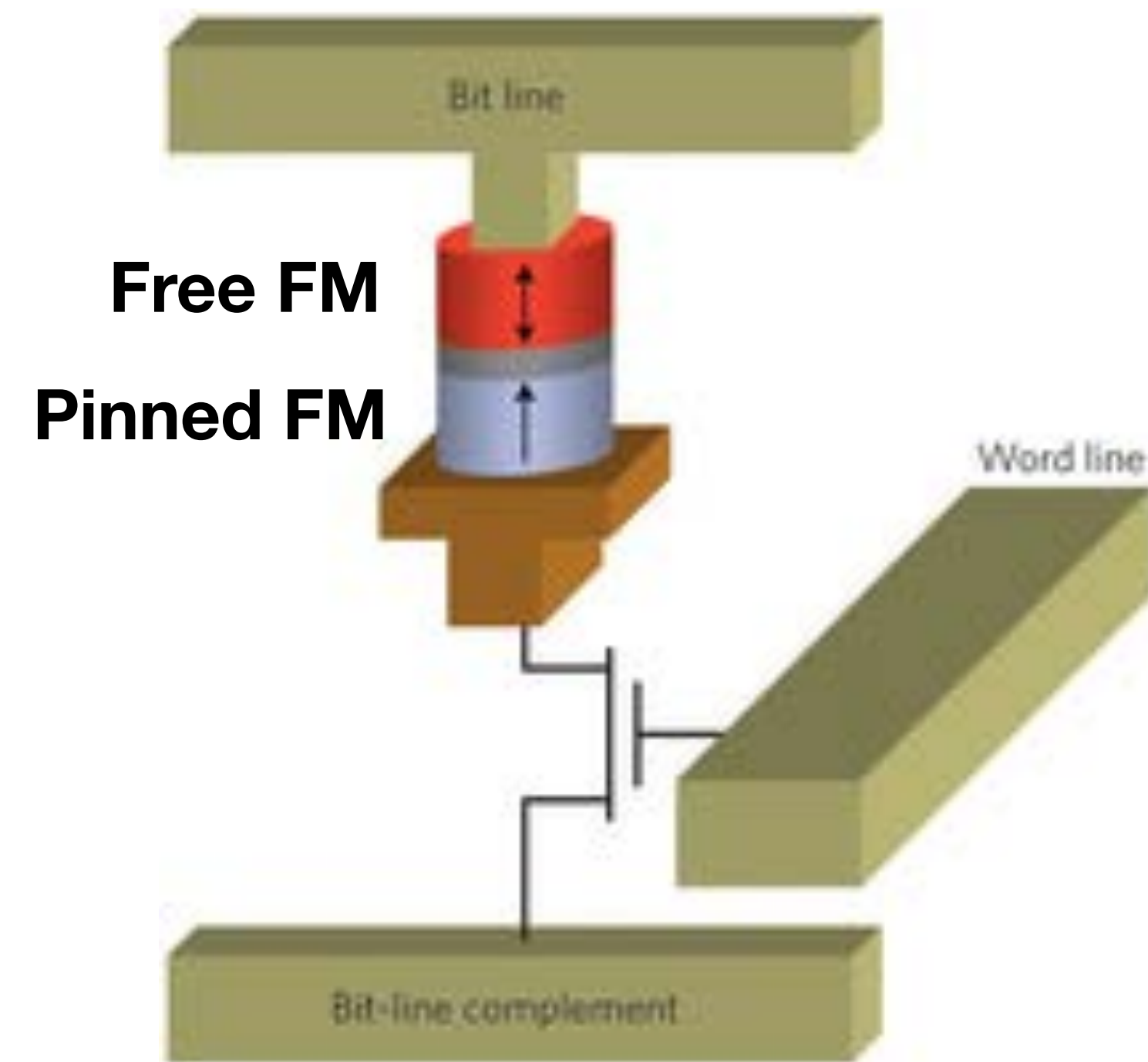


- Intel: proposal for a magneto electronic spin orbit (MESO) logic device that combines magnetolectric switching of a ferromagnet with spin-orbit torque detection of the device state.
- Compared with current CMOS technology, this scheme is predicted to have better performance: switching energy lower by a factor of 10 to 30, lower switching voltage by a factor of 5 and enhanced logic density by a factor of 5.
- Non-volatility enables ultralow standby power.



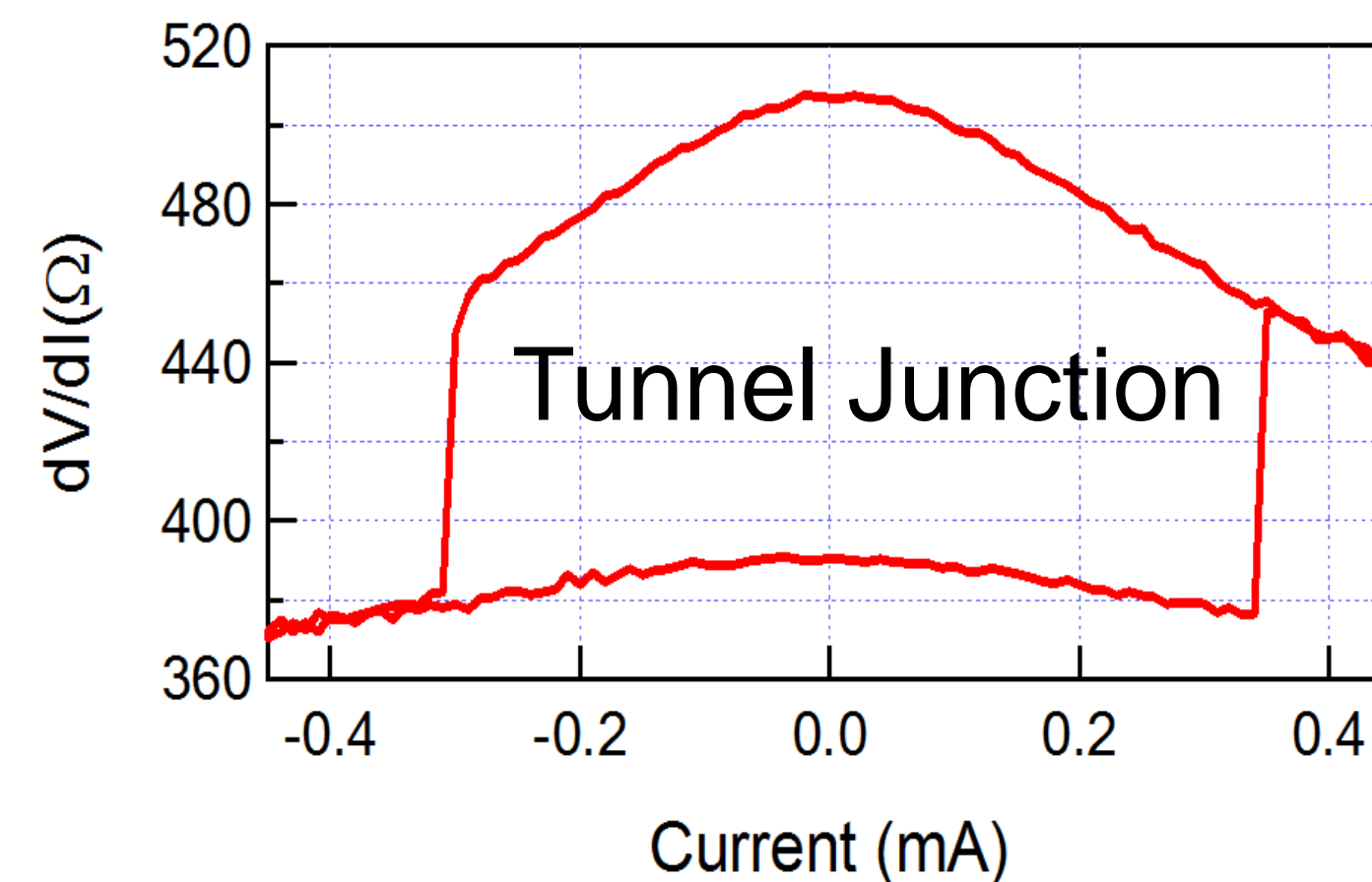
# Spintronics for universal memory

- MRAM: bits consist of arrays of magnetic tunnel junctions.
- Write: “spin-transfer torque”; read: TMR.
- Density competitive with DRAM ( $< 20$  nm), speed with FLASH ( $\sim 10$  ns), possibly with SRAM ( $\sim 2$  ns projected).
- Non-volatile, little wear out (at low write voltage), low standby power.
- Challenges: reduce the “write” current density ( $< 20\mu\text{A}$ ), “write” voltage  $\sim 400$  mV, industry compatible processing...

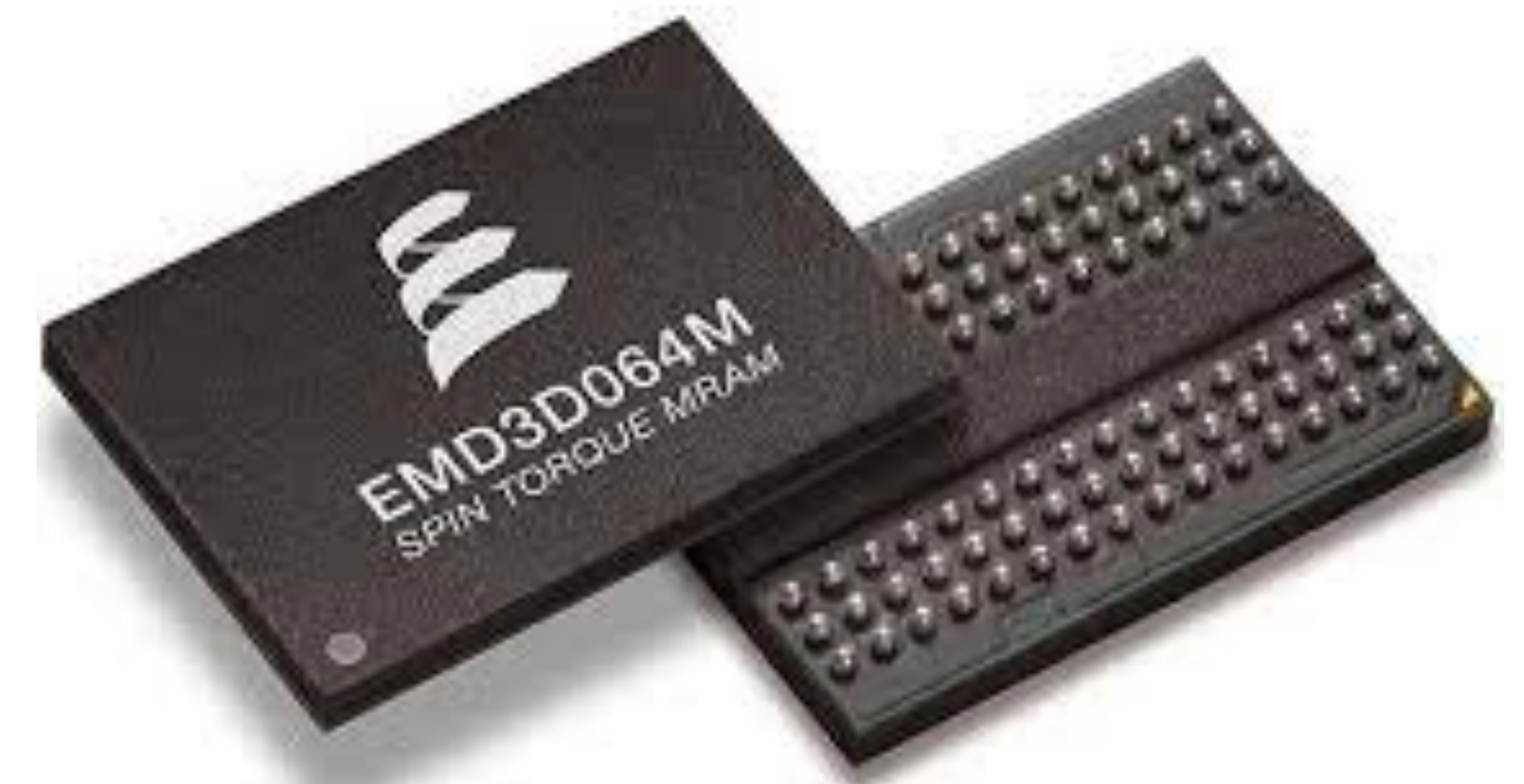


Nature Nano 10, 187 (2015)

See lecture by J. Sun



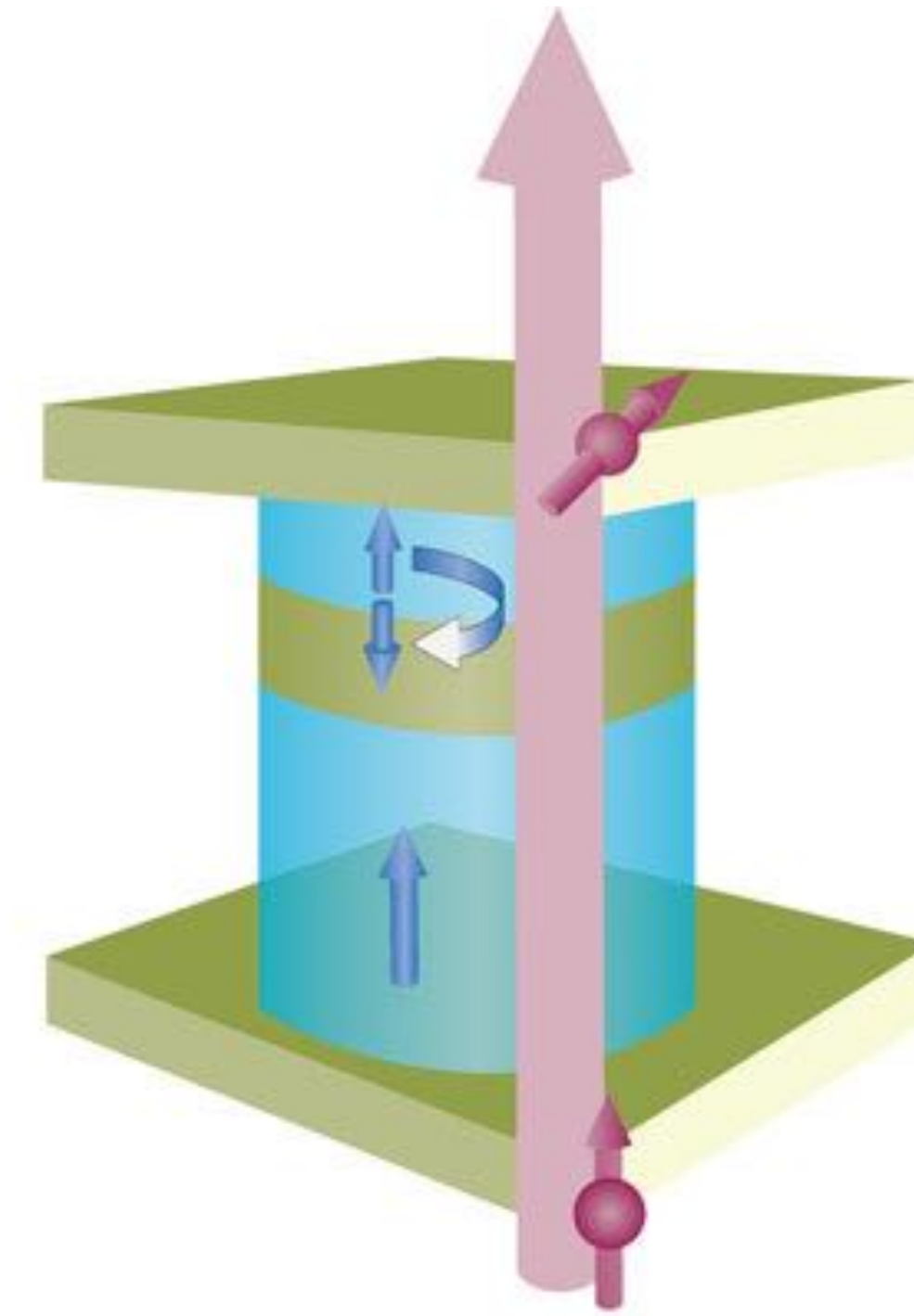
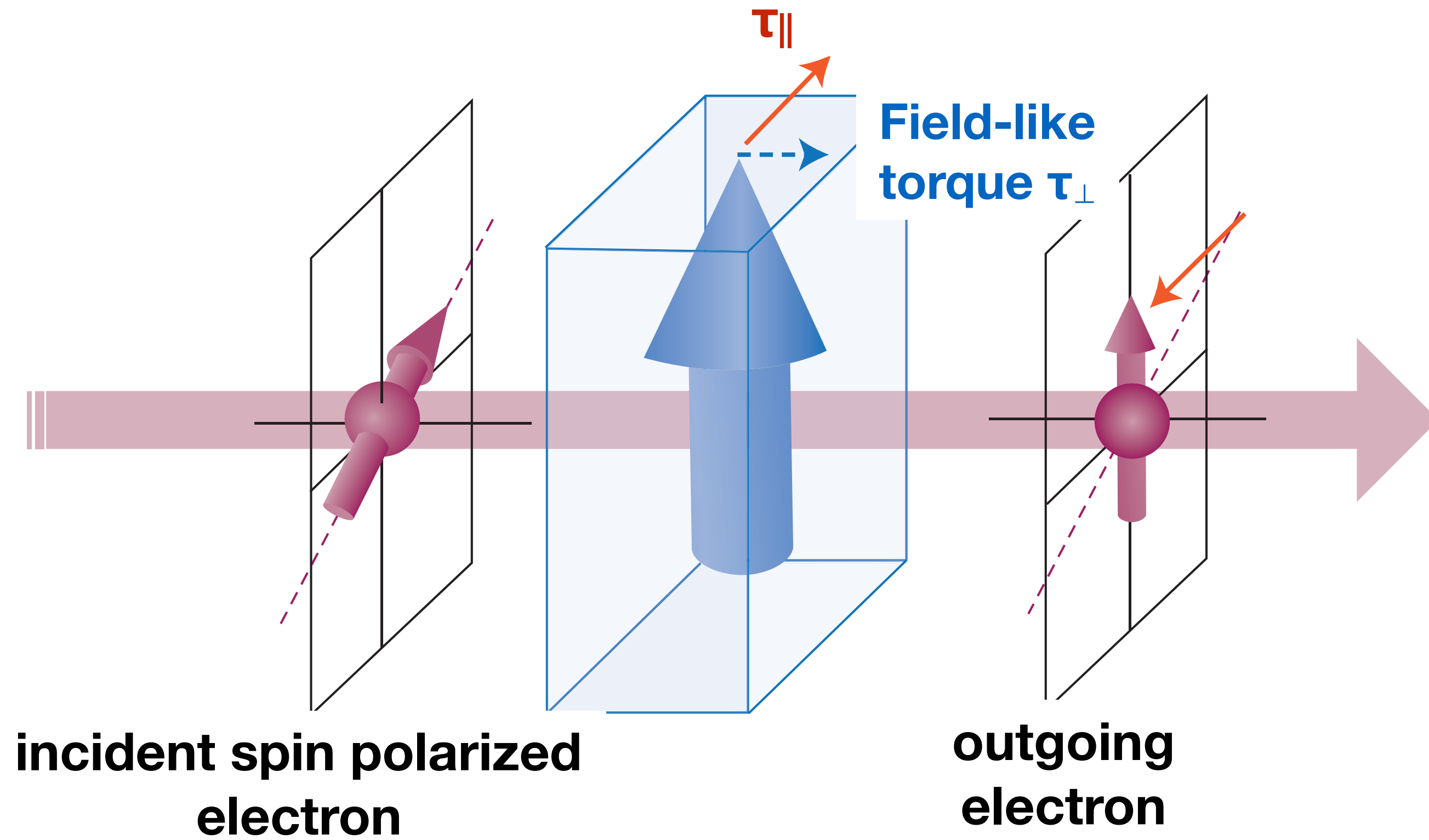
Huai et al., APL **84**, 3118 (2004).  
Fuchs et al., APL **85**, 1205 (2004).



([EVERSPIN.com](http://EVERSPIN.com))

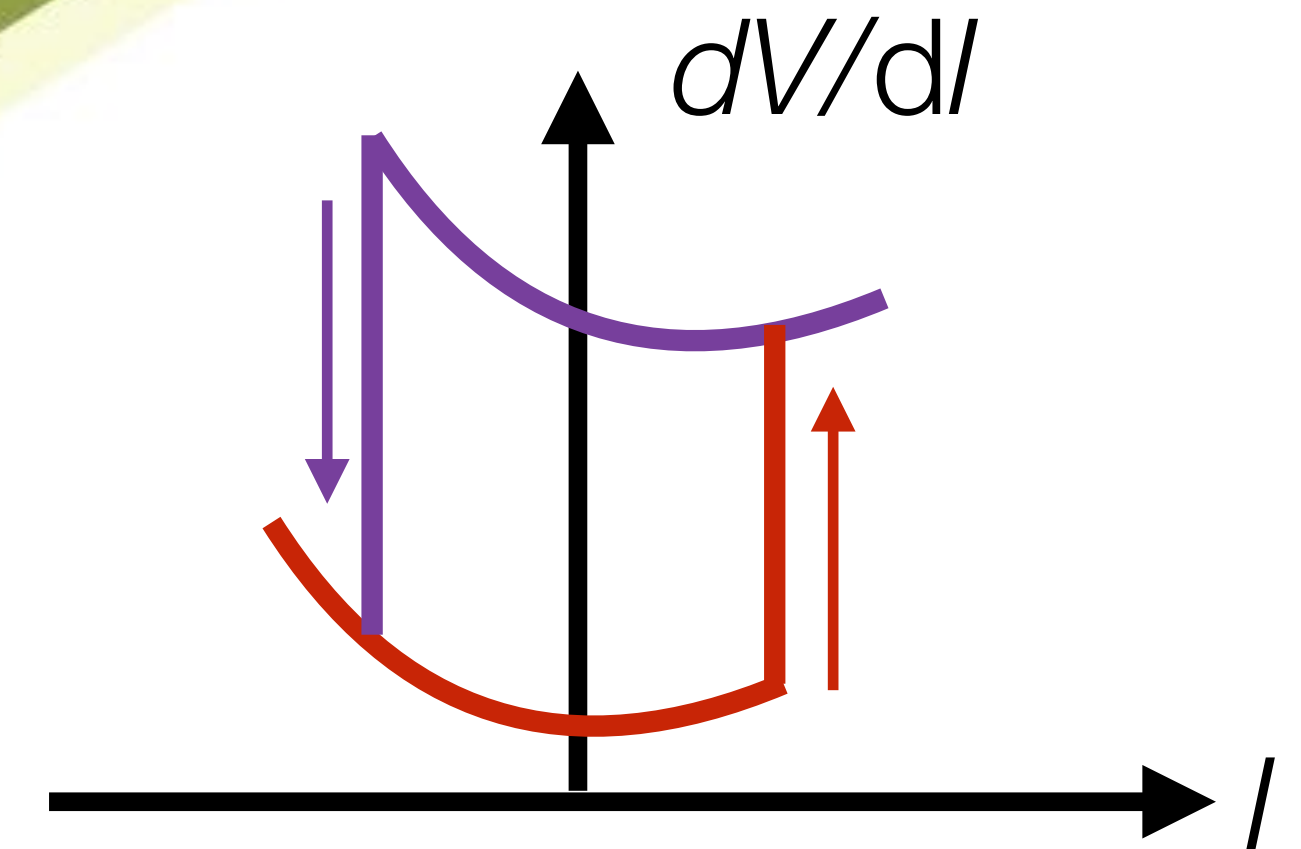
# “Reading” & “writing” MRAM

## Spin transfer torque



Nanopillar magnetic tunnel junction

- Read the state of the memory cell using TMR
- Write the state of memory cell using the current through the cell.

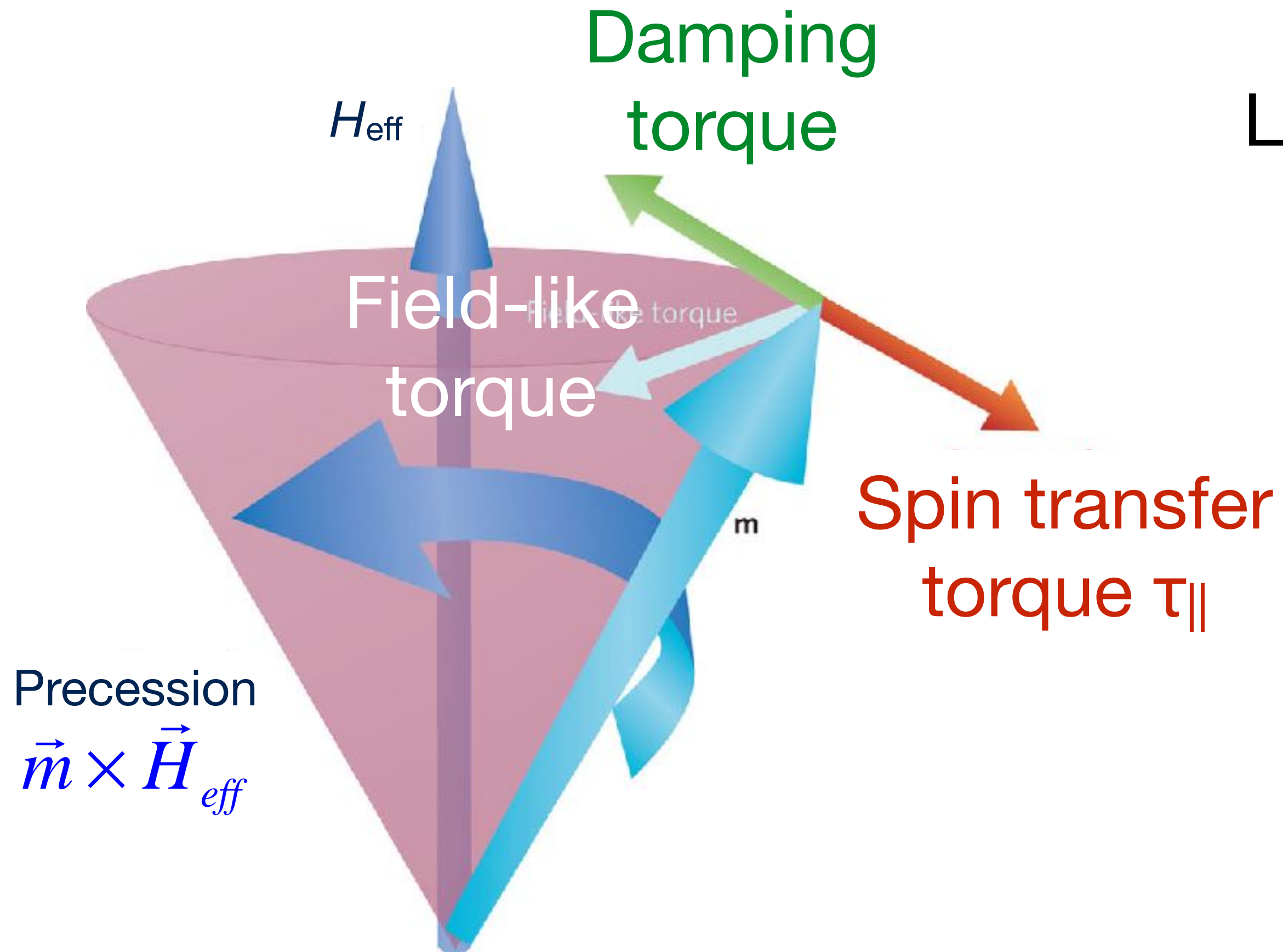


[Graphics: Brataas, Kent, Ohno, Nature Mater. **11**, 372 (2012)]

Interaction between a spin polarized current and a ferromagnet results in a “spin transfer torque” that can make the magnetization precess and even reverse orientation.



# Physics of spin transfer torque



Landau-Lifshitz-Gilbert-Slonczewski equation

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \gamma \frac{\partial \vec{m}}{\partial t} + \vec{\tau}$$

Spin-transfer torque:

$$\vec{\tau}_{||} = -\frac{\gamma \hbar}{2eM_s V} \vec{m} \times (\vec{m} \times \vec{J}_s)$$

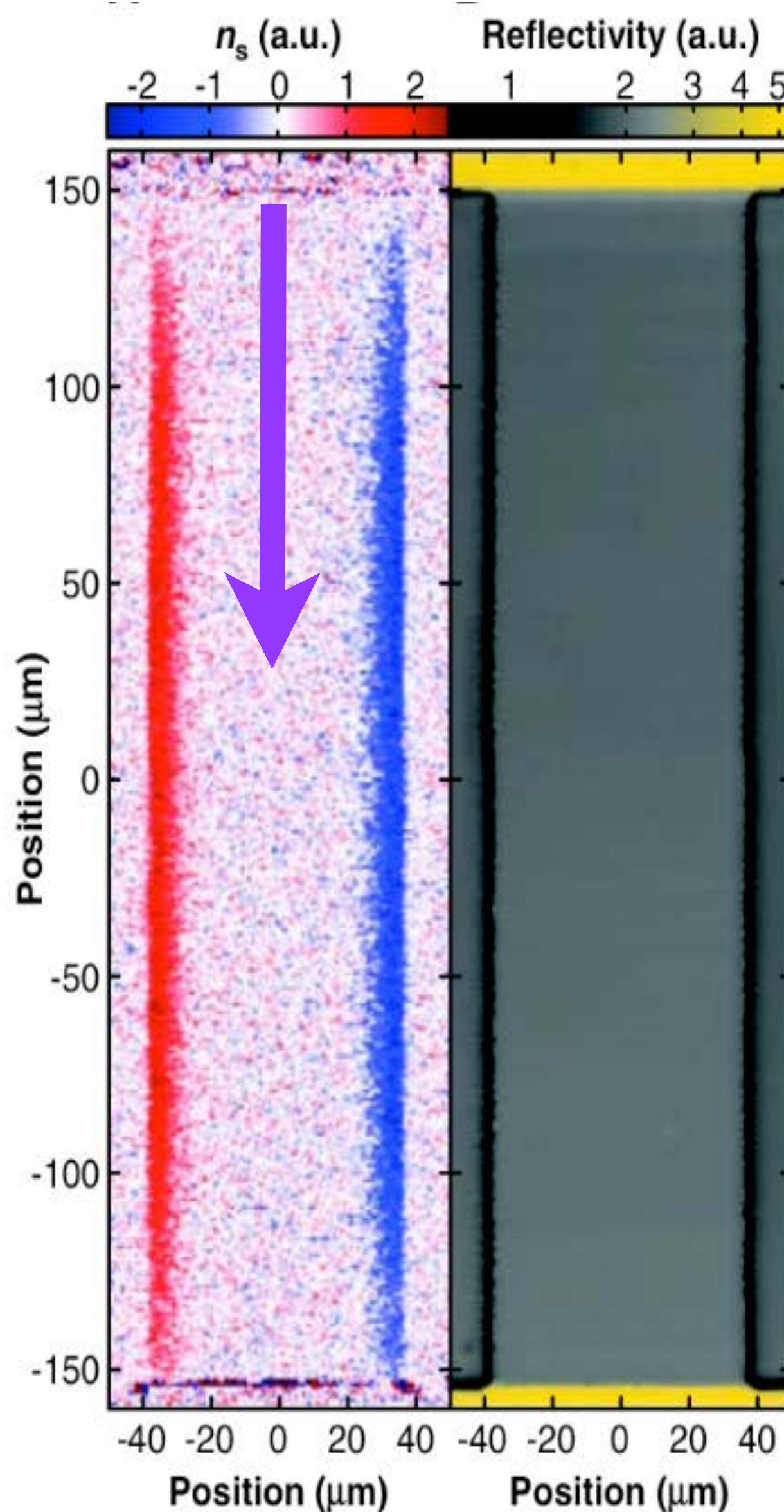
Field-like torque:

$$\vec{\tau}_{\perp} = -\frac{\gamma \hbar}{2eM_s V} \beta_s \vec{m} \times \vec{J}_s$$

[Graphic: Brataas, Kent, Ohno, Nature Mater. **11**, 372 (2012)]

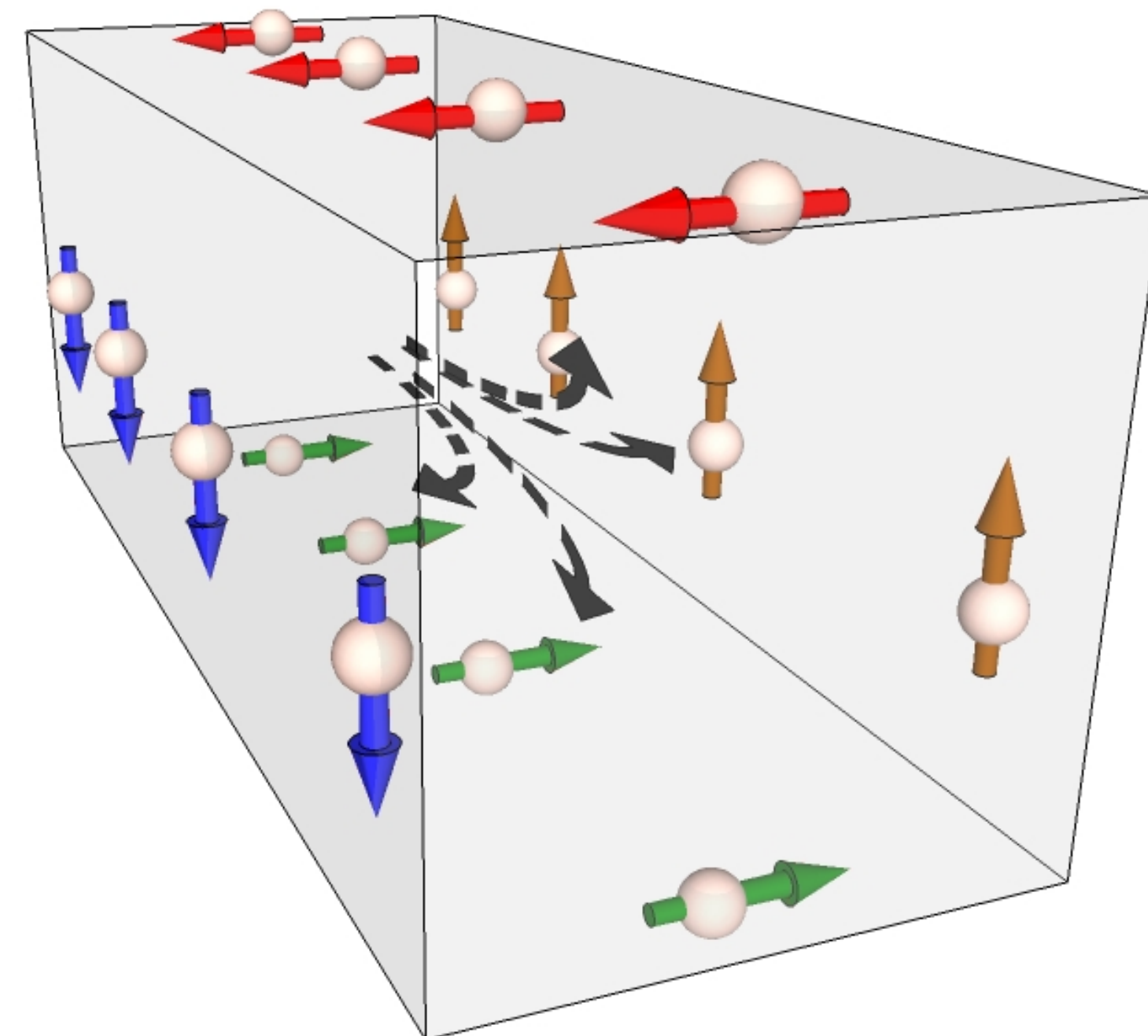
[L. Berger, *Phys. Rev. B* **54**, 9353–9358 (1996); J. C. Slonczewski, *J. C. J. Magn. Mater.* **159**, L1–L7 (1996)]

# Polarizing spins without magnetism: spin Hall effect



Kato *et al*, Science (2004)

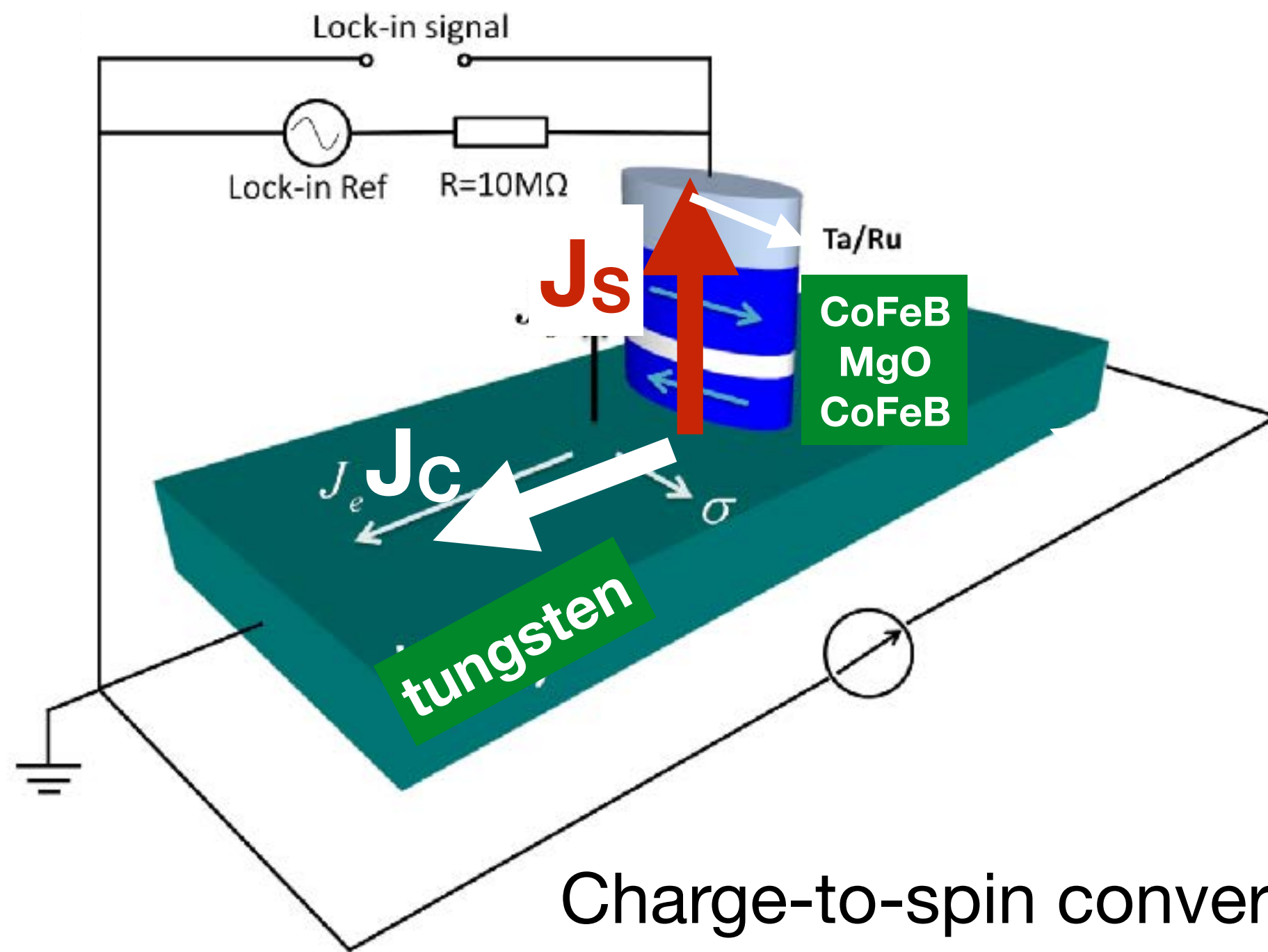
- The spin-orbit interaction in semiconductors and in heavy metals: spin polarization and accumulation via asymmetric spin-dependent scattering or via band structure
- Known as spin Hall effect — first discovered in GaAs and now measured in many semiconductors & heavy metals



Graphic: D. C. Ralph



# Spin transfer torque from spin Hall effect



Charge-to-spin conversion efficiency given by:

$$\theta_{\parallel} = \frac{J_s}{J_e} \frac{2e}{\hbar} = \frac{\sigma_{s,\parallel}}{\sigma} \frac{2e}{\hbar}$$

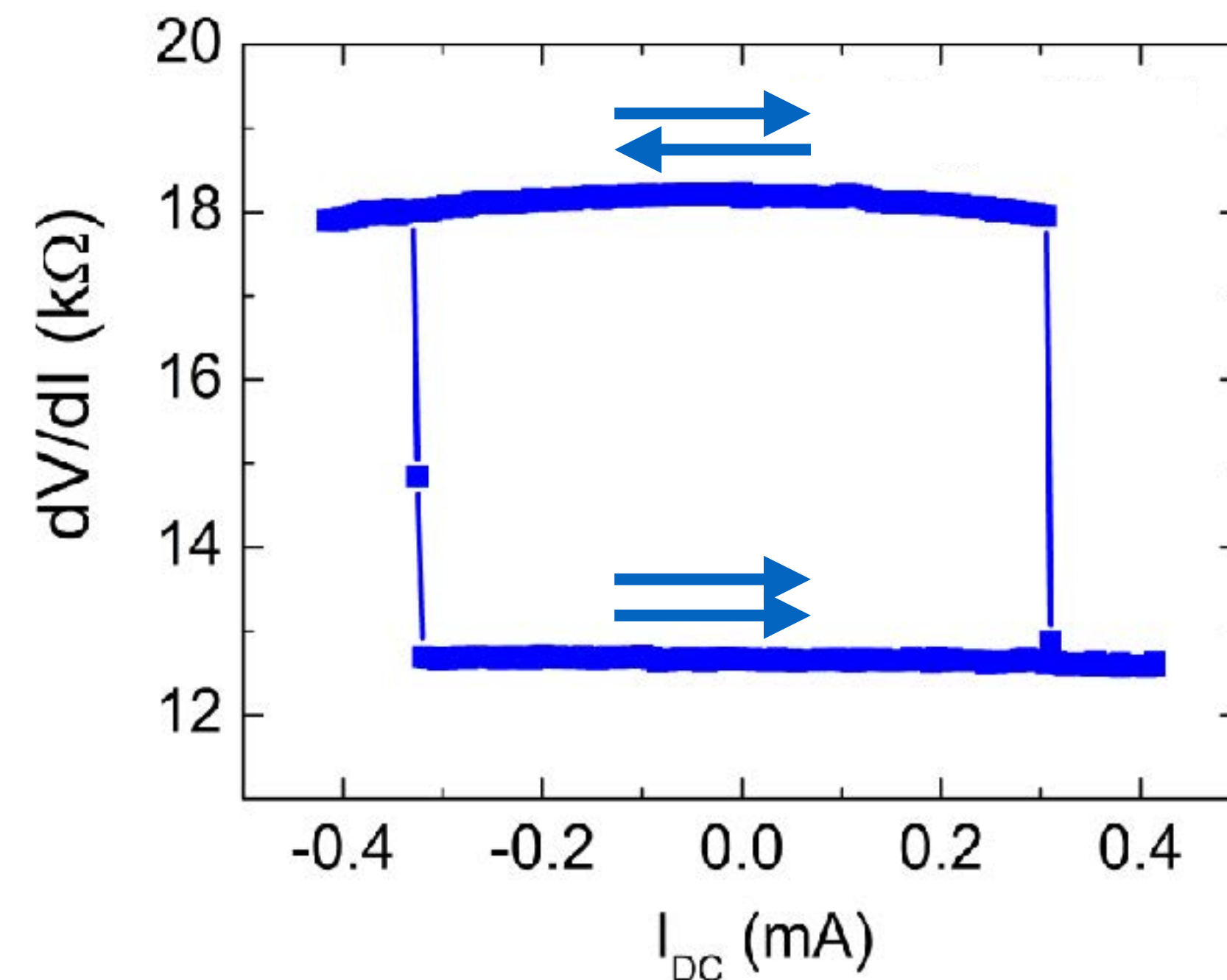
“spin Hall ratio”, “spin Hall angle”

“spin torque ratio”

$\sigma_{s,\parallel}$  “spin current conductivity”

The spin polarized electrons needed to exert a spin transfer torque can be created without a ferromagnet: use the “spin Hall effect” due to spin-orbit interaction in a heavy metal.

Liu *et al.*, *Phys. Rev. Lett.* **106**, 036601 (2011); *Science* **336**, 555 (2012)  
Miron *et al.*, *Nature* **476**, 189 (2011)



# Charge-to-spin conversion efficiency

$$\theta_{\parallel} = \frac{J_s}{J_e} \frac{2e}{\hbar} = \frac{\sigma_{s,\parallel}}{\sigma} \frac{2e}{\hbar}$$

$$10^5 \frac{\hbar}{2e} (\Omega \cdot \text{m})^{-1}$$

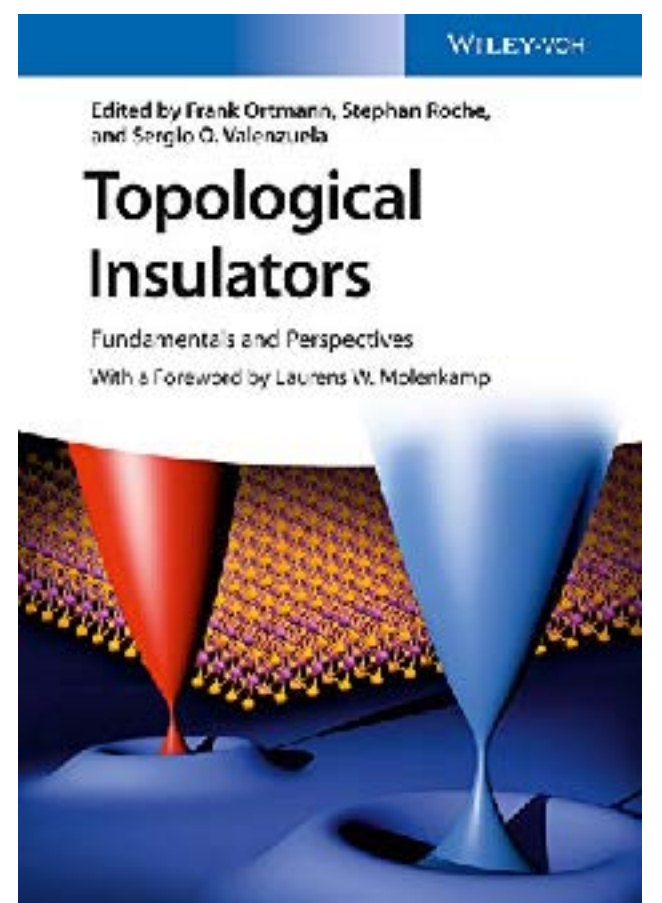
Material	$\sigma_{s,\parallel}$	$\theta_{\parallel}$
Pt [1]	3.4	0.08
$\beta$ -Ta [2]	0.8	0.15
Cu(Bi) [3]		0.24
$\beta$ -W [4]	1.8	0.3

**Can we find materials that have a larger charge-to-spin conversion efficiency than heavy metals?**

- [1] Liu *et al.*, *PRL* **106**, 036601 (2011)  
[2] Miron *et al.* *Nature* **476**, 189 (2011)  
[3] Niimi *et al.*, *PRL* **109**, 156602 (2012)  
[4] Pai *et al.*, *APL* **122**, 101404 (2012)



- Spintronics: overview of concepts & devices
- Topological insulators: concepts, materials, phenomena
- Topological spintronics: concepts, materials, phenomena, devices





# Dirac equation in free space

Quantum mechanics + special theory of relativity: Dirac equation & electron spin ( $S = 1/2$ )

$$\text{3D: } i\hbar \frac{\partial}{\partial t} \psi = [c\vec{\alpha} \cdot \vec{p} + \beta mc^2] \psi \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \vec{\beta} = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}$$

In 3D, Dirac equation preserves continuous Lorentz symmetry as well as discrete symmetries (time-reversal, parity, charge conjugation).

$$\text{2D: } i\hbar \frac{\partial}{\partial t} \psi = \left[ \left( -p_x \sigma_y + p_y \sigma_x \right) c + \sigma_z mc^2 \right] \psi$$

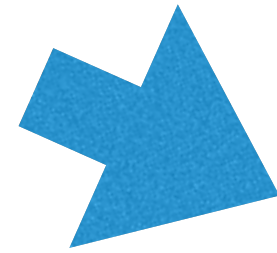
In 2D, the mass term in the Dirac equation breaks both time-reversal and parity symmetries.



# From Dirac equation to spin- and spin-orbit coupling

Non-relativistic approximation to Dirac equation (Pauli): 2nd order perturbation theory leads to spin-orbit coupling

$$H_D = c\vec{\alpha} \cdot \vec{p} + \beta mc^2 + V(r)$$



$$H_{so} = -\frac{\hbar^2}{4m_0^2 c^2} \vec{\sigma} \cdot (\vec{p} \times \nabla V)$$

For a Coulomb potential [ $V(r) \sim 1/r$ ], we get:

$$H_{so} = \frac{e^2}{2mc^2 r^3} \frac{\hbar}{2} \vec{\sigma} \cdot \vec{L}$$

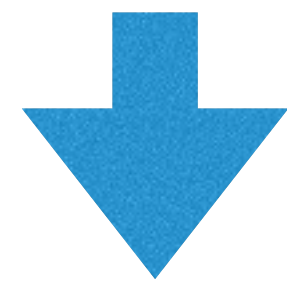
Spin-orbit interaction: relativistic transformation of electric field into a magnetic field

# Spin-orbit Coupling in Crystalline Solids

- Solid state crystals: energy scale of spin-orbit interaction  $\gg$  free atoms
- Key to understanding band structure & magnetism
- Also leads to remarkable spin transport phenomena

$$H_{so} = -\frac{\hbar^2}{4m_0^2 c^2} \vec{\sigma} \cdot (\vec{p} \times \nabla V)$$

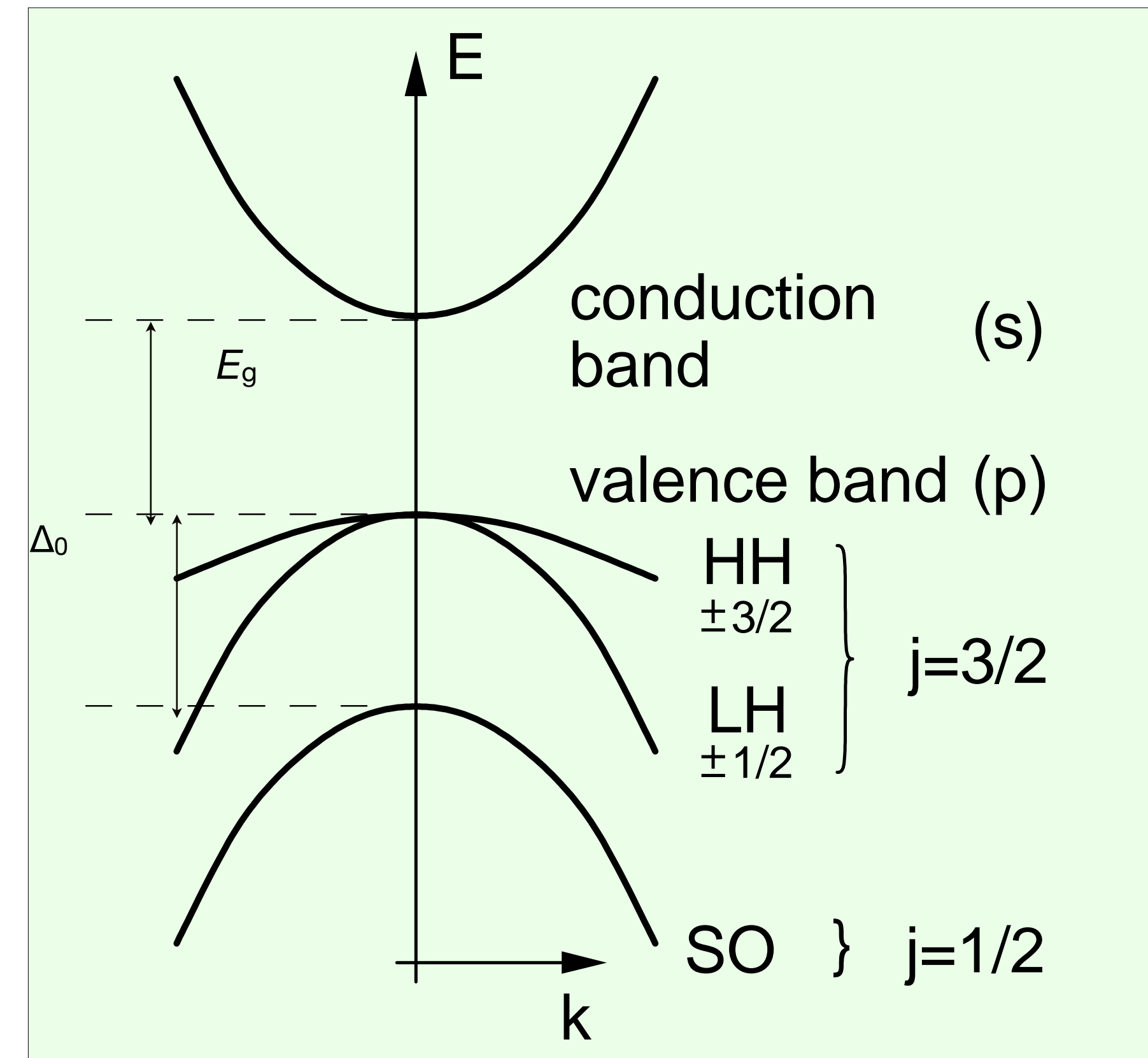
crystal  
momentum



$$H_{so} = \lambda \vec{\sigma} \cdot (\vec{k} \times \nabla V)$$

electric field  
(internal or external)

$$\lambda = P^2 \left[ \left( E_g \right)^{-2} - \left( E_g + \Delta_0 \right)^{-2} \right]$$



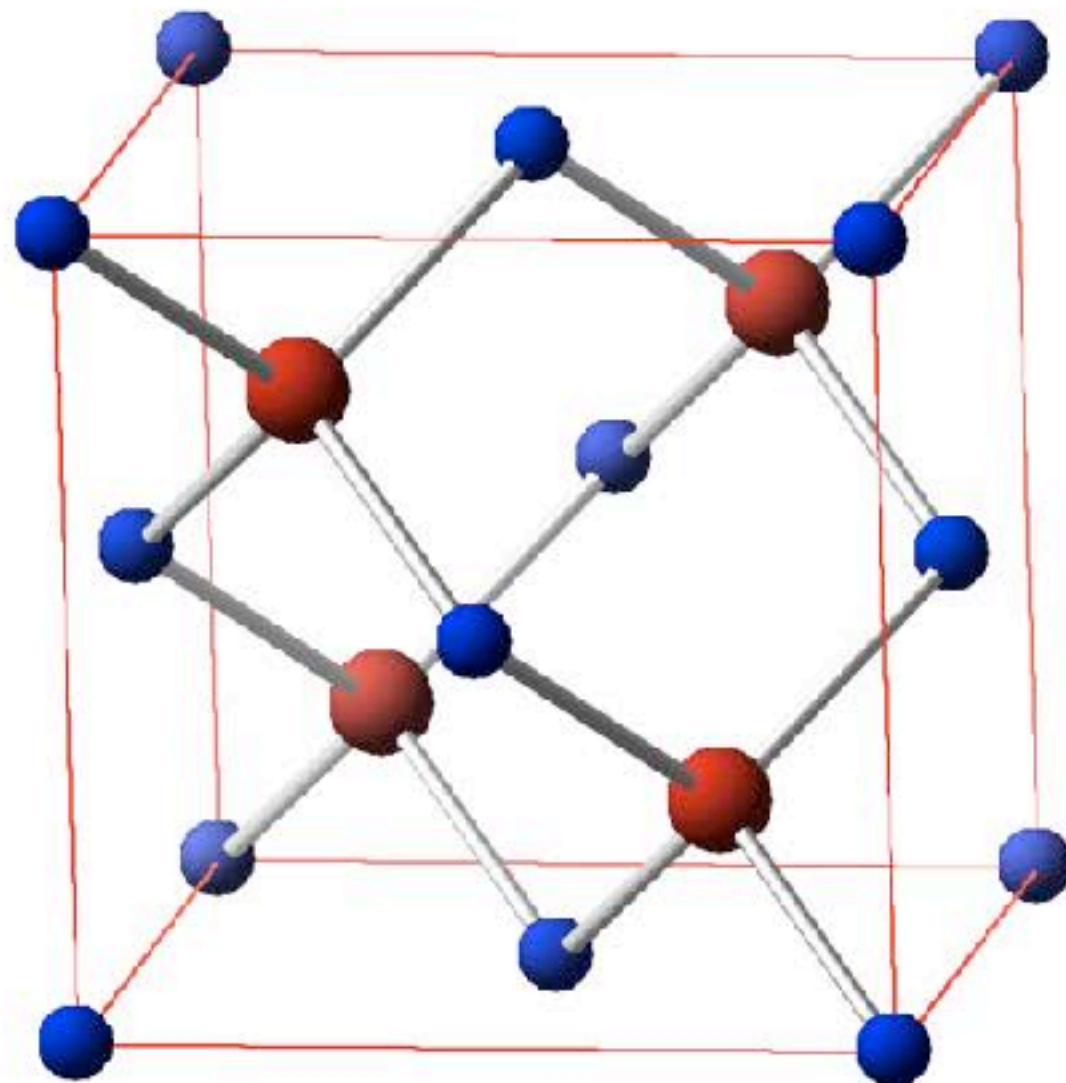


# The Landau paradigm

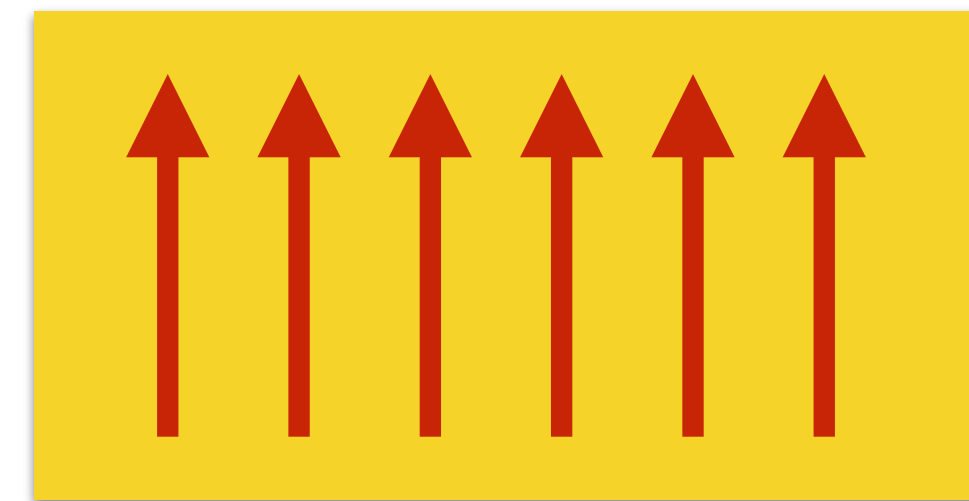
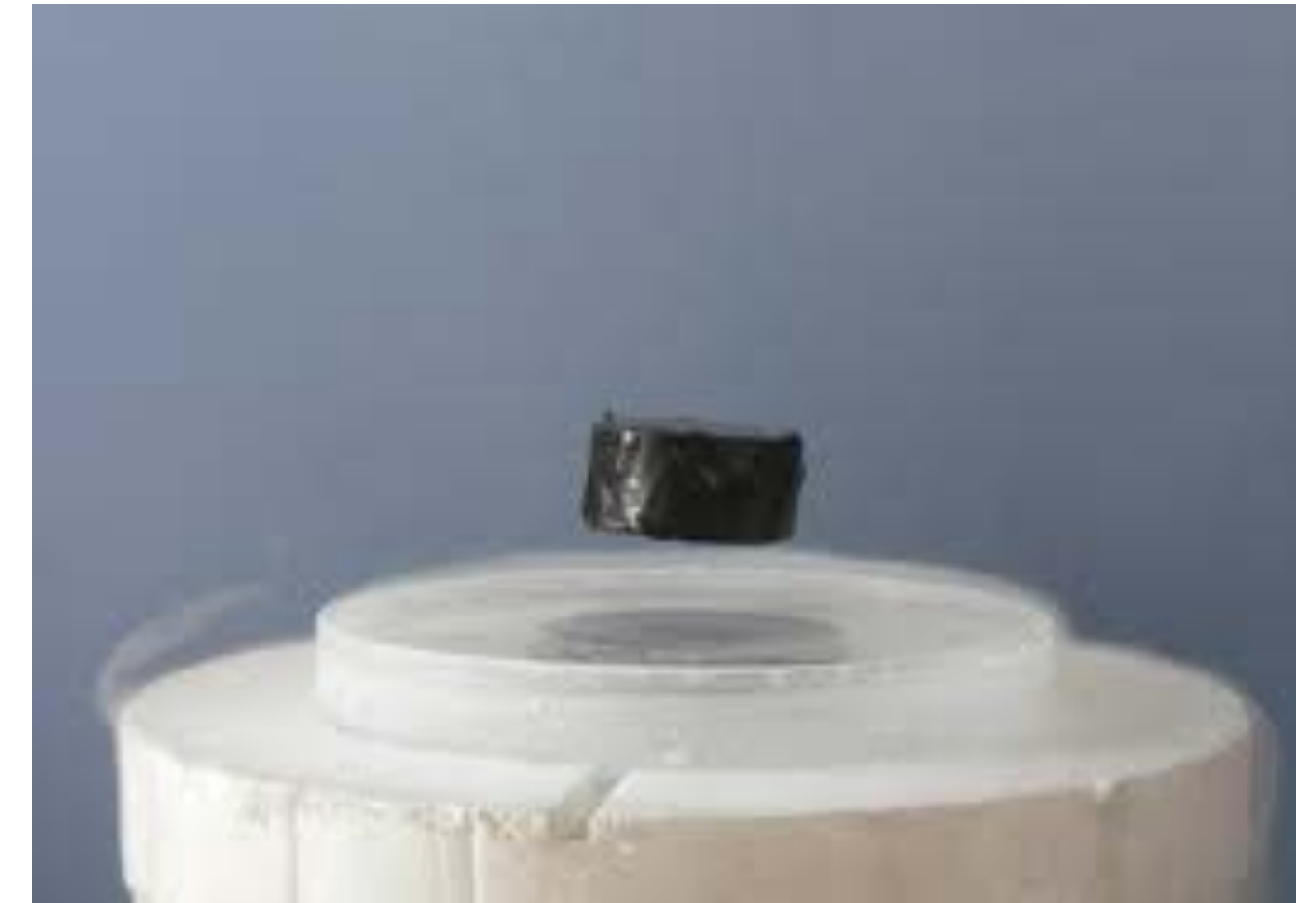


Phases of matter are created by  
spontaneously broken symmetry  
and emergence of an order  
parameter

Crystalline solids:  
breaking of full translation &  
rotational invariance



Superconductivity:  
breaking of gauge invariance



Ferromagnetism:  
breaking of time reversal invariance

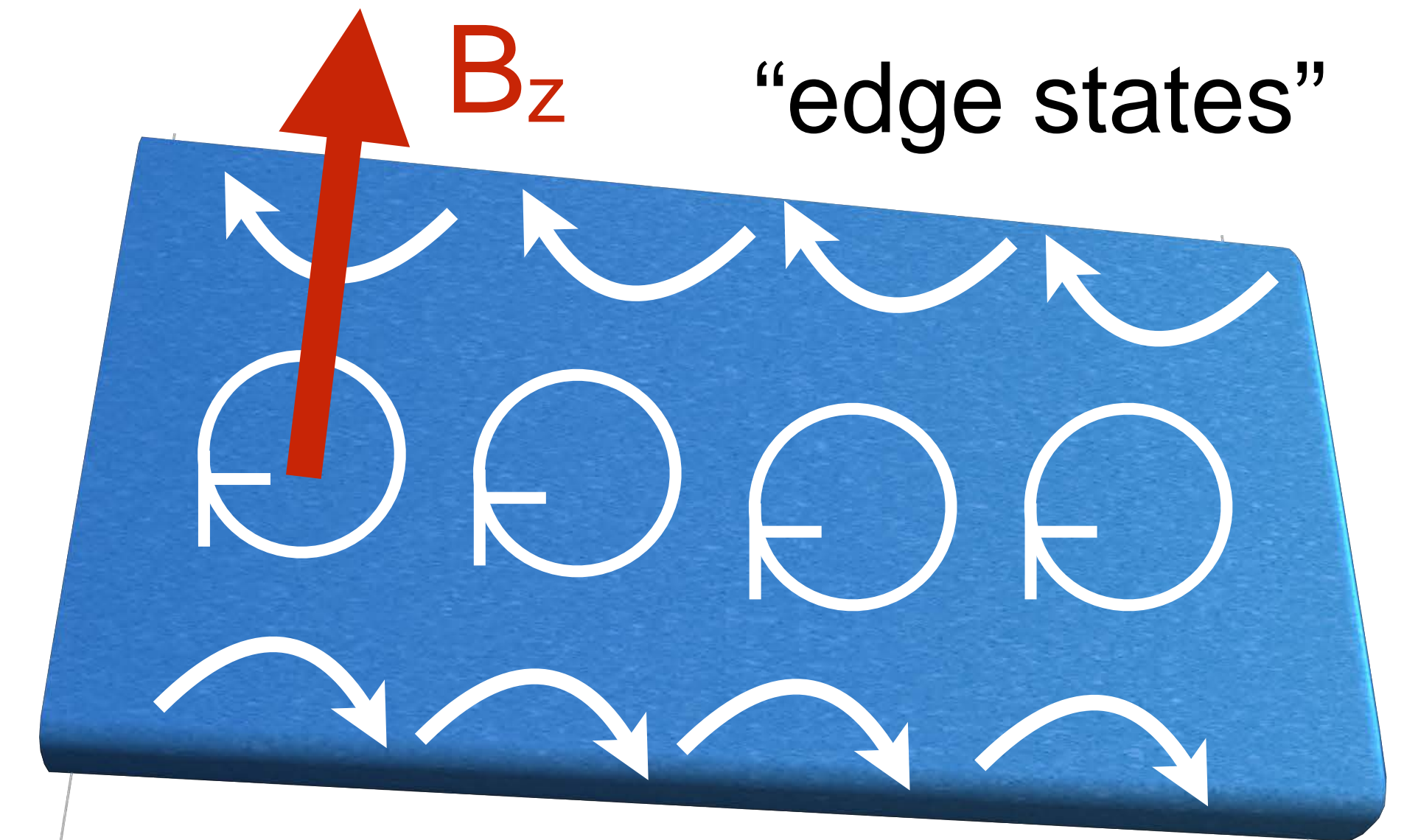
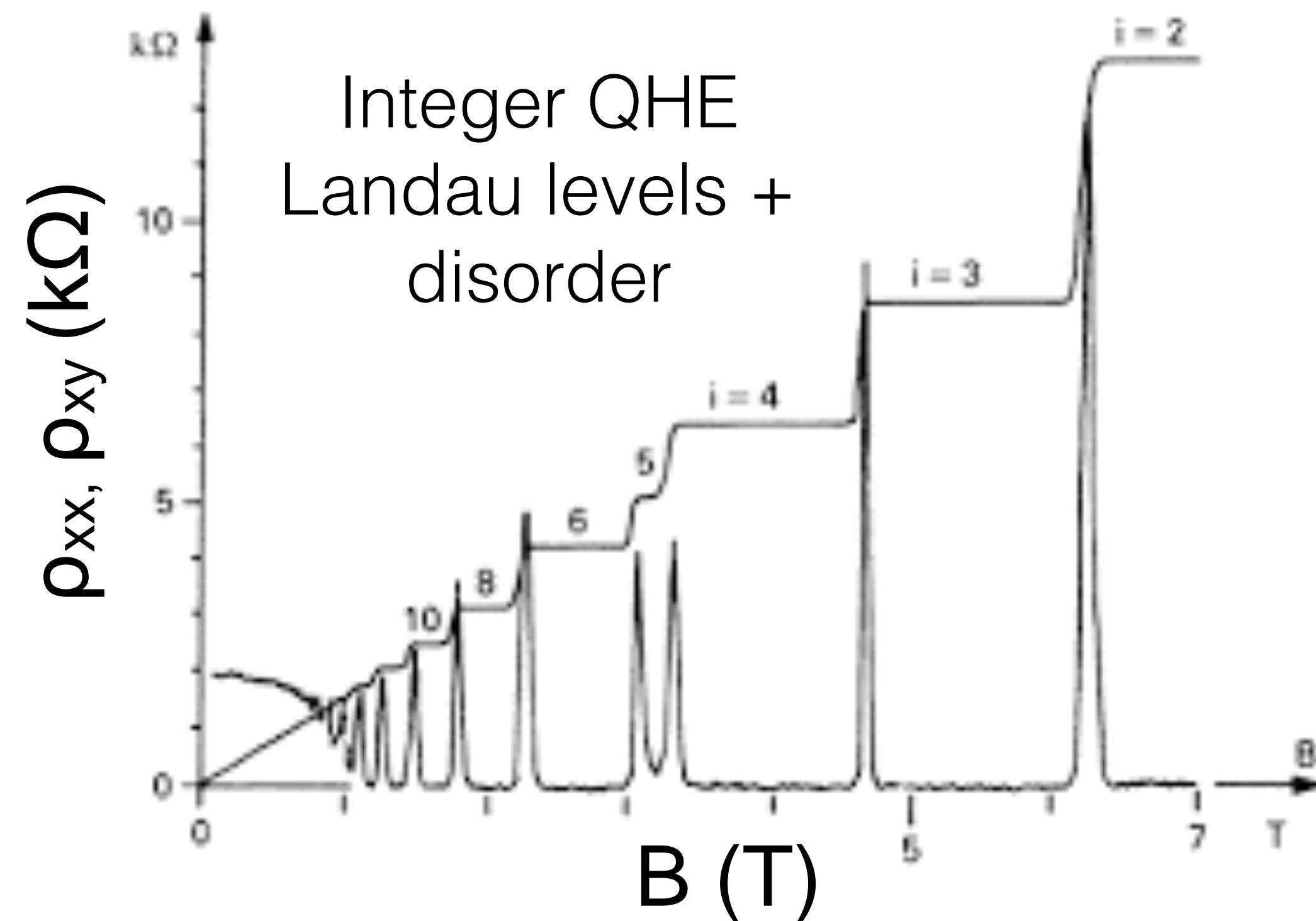
# Beyond Landau: the quantum Hall effect



$$\rho_{xy} = \frac{h}{ne^2} \quad \rho_{xx} = 0$$



Von Klitzing, Dorda, Pepper (1980)



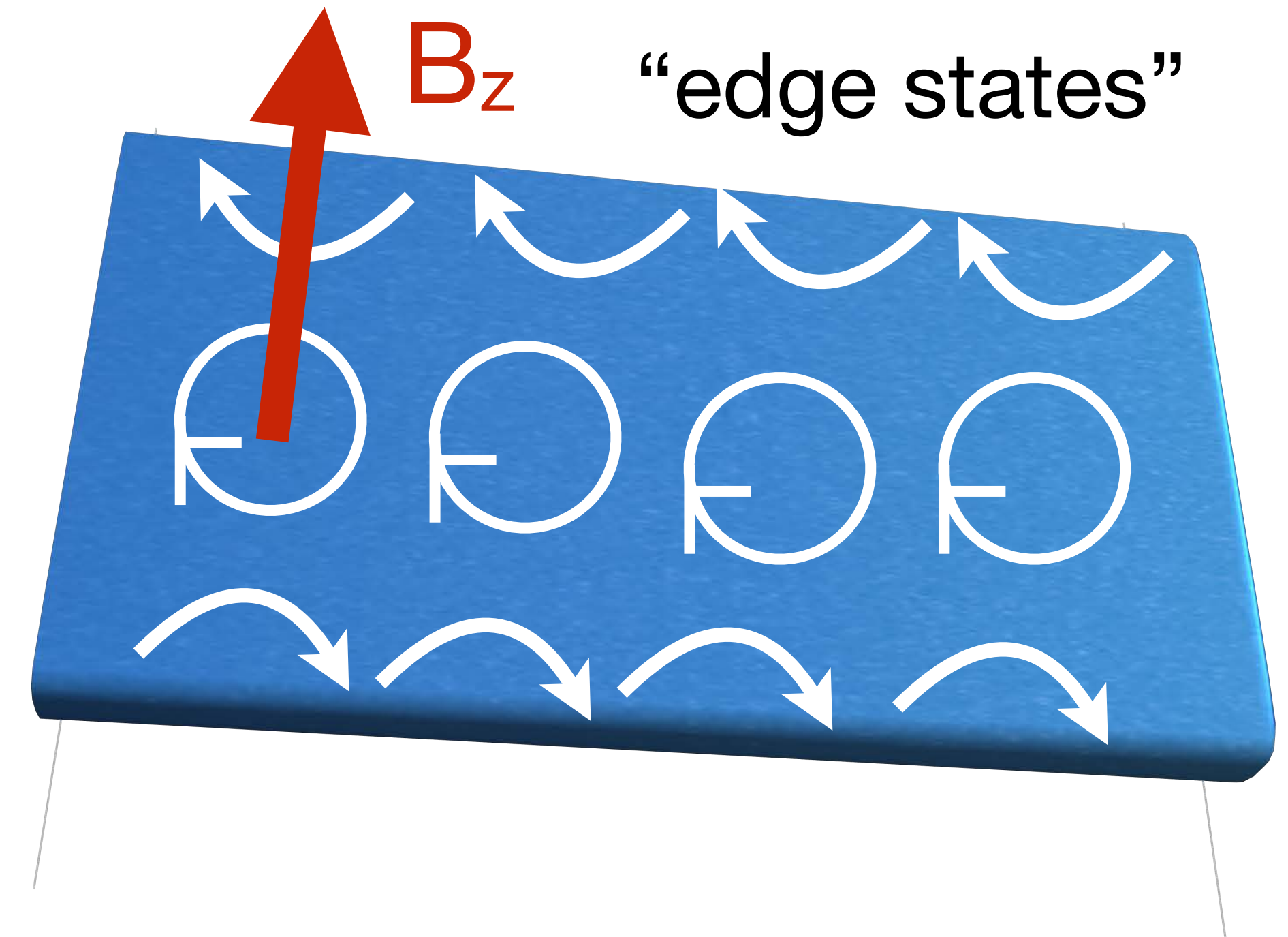
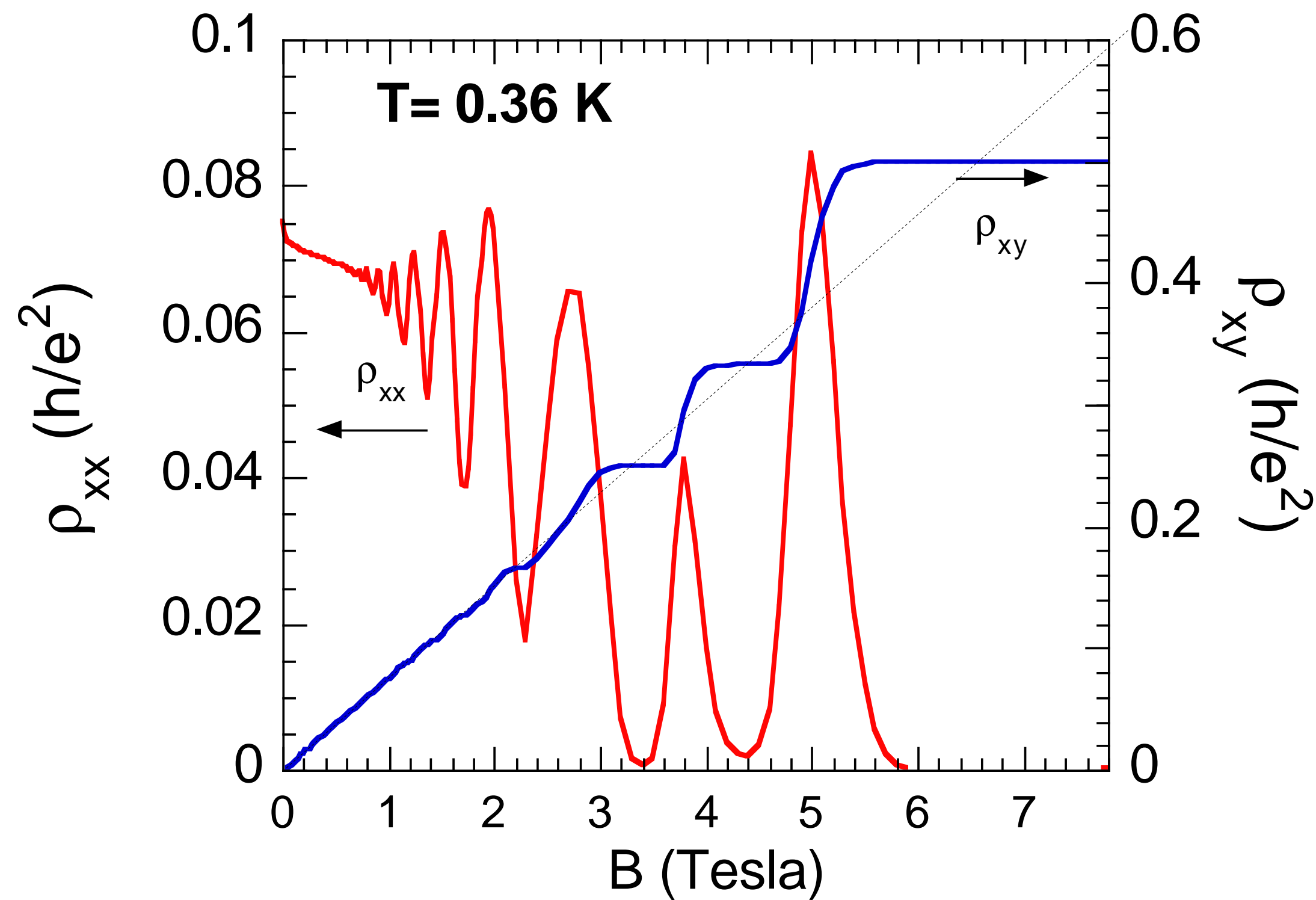
2D conductor in a perpendicular magnetic field supports ballistic (dissipationless) edge state transport with precisely quantized Hall resistance.

Requires well-defined 'Landau levels' but details about the sample and material do not matter.



# Quantum Hall effect & edge states

Integer Quantum Hall Effect in ZnSe 2DEG



Quantum Hall effect: ballistic edge state transport & precisely quantized Hall resistance/conductance, requires well-defined Landau levels ( $\omega_c \tau > 1$ ), but otherwise does not involve any details about the sample and material.

# Quantum Hall effect & topology

$$\sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} = n \frac{e^2}{h} \quad \sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} = 0$$

$n$  = “Chern number”

$$n = \frac{1}{2\pi} \int_{BZ} d^2k \vec{F}(\vec{k}) = \frac{1}{2\pi} \int_C \vec{A} \cdot d\vec{k}$$

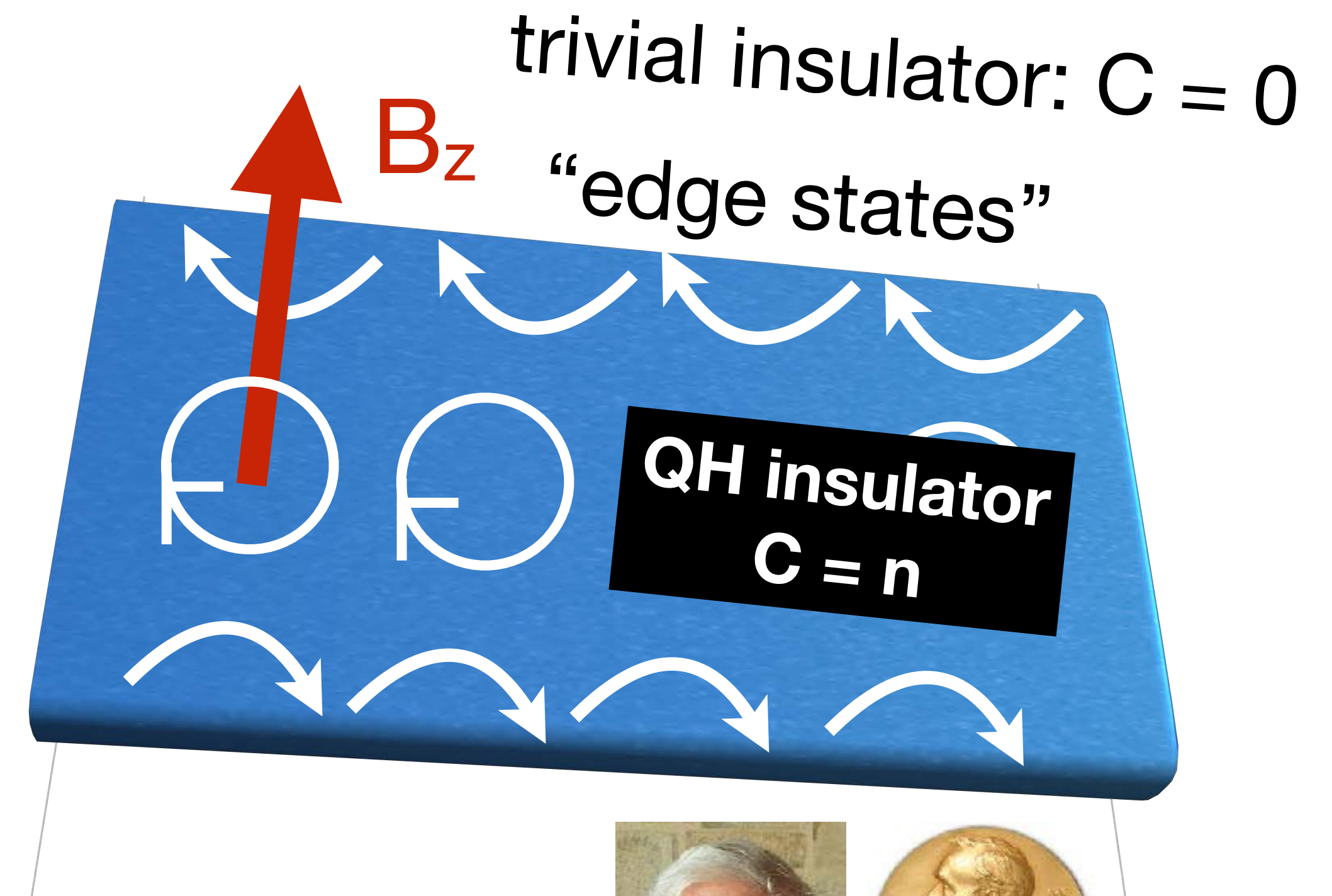
Berry curvature:

Berry connection:

$$\vec{F} = \nabla_k \times \vec{A} \quad \vec{A} = -i \left\langle u(\vec{k}) \left| \nabla_k \right| u(\vec{k}) \right\rangle$$

Why is quantization of  $\rho_{xy}$  ( $\sigma_{xy}$ ) independent of details?

- Thouless, Kohmoto, Nightingale, den Nijs [PRL **49**, 405 (1982)]: quantized  $\sigma_{xy}$  is a **topological property**.
- Quantum Hall insulator has a Chern number distinct from a “trivial” insulator.



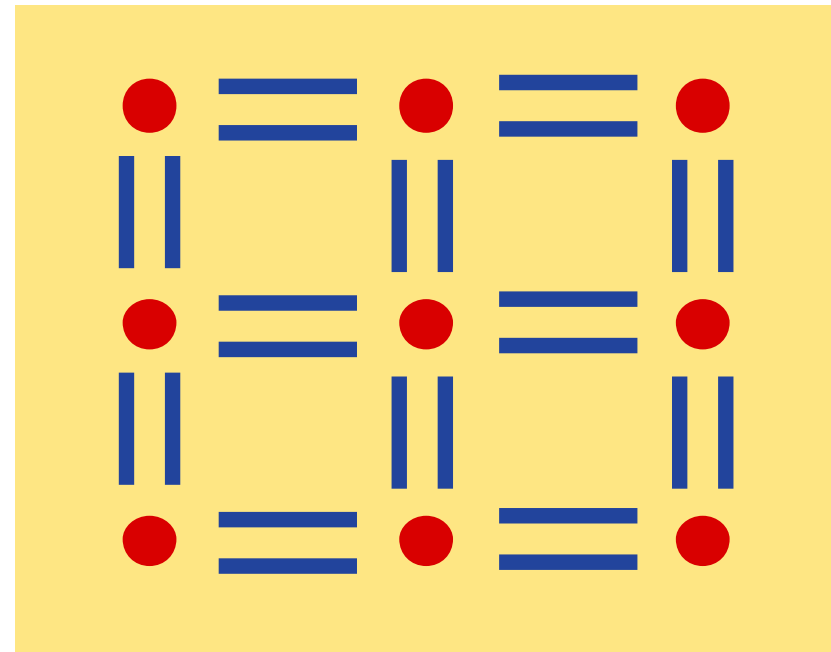
2016



# Topology and matter

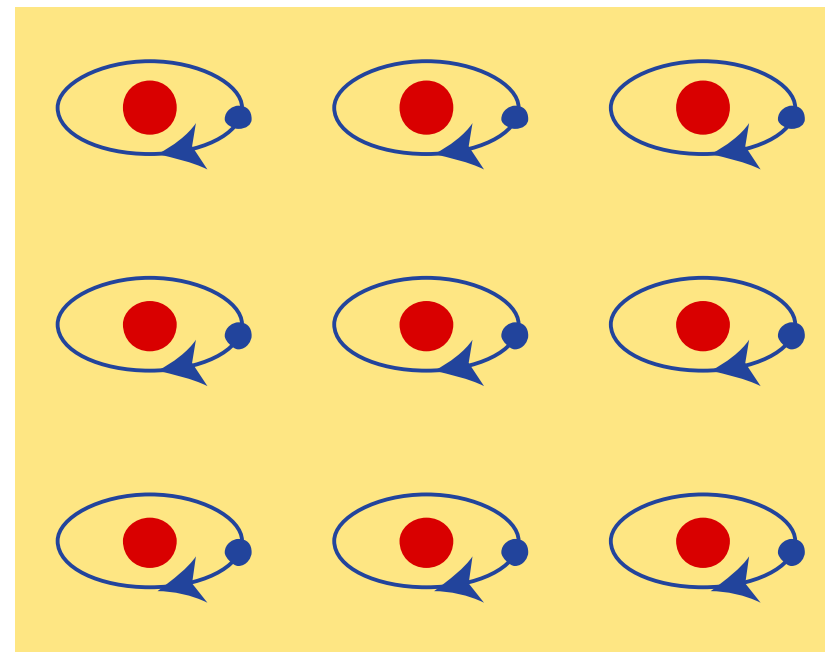
Covalent Insulator

e.g. intrinsic semiconductor

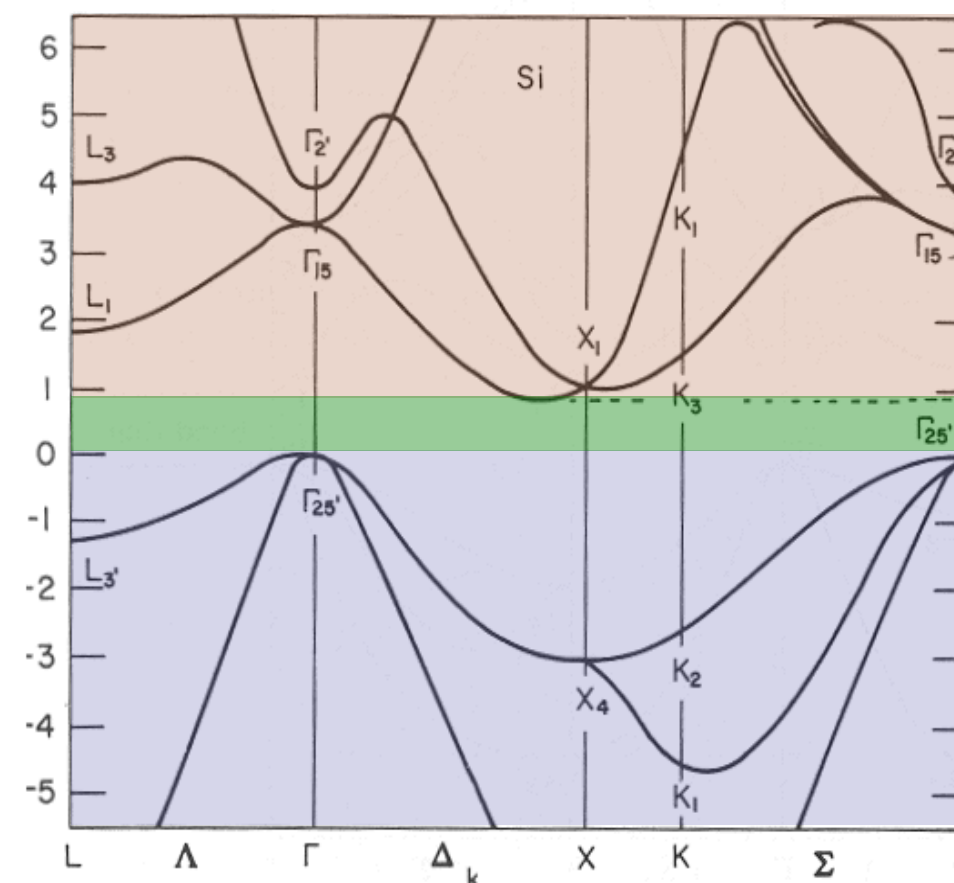


Atomic Insulator

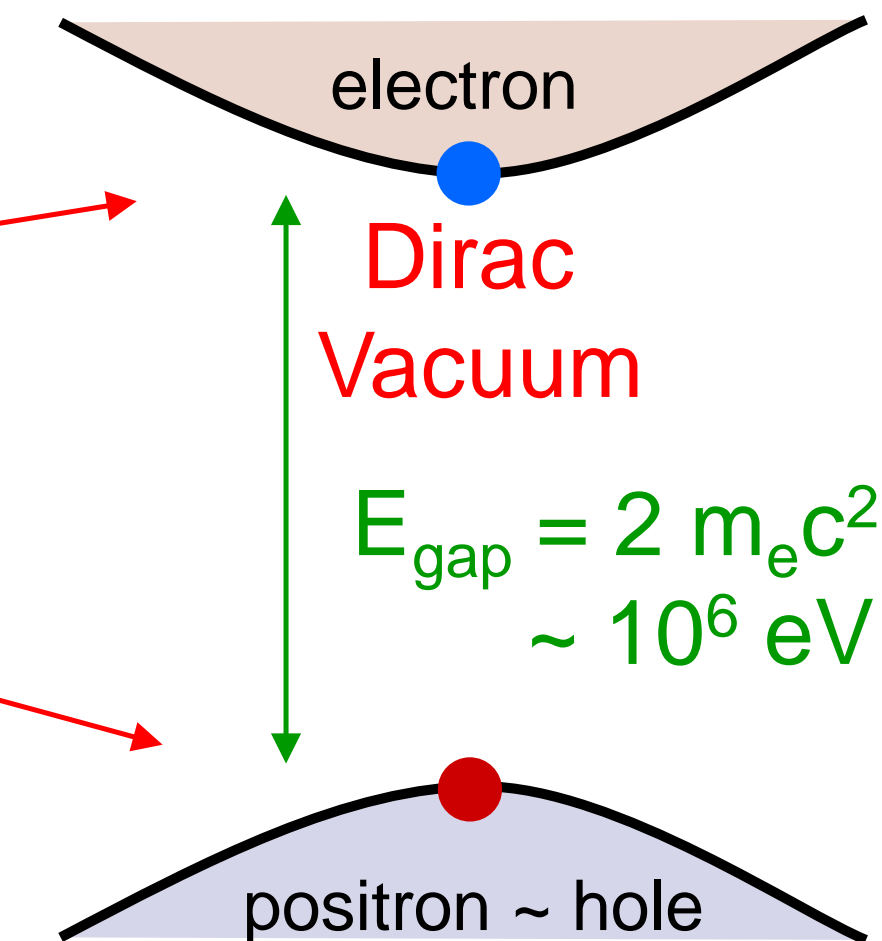
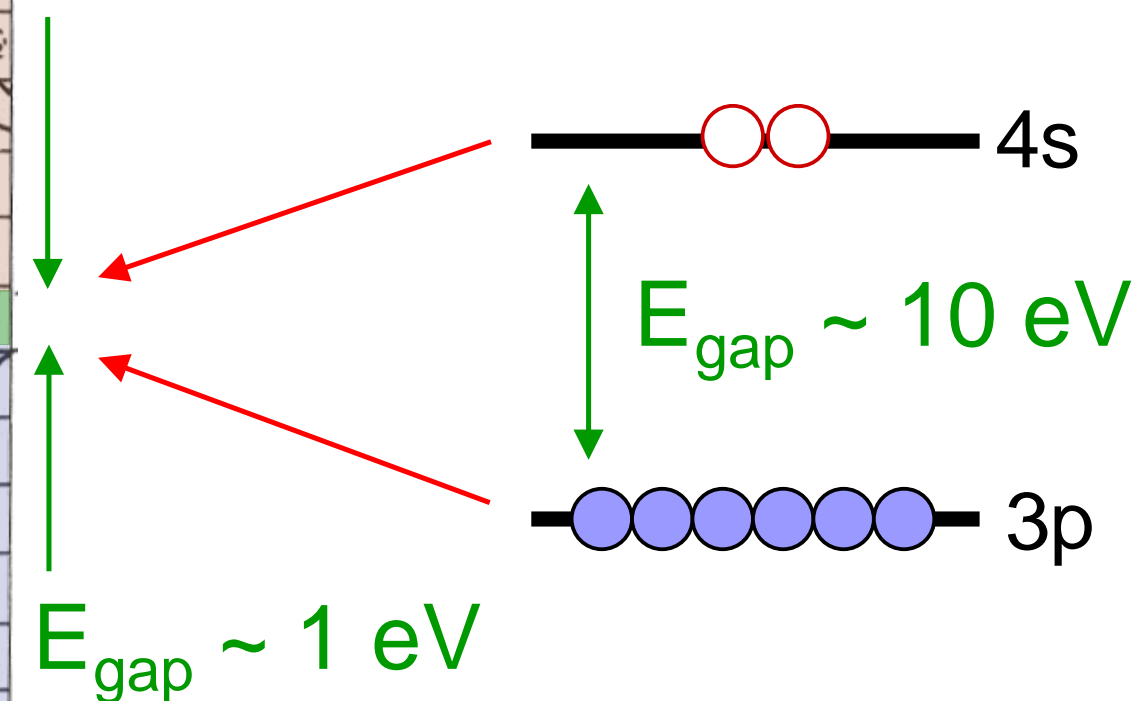
e.g. solid Ar



The vacuum



Silicon



graphic:  
Hasan & Kane, RMP

From a topological perspective, all cases above are topologically equivalent: one can continuously tune the Hamiltonian to go from one to the other without closing the gap.

# Imagining edge states at $B = 0$

VOLUME 61, NUMBER 18

PHYSICAL REVIEW LETTERS

31 OCTOBER 1988

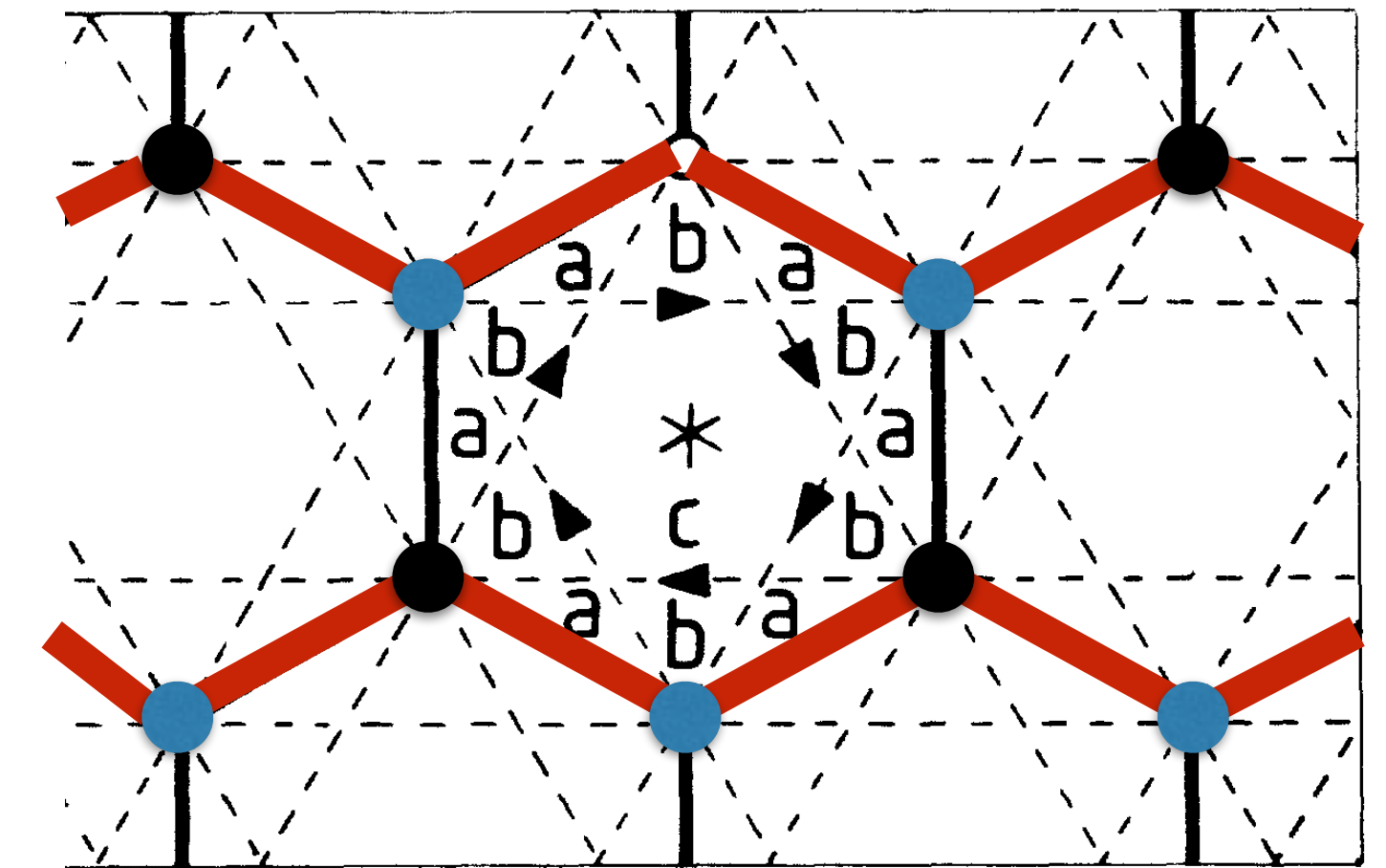
## Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the “Parity Anomaly”

F. D. M. Haldane

*Department of Physics, University of California, San Diego, La Jolla, California 92093*

(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance  $\sigma^{xy}$  in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called “parity anomaly” of (2+1)-dimensional field theories.



Haldane:

- In a 2D semimetal, inversion symmetry + time-reversal symmetry can create a degeneracy at isolated points in the band structure
- A clever arrangement of magnetic flux can break time-reversal symmetry while still keeping  $B = 0$ , opening a gap and resulting in a ‘Chern insulator’ with  $\sigma_{xy} = \pm e^2/h$ .



2016



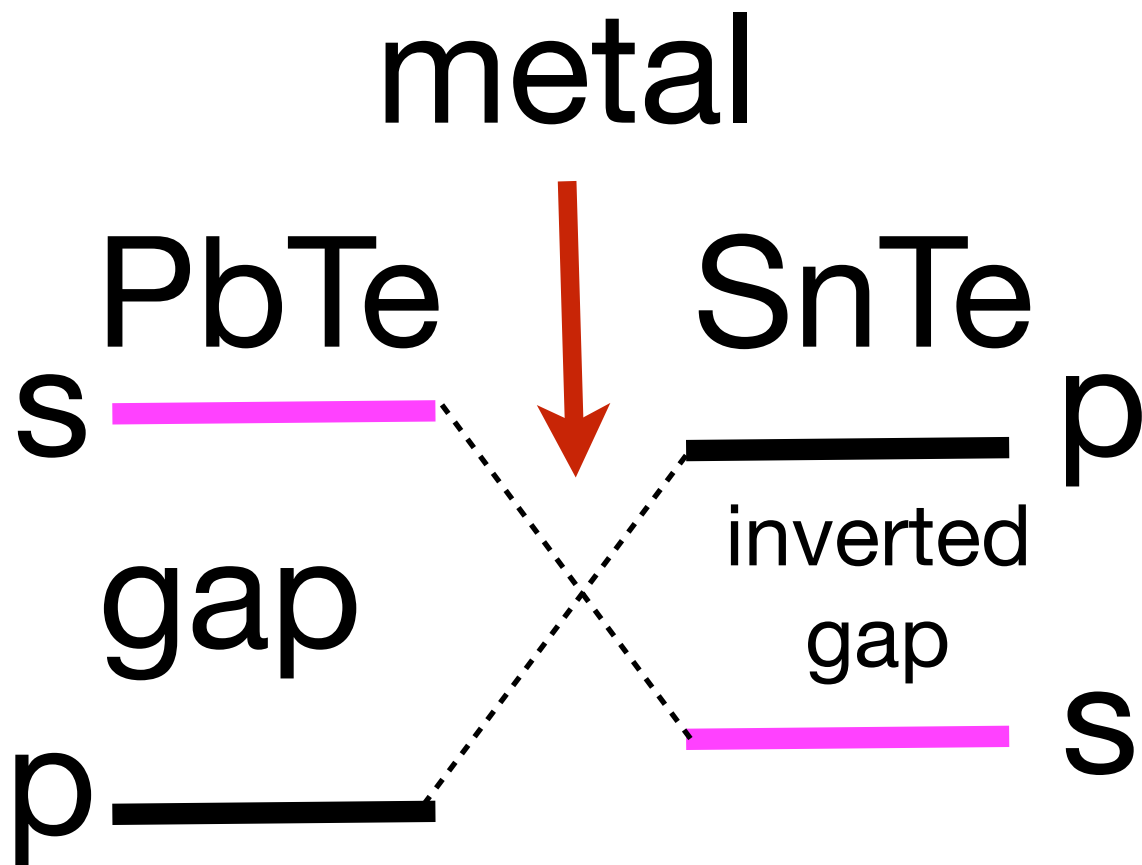
# Spin-orbit coupling and interface states: early work

## Two-dimensional massless electrons in an inverted contact

B. A. Volkov and O. A. Pankratov  
*P. N. Lebedev Physics Institute, Academy of Sciences of the USSR*

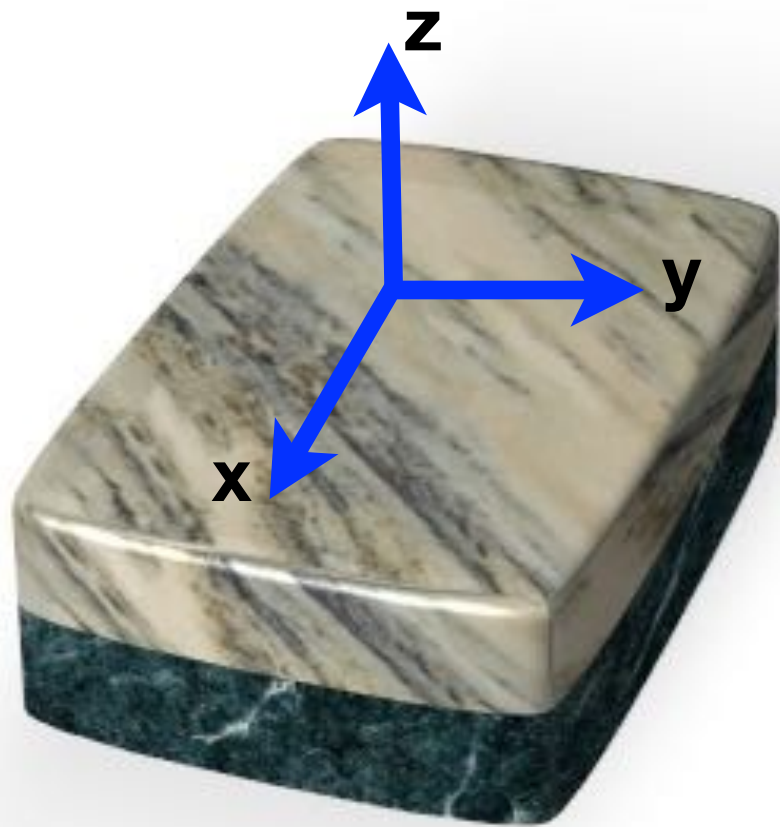
(Submitted 20 June 1985)  
 Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 4, 145–148 (25 August 1985)

A new type of semiconductor structures based on the contact of two materials with mutually inverted bands is proposed. A qualitative feature of this contact is the presence of electron states which have a two-dimensional linear spectrum and which do not depend on the transition region. The properties of an inverted contact in an external magnetic field are determined.



$$\begin{pmatrix} \Delta(z) & \sigma \hat{p} \\ \sigma \hat{p} & -\Delta(z) \end{pmatrix} \psi = [\varepsilon - \vartheta(z)] \psi$$

$$\varepsilon = \hbar v_F k_{\perp}$$



PbTe (HgTe)  
 SnTe (CdTe)

