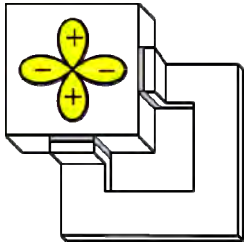


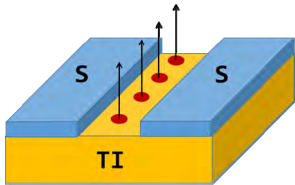
LECTURE 1: Monday, June 3



Phase-sensitive measurements on superconducting quantum materials and hybrid superconductor devices

Josephson physics and techniques useful for exploring superconductor materials and devices, focusing on probing unconventional superconductors and junctions

LECTURE 2: Tuesday, June 4



S-TI-S Josephson junction networks: a platform for exploring and exploiting topological states and Majorana fermions

A specific device architecture that may support Majorana fermions and shows promise for manipulating them for quantum computation processes

Josephson Interferometry: what it tells you

$$I_c(\Phi) = \max \int_{-w/2}^{w/2} dy \, t J_c(y) \cos \left(\phi_0 + \phi_{op}(y) + \frac{2\pi}{\Phi_0} \left(\Phi + \int_0^y dy' d_m \delta B(y') \right) \right)$$

Critical
current
variation

Gap anisotropy
Domains
Charge traps

Current-
phase
relation

Non-sinusoidal terms
 π -junctions
Exotic excitations
e.g. Majorana fermions

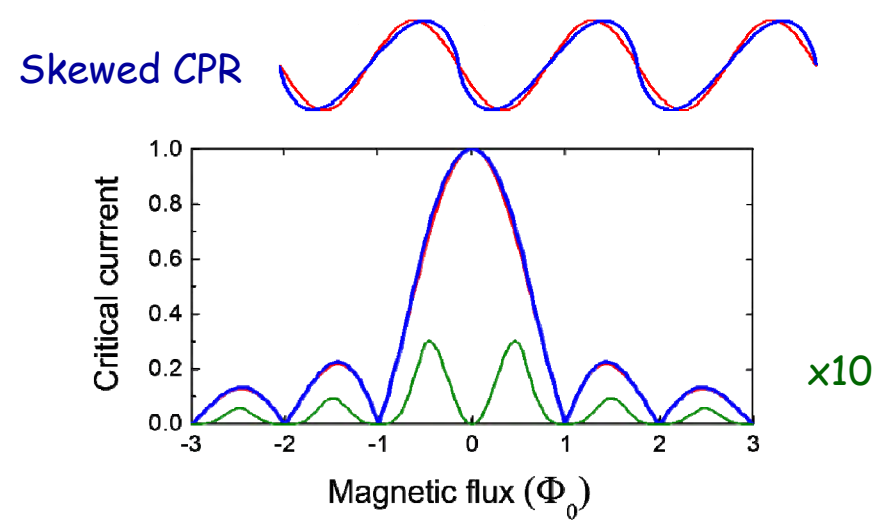
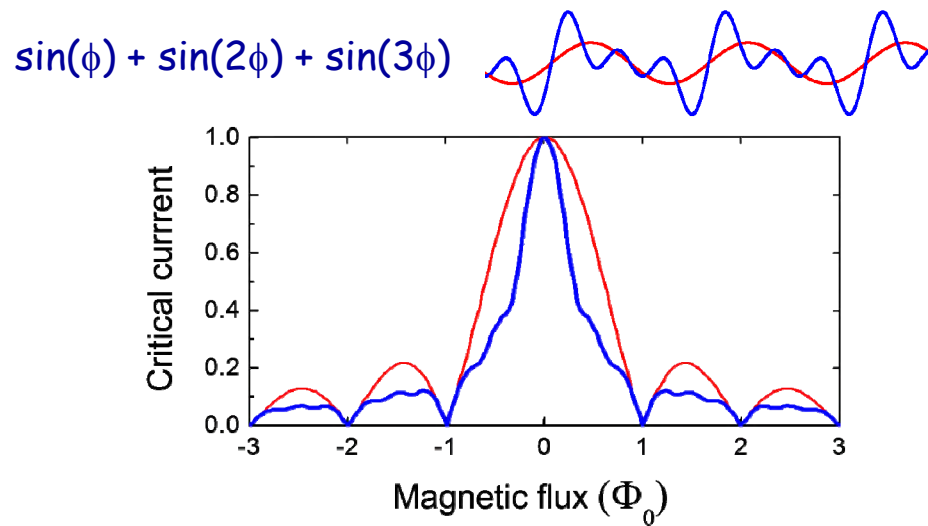
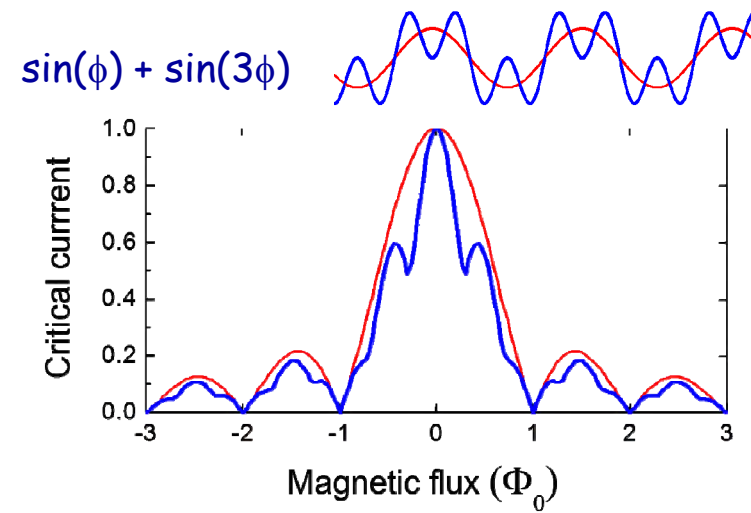
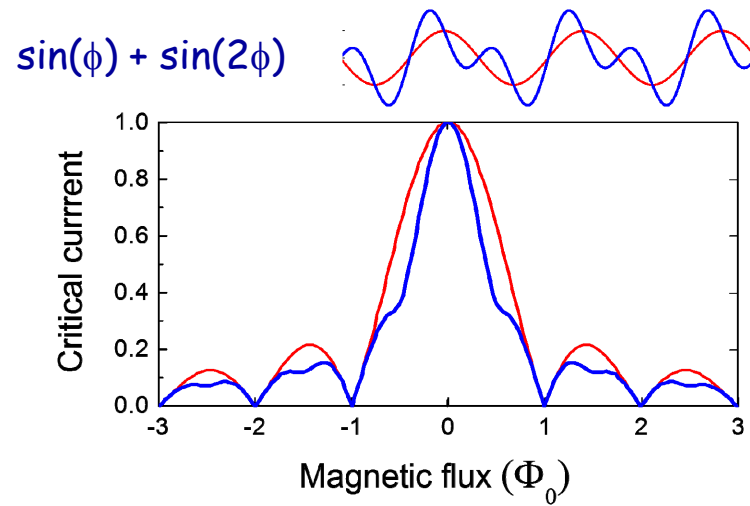
Order
parameter
symmetry

*Unconventional
superconductivity*

Magnetic
field
variations

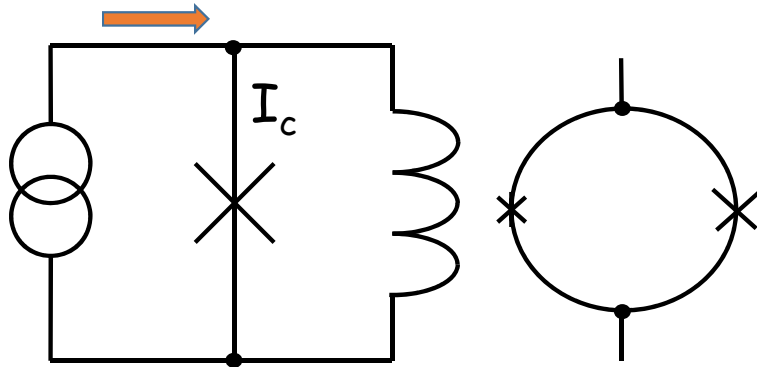
Flux focusing
Trapped vortices
Magnetic particles

Effect of non-sinusoidal CPR on diffraction patterns



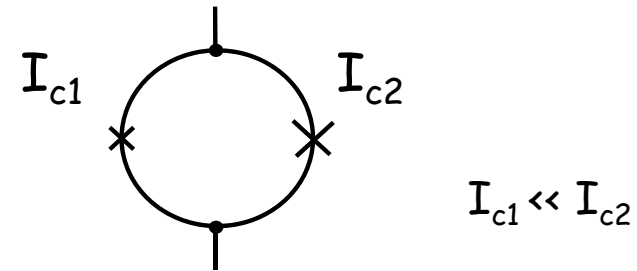
Direct measurement of the Current-Phase Relation

Interferometer technique (Waldram)



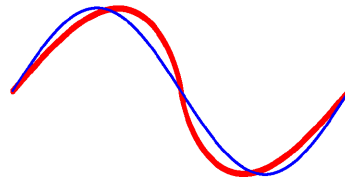
Junction in SC loop (rf-SQUID)
Inject current \rightarrow divides according to phase
Detect flux with SQUID
Extract CPR

Asymmetric dc SQUID technique



Junction embedded in dc SQUID
Apply flux \rightarrow induces circulating current
Measure critical current vs. flux
Modulation is dominated by the phase evolution of the small junction

Developed to study superconducting microbridges \rightarrow skewed CPR from an inductance that gives extra phase



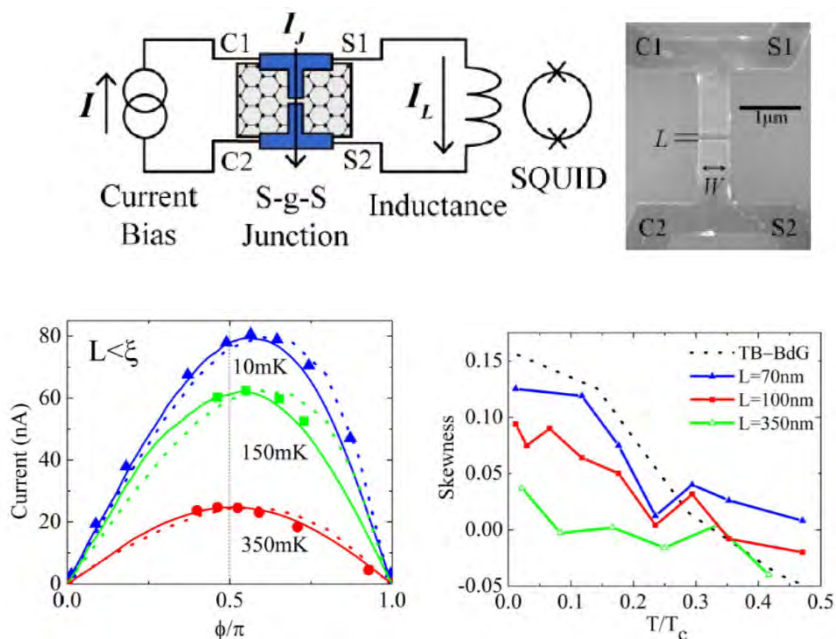
Superconductor-Graphene-Superconductor Junctions

High transparency states give higher harmonic contributions to the CPR, inducing skewness

$$I_c = \frac{e\Delta}{h} \sum_{i=0}^{\infty} \frac{T_n \sin(\phi)}{\sqrt{1 - T_n \sin^2(\phi/2)}} \quad \rightarrow \quad I_c(\phi) = \frac{e\Delta}{h} \frac{2W}{L} \cos(\phi/2) \tanh^{-1}(\sin(\phi/2))$$

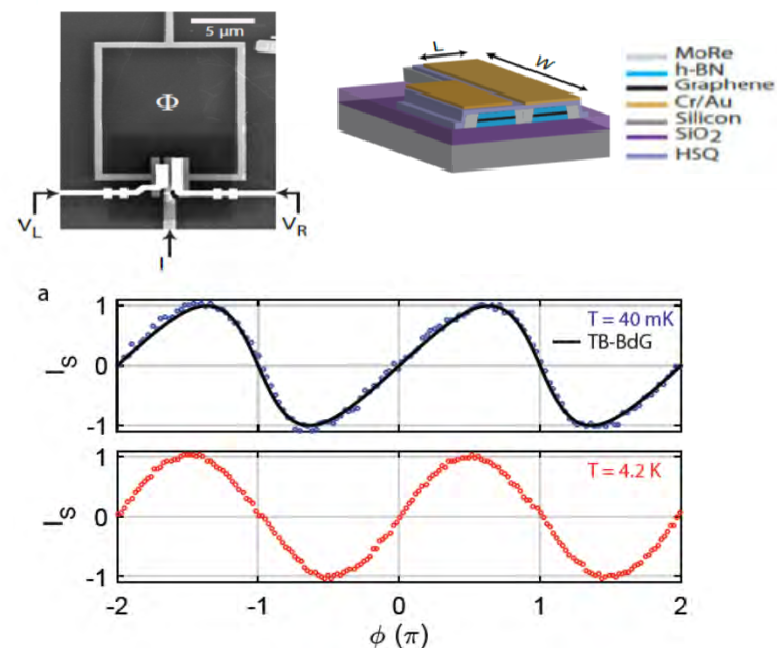
Titov and Beenaker,
PRB 94, 041401 (2006)

Interferometer technique (Urbana)



C. English et al., PRB 94, 115435 (2016)

Asymmetric SQUID technique (Delft)



Nanda et al., arXiv:1612.06895v2

S-TI-S Josephson junction networks: a platform for exploring and exploiting topological states and Majorana fermions

Agenda

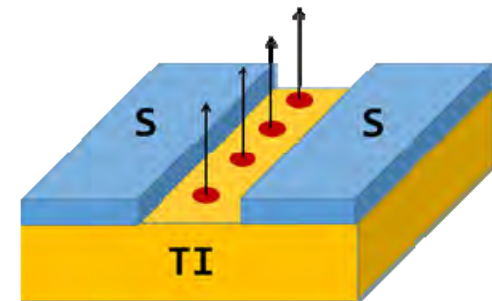
1. Topological insulators and Majorana fermions --- very little, but enough
2. Why junctions are better than nanowires (for Sergey)
3. The model --- what we think should happen
4. The experimental picture --- what we see
5. Mysteries --- what we still need to understand
6. Functionalization

Imaging --- finding Majorana

Braiding

Parity readout

Quantum processors



I am not going to talk about ...

History of Majorana fermions



Ettore Majorana

A particle that is its own anti-particle

Manifestation in condensed matter

Half of a fermion at zero-energy

Topology



Materials defined by topological order of band structure rather than symmetry --- support Majorana fermions

e.g. topological insulators

Classifications of topological states (Ryu et al.)

| TABLE - "Ten Fold Way" ["CARTAN Classes"] | | | | | Examples | |
|-------------------------------------------|----|----|------|----------------------------------------------|----------------------------------------------------------------------|----------------------|
| Name (Cartan) | T | C | S=TC | Time evolution operator $U(t) = \exp\{iHt\}$ | Anderson Localization NLSM Manifold G/H (compact (fermionic) sector) | SU(2) spin conserved |
| A (unitary) | 0 | 0 | 0 | $U(N)$ | $U(2n)/U(n) \times U(n)$ | yes/no |
| AI (orthogonal) | +1 | 0 | 0 | $U(N)/O(N)$ | $Sp(4n)/Sp(2n) \times Sp(2n)$ | yes |
| AII (symplectic) | -1 | 0 | 0 | $U(2N)/Sp(2N)$ | $SO(2n)/SO(n) \times SO(n)$ | no |
| AIII (chiral unitary) | 0 | 0 | 1 | $U(N+M)/U(N) \times U(M)$ | $U(n)$ | yes/no |
| BDI (chiral ortho) | +1 | +1 | 1 | $SO(N+M)/SO(N) \times SO(M)$ | $U(2n)/Sp(2n)$ | yes/no |
| CII (chiral symplectic) | -1 | -1 | 1 | $Sp(2N+2M)/Sp(2N) \times Sp(2M)$ | $U(n)/O(n)$ | no |
| D | 0 | +1 | 0 | $O(N)$ | $O(2n)/U(2n)$ | no |
| C | 0 | -1 | 0 | $Sp(2N)$ | $Sp(2n)/U(2n)$ | yes |
| DIII | -1 | +1 | 1 | $O(2N)/U(2N)$ | $O(n)$ | no |
| CI | +1 | -1 | 1 | $Sp(2N)/U(2N)$ | $Sp(2n)$ | yes |

Topologically-protected quantum computing via braiding



Encode information in the "parity" of non-local quantum states to avoid decoherence in quantum computing

Exploit non-Abelian statistics of Majorana fermions

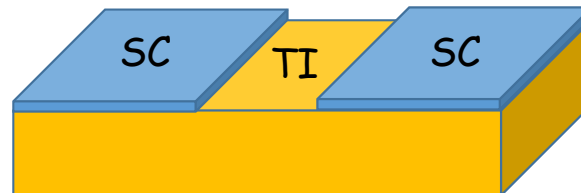
Topological systems that could support Majorana fermions

Intrinsic chiral materials:

- 5/2-quantum Hall state
- Topological superconductors, e.g. Sr_2RuO_4 , $\text{Nb}_x\text{Bi}_2\text{Se}_3$, ...

Engineer new chiral materials via proximity-coupling:

- spin-orbit semiconductor nanowires + superconductor + magnetic field
- spin-orbit semiconductor (InAs) + superconductor + ferromagnet
- non-centrosymmetric superconductor + ferromagnet
- chains of magnetic atoms on a superconductor
- (your idea here)
- * topological insulator + superconductor



Detecting Majorana fermions --- a grand challenge

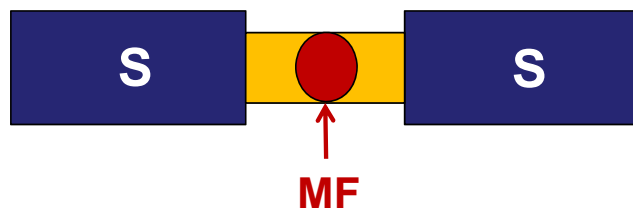
Many proposed schemes testing various properties of MF states:

spectroscopy --- probe zero energy states via quasiparticle tunneling

noise signatures --- 5/2-quantum Hall states

vortex interference --- Aharonov-Casher effect (Vishveshwara, etc.)

★ Josephson current-phase relation --- $I \sim \sin(\phi/2) = 4\pi$ -periodicity

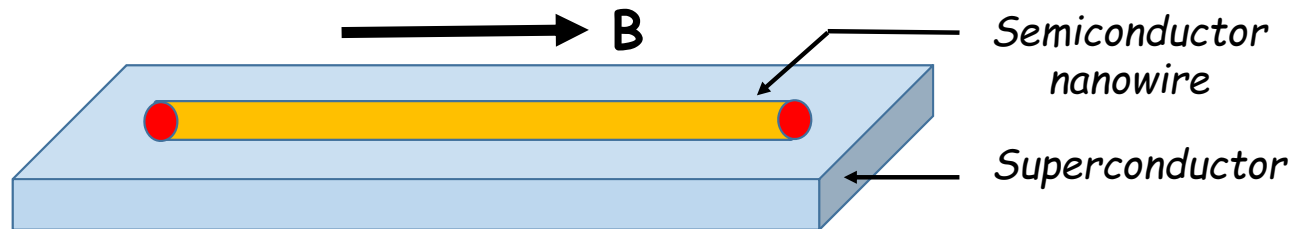


Kitaev (2000) + others

*Tunneling by a split fermion
rather than by Cooper pairs*

★ Josephson interferometry --- critical current vs. magnetic field

Most attention to date --- semiconductor nanowires

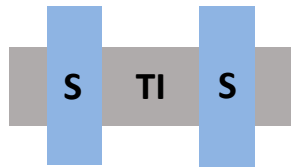


Advantages:

- System can be tuned into the topological state with a magnetic field
- Majorana fermions are stabilized at the ends of the wire
- Parity lifetimes are expected to be long (few other states around)
- Can probe zero energy states via quasiparticle tunneling spectroscopy

Disadvantages:

- Need a large oriented magnetic field to induce the topological state
- Majorana fermions are stabilized at the ends of the wire
- Challenging to manipulate/braid Majorana fermions



Why lateral S-TI-S junctions?

Not the favorite system of most because of the complexity:

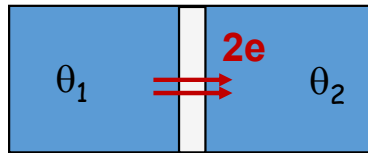
- 2D width (multiple channels)
- Multiple surfaces (top, edges, bottom)
- Conducting bulk states and trivial surface states in the TI

Advantages:

- Supports topological excitations without a strong magnetic field.
Allows phase-sensitive techniques, e.g. Josephson interferometry
- Allows access to barrier for probes and imaging
- Expandable into networks
- Enables multiple modes of operation to move and control Majorana fermions by phase, current, or voltage
- Schemes proposed to braid and perform logical operations.

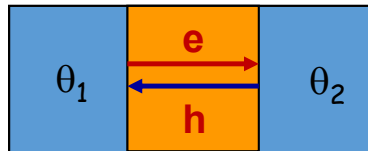
Trade-off stability for functionalization !

Key component: S-TI-S lateral Josephson junction



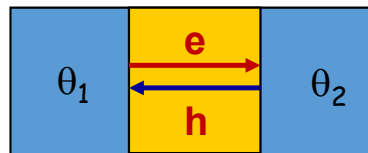
S-I-S
insulator barrier

Correlated tunneling of electrons
"Cooper-pair tunneling"



S-N-S
normal metal barrier

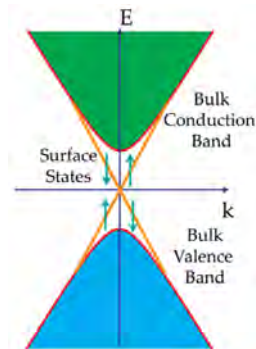
Coherent electron-hole pair transport
with Andreev reflection at SC interface
"Andreev bound states"



S-TI-S
topological insulator
barrier

Coherent electron-hole pair transport
via topological surface states
"Andreev bound states" +
"Majorana bound states" (zero-energy)

3D
topological
insulator



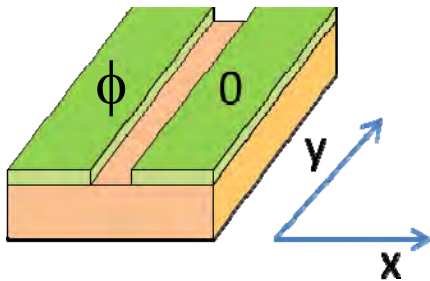
Insulating bulk (nearly)

High-mobility surface states protected by topology

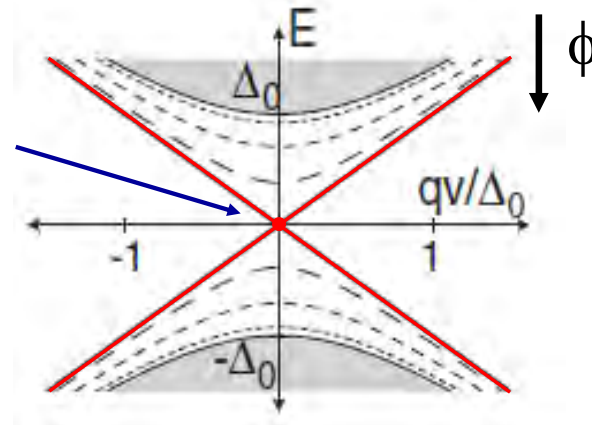
Spin-momentum locking prevents backscattering

Andreev Bound States (ABSs) in S-TI-S Josephson junctions

L. Fu and C. Kane, Phys. Rev. Lett. 100, 096407 (2008)



Zero-energy
Majorana
Bound states
for $\phi = \pi$



ABS energy levels

$$E_{\pm}(q, \phi) = \pm \sqrt{v^2 q^2 + \Delta^2 \cos^2(\phi/2)}$$

$$E_{\pm}(\phi) = \pm \Delta \cos(\phi/2) \quad \text{for } q=0$$

Supercurrent
contribution

$$I_{\pm}(\phi) = \frac{2e}{\hbar} \frac{\partial E_{\pm}}{\partial \phi} = \mp \left(\frac{e\Delta_0}{2\hbar} \right) \frac{\sin \phi}{\sqrt{\frac{v^2 q^2}{\Delta_0^2} + \cos^2\left(\frac{\phi}{2}\right)}}$$

Low-energy ABS:

$$I \propto \sin \phi$$

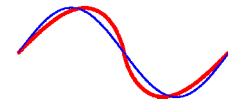
for q small, high transparency (skewed CPR)

Majorana states:

$$I \propto \sin \frac{\phi}{2}$$

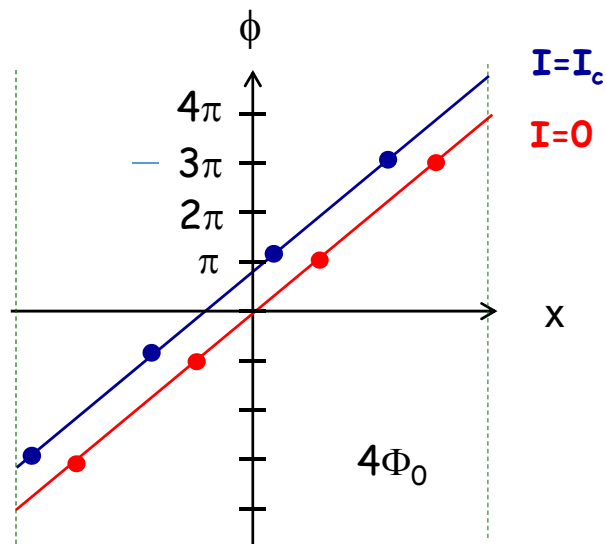
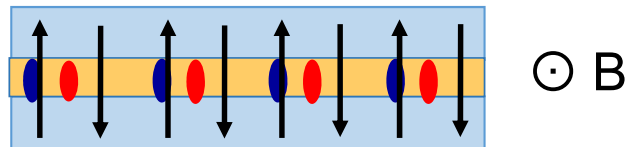
for $q=0$

(4π -periodic CPR)



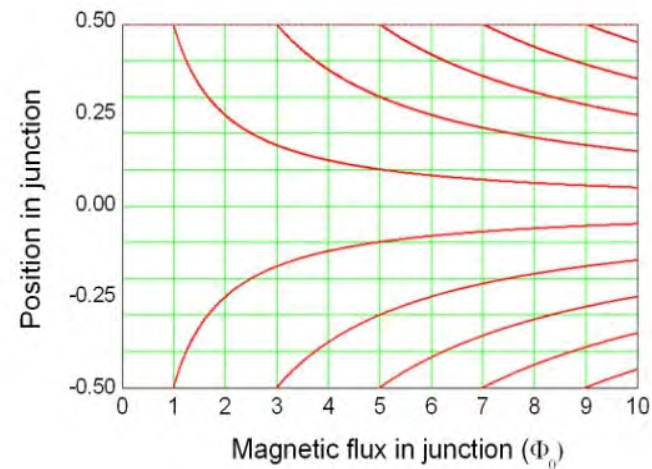
Current-controlled devices: lateral junctions in a magnetic field

Perpendicular magnetic field induces a phase gradient and circulating currents

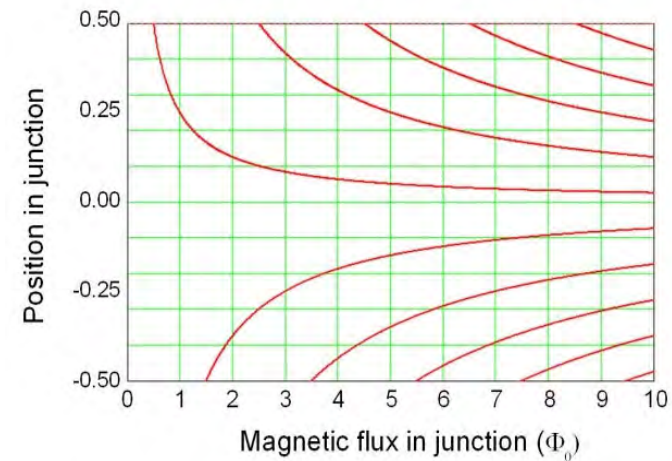


Majorana fermions enter junction attached to Josephson vortices --- located where the phase difference is an odd multiple of π

Zero current: MFs enter symmetrically



At critical current: MFs enter alternatively

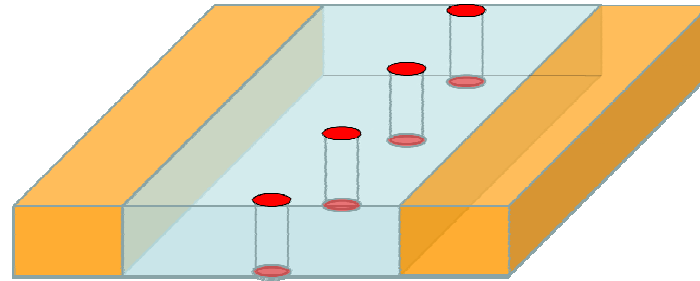


Where is the other Majorana?

Symmetric configuration:

MFs on top and bottom surface
connected by a vortex,
series of nanowires in the barrier

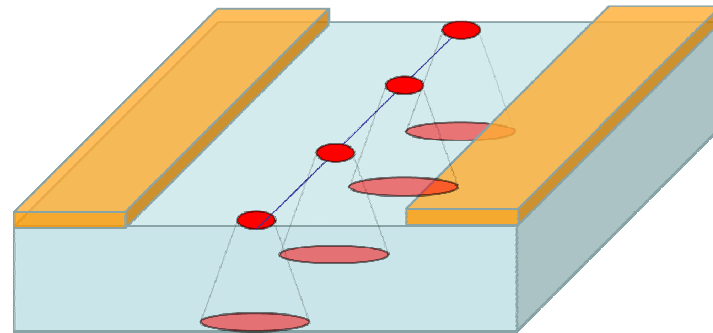
supercurrent on both surfaces



Asymmetric configuration: lateral junction

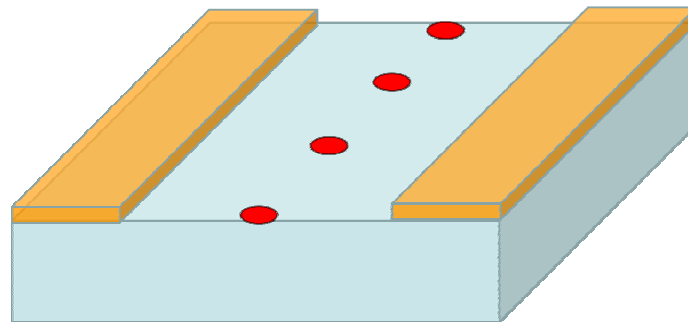
MFs on localized on top surface,
delocalized on bottom surface

supercurrent only on top surface



Extreme asymmetric configuration:

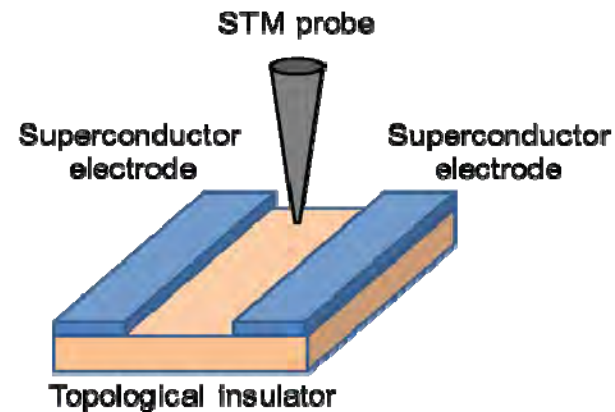
Consider only MFs on top surface,
partners are fully delocalized



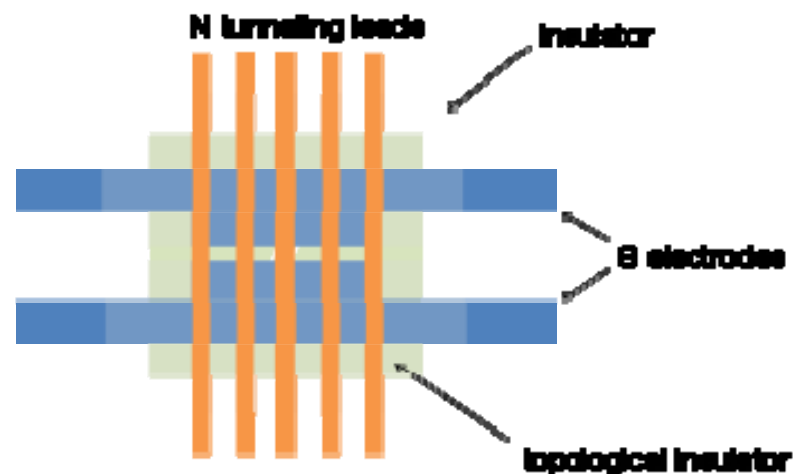
Detecting localized Majorana fermions via tunneling spectroscopy

Can scan an STM to map out the location of Majorana fermions

(experiments underway with Michigan State - Tessmer)

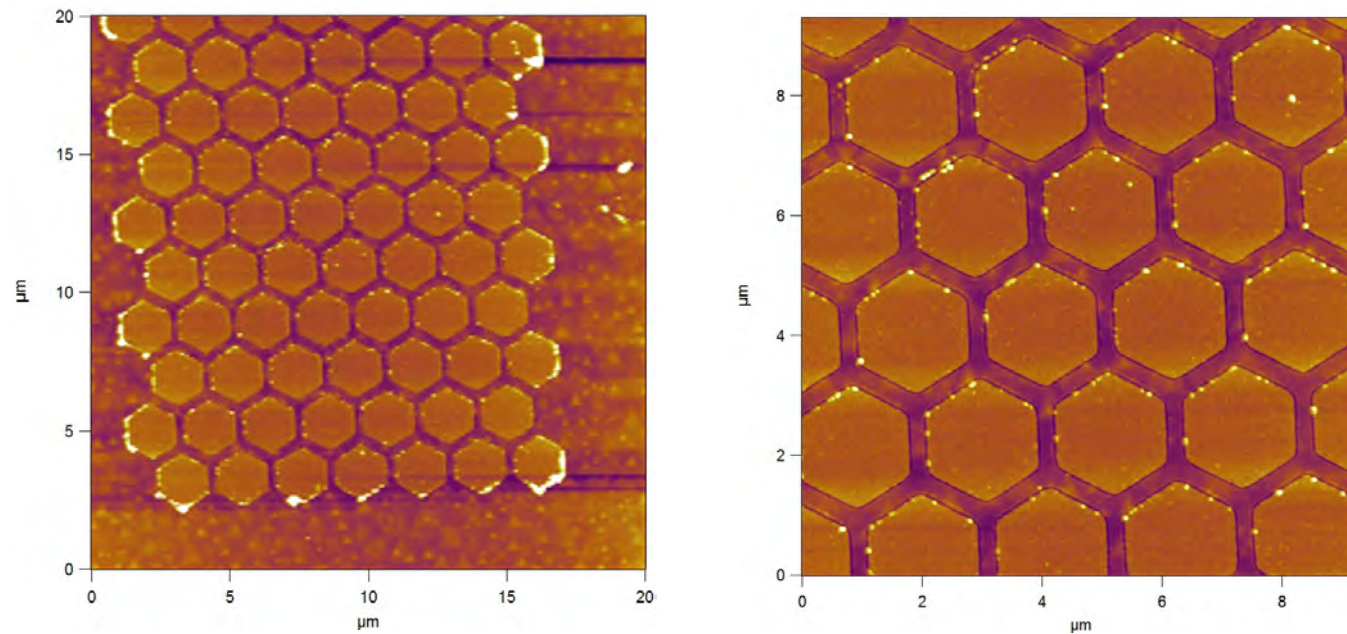


Or move Majorana fermions bound to Josephson vortices under fixed tunnel junctions



S-TI-S arrays for STM and SSM imaging

AFM images of Nb hexagons on Bi_2Se_3



This will be a suitable platform for multiple braiding operations by controlling magnetic fields and island phases

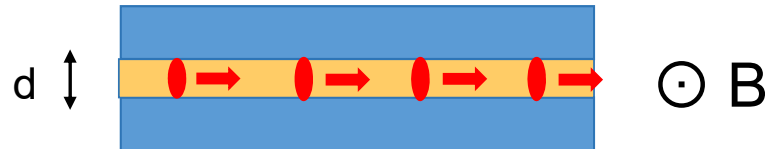
Voltage-controlled devices: moving Majorana fermions

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

Phase winds according to the Josephson relation:

$$v = \frac{V}{Bd}$$

Majorana fermions move laterally through junction at speed:



For $V = 1 \mu\text{V}$ and $d = 100 \text{ nm}$ and $B = 10 \text{ mT}$, $v = 1 \text{ km/s}$!

Provides way to move Majorana fermions fast along lateral junctions

Can manipulate MFs in multiply-connected junction networks for braiding

Majorana Fermion Surface Code for Universal Quantum Computation

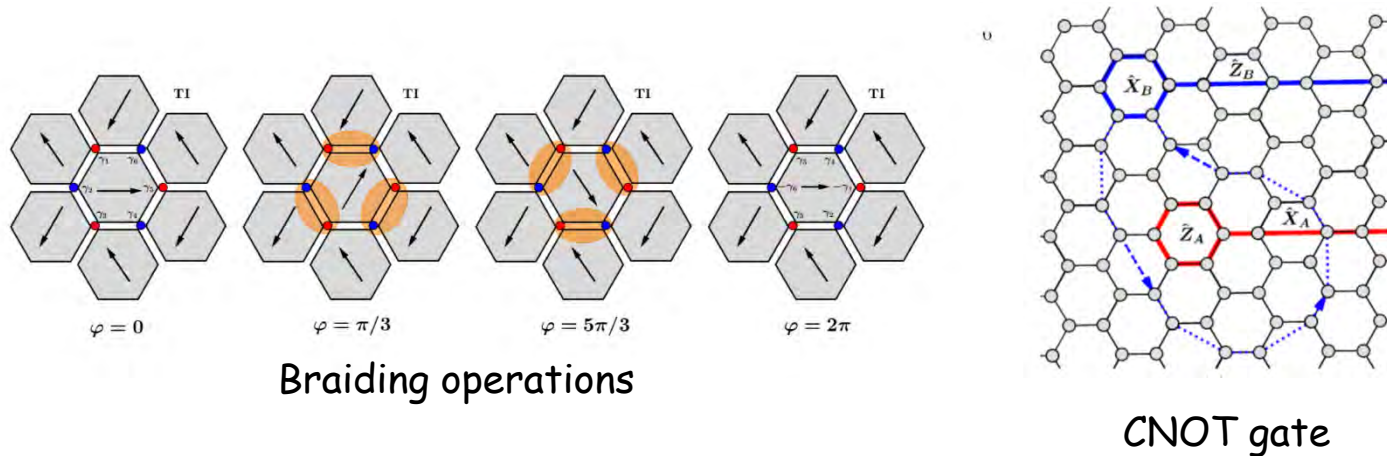
Sagar Vijay, Timothy H. Hsieh, and Liang Fu

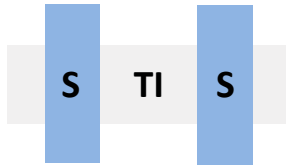
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 8 April 2015; published 10 December 2015)

We introduce an exactly solvable model of interacting Majorana fermions realizing Z_2 topological order with a Z_2 fermion parity grading and lattice symmetries permuting the three fundamental anyon types. We propose a concrete physical realization by utilizing quantum phase slips in an array of Josephson-coupled mesoscopic topological superconductors, which can be implemented in a wide range of solid-state systems, including topological insulators, nanowires, or two-dimensional electron gases, proximitized by s -wave superconductors. Our model finds a natural application as a Majorana fermion surface code for universal quantum computation, with a single-step stabilizer measurement requiring no physical ancilla qubits, increased error tolerance, and simpler logical gates than a surface code with bosonic physical qubits. We thoroughly discuss protocols for stabilizer measurements, encoding and manipulating logical qubits, and gate implementations.

Braid MFs by phase-control of islands





Experiments: completed and in progress

Transport in Nb-Bi₂Se₃-Nb junctions

- Phase transition in the location of the topological surface state

Josephson interferometry in Nb-Bi₂Se₃-Nb junctions and SQUIDs

- Node-lifting of the magnetic field modulation patterns
- Non-sinusoidal components in the current-phase relation
- Evidence for 4π -periodicity that could arise from Majorana states

Functional steps

- Schemes for braiding
- Schemes for reading the parity

UIUC research team and collaborators

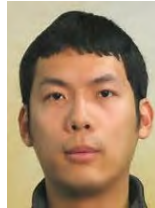
Experiments

Dale Van
Harlingen



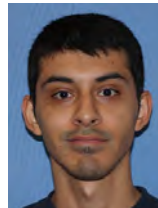
UIUC

Guang
Yue



Postdoc

Gilbert
Arias



Grad student

Jessica
Montone



Grad student

Alexey
Berzryadin



UIUC

Stuart
Tessmer



Michigan State

Erik
Huemiller



Inprentus

Can
Zhang



Intel

Aaron
Finck



IBM Research

Cihan
Kurter



Missouri S&T

Vlad
Orlyanchik



Accio Energy

Martin
Stehno



U. of Würzburg

Samples

Seongshik
Oh



Rutgers

Yew San
Hor



Missouri S&T

Theory

Pouyan
Ghaemi



CCNY

Smitha
Vishveshwara



UIUC

Taylor
Hughes



UIUC

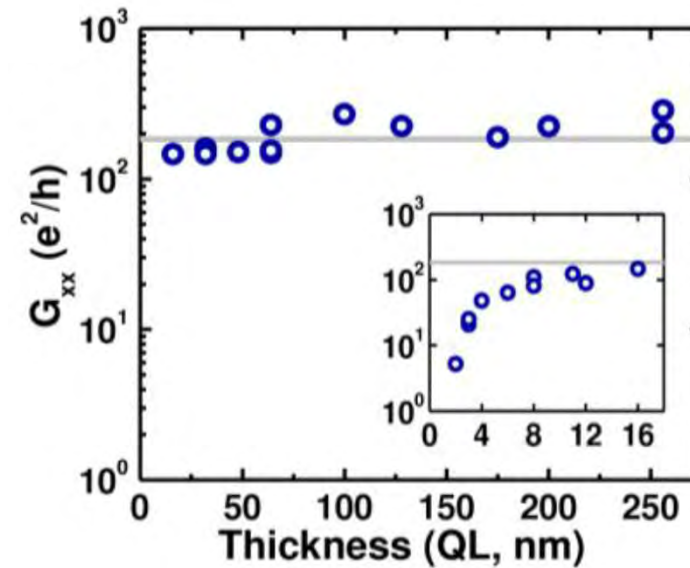
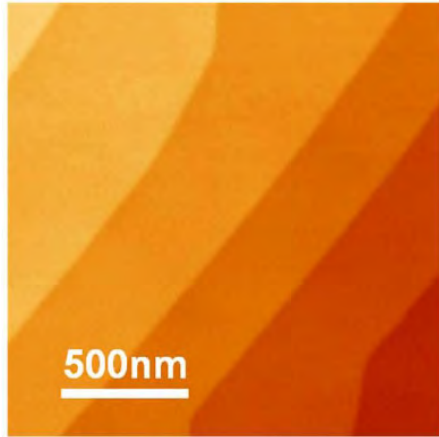
Suraj
Hedge



UIUC

Bi_2Se_3 Materials Characteristics

- Bi_2Se_3 film MBE-grown on Al_2O_3



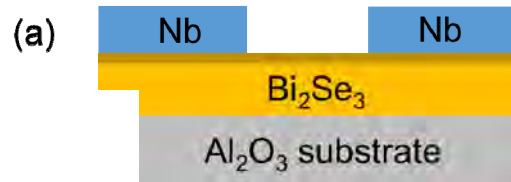
Bulk is insulating - conductance is dominated by **two surface channels**:

1. Trivial 2DEG (2-3 quintuple layers)
2. Topological surface state

This suggests that:

1. Most of the supercurrent is carried by the top surface.
2. Bulk conductance does not play a large role in the supercurrent properties.

Nb/Bi₂Se₃/Nb Josephson Junctions



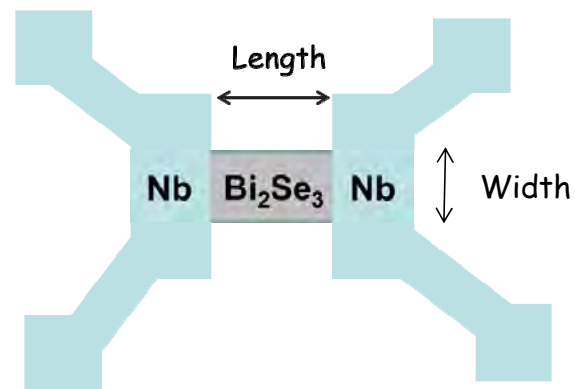
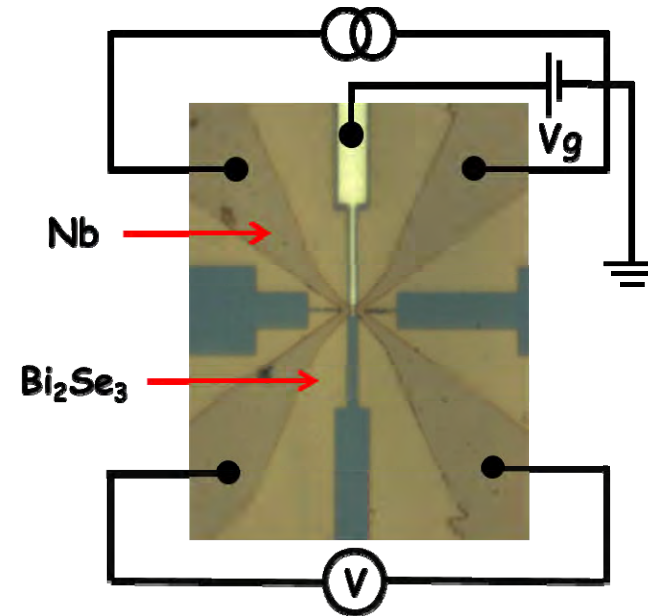
- E-beam lithography, W
- Ion milling
- Evaporation and sputtering

Typical Dimensions:

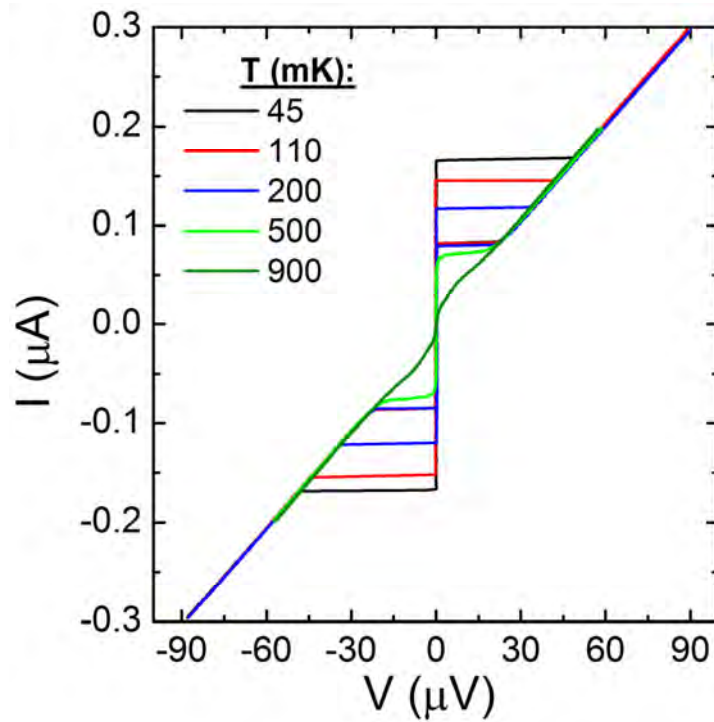
Length = 100-300nm

Width = 300nm-1 μ m

Top gate dielectric ~ 35-40nm
(ALD Al₂O₃/HfO₂)

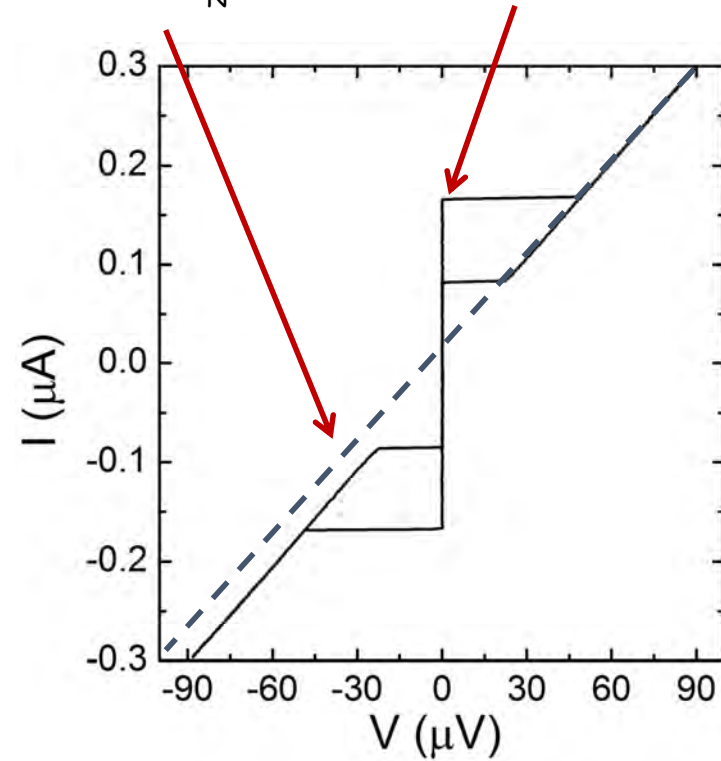


Supercurrents in S-TI-S junctions

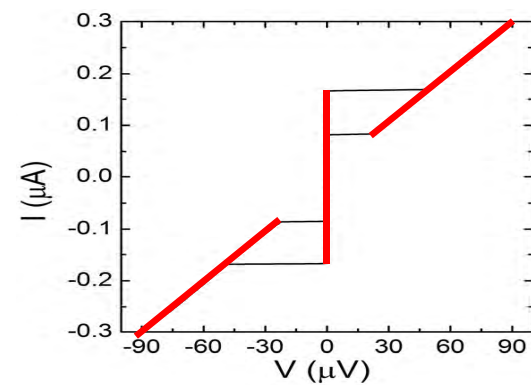
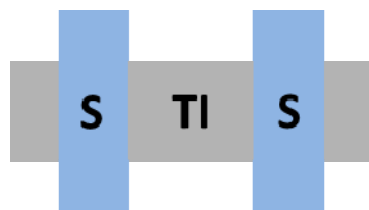


Normal State
Resistance: R_N

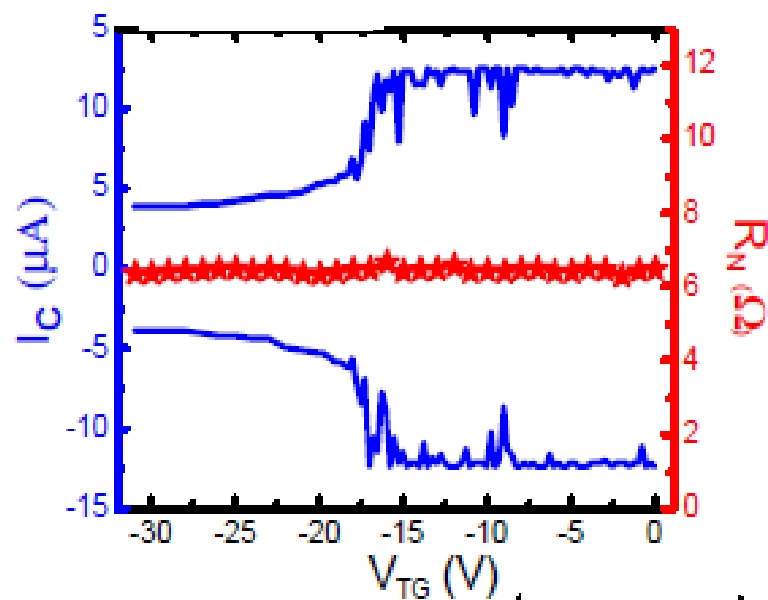
Critical Current: I_C



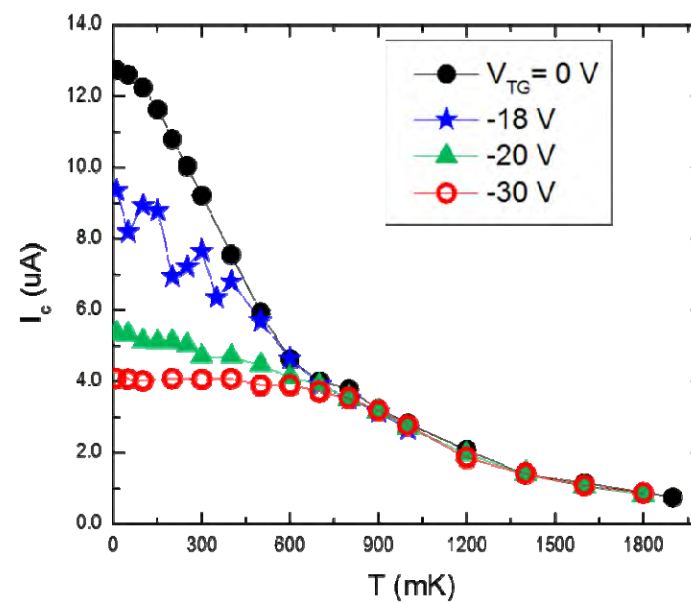
Supercurrents



Gate voltage dependence



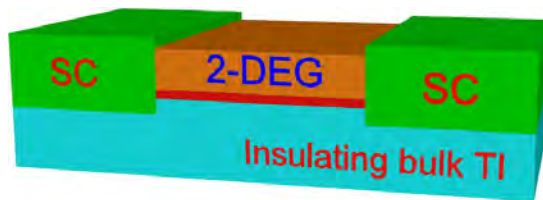
Temperature dependence



Theoretical Model – topological phase transition

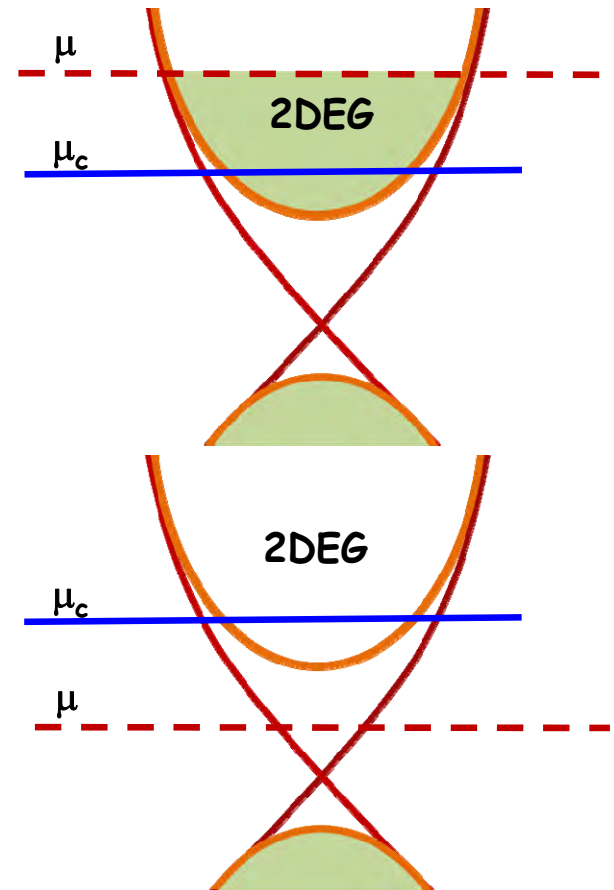
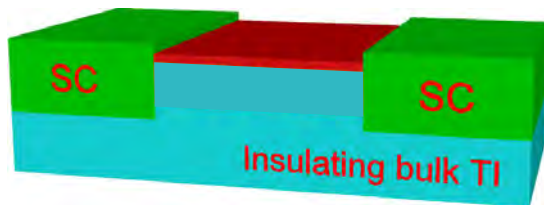
Low gate voltage:

- Fermi energy in conductance band
- Topological surface state buried

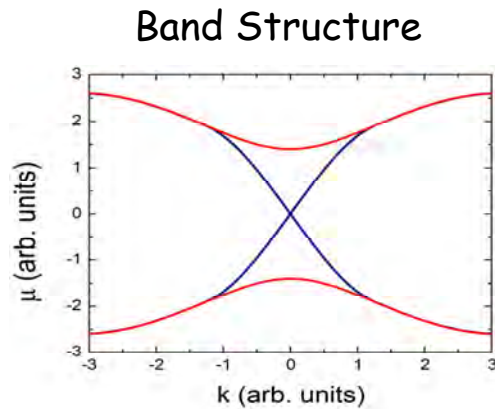


High (negative) voltage:

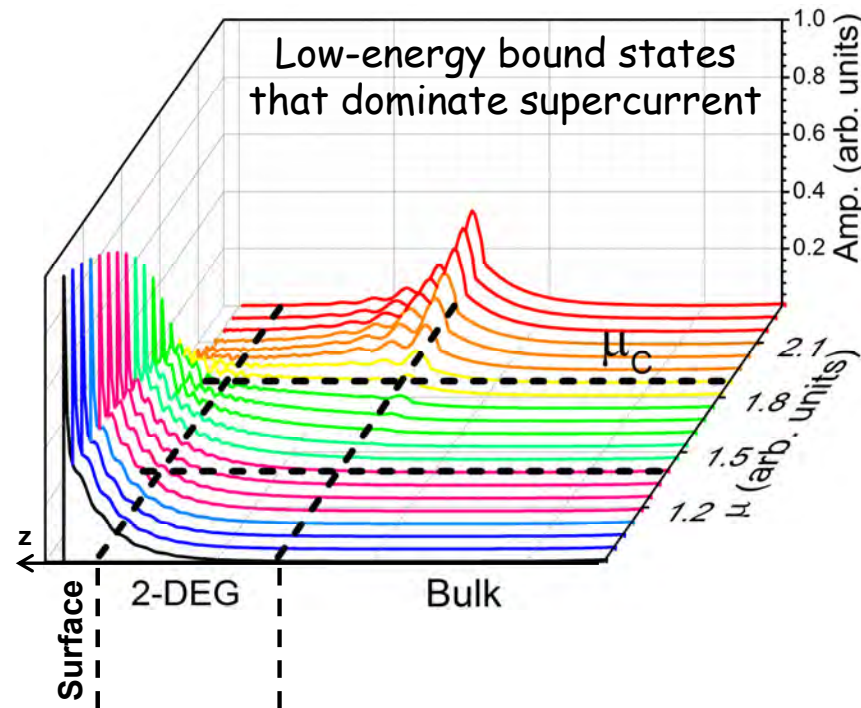
- Fermi energy in the band gap
- Topological surface state on surface



Numerical Solutions --- low-energy Andreev Bound States



As μ decreases, the ABS move from the interface between the 2DEG and the insulating region to the free surface \rightarrow topological phase transition

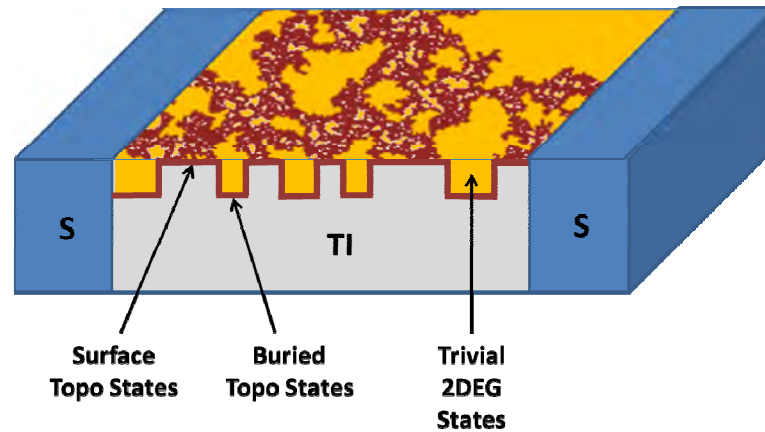


These states carry the majority of the supercurrent which drops because:

1. The transparency is higher when buried - 2DEG protects the states
2. The transport becomes most diffusive on the surface due to scattering
3. The 2DEG contribution to the supercurrent turns off when depleted

Physical Picture

Topological surface state winds through 2DEG at intermediate gating

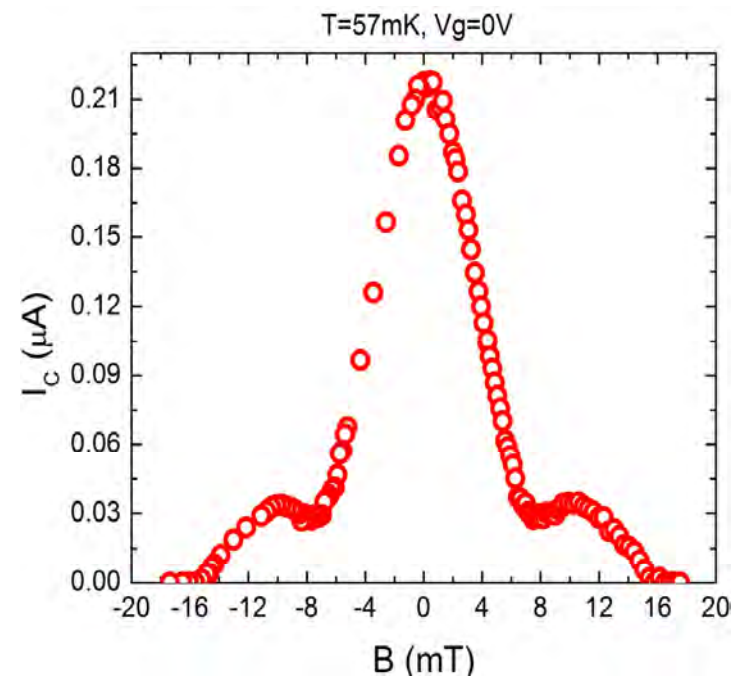
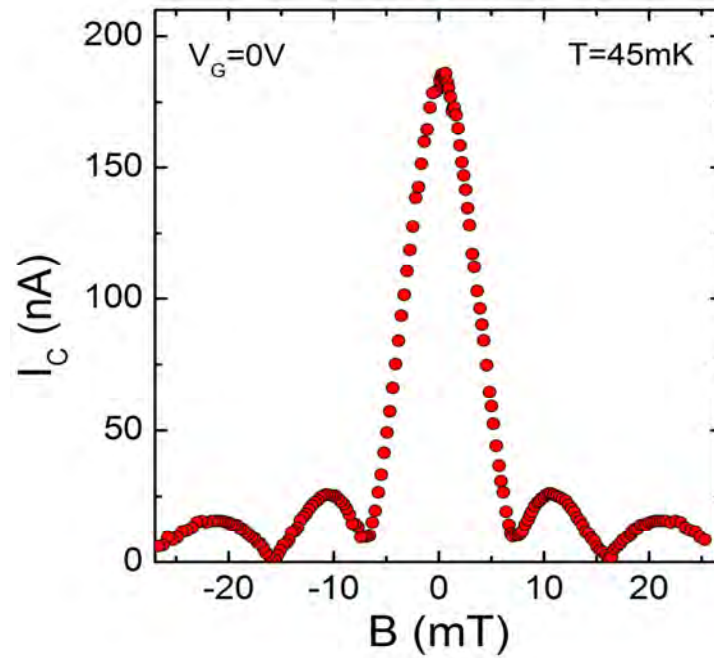


dynamically-meandering topological surface state

Most likely these arise from charge fluctuations in the gate than change the local carrier density and induce the phase transition

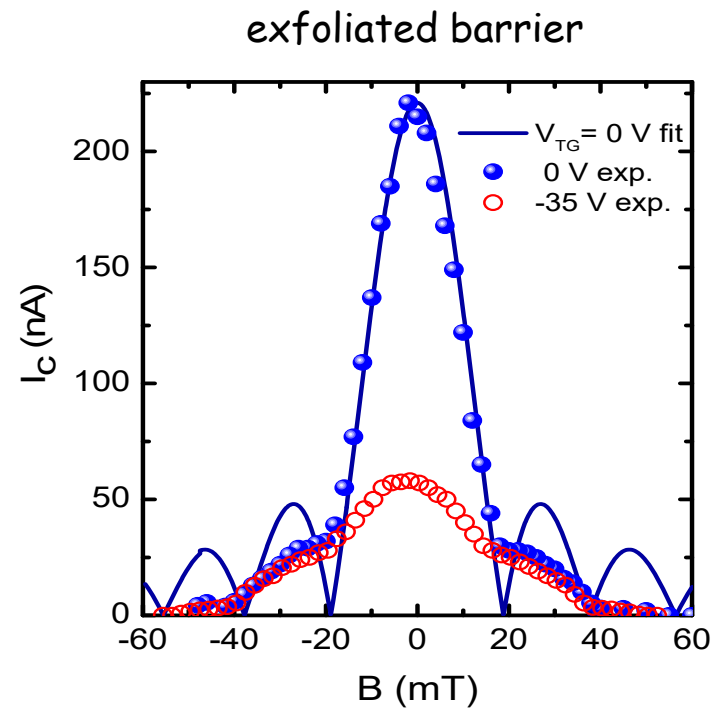
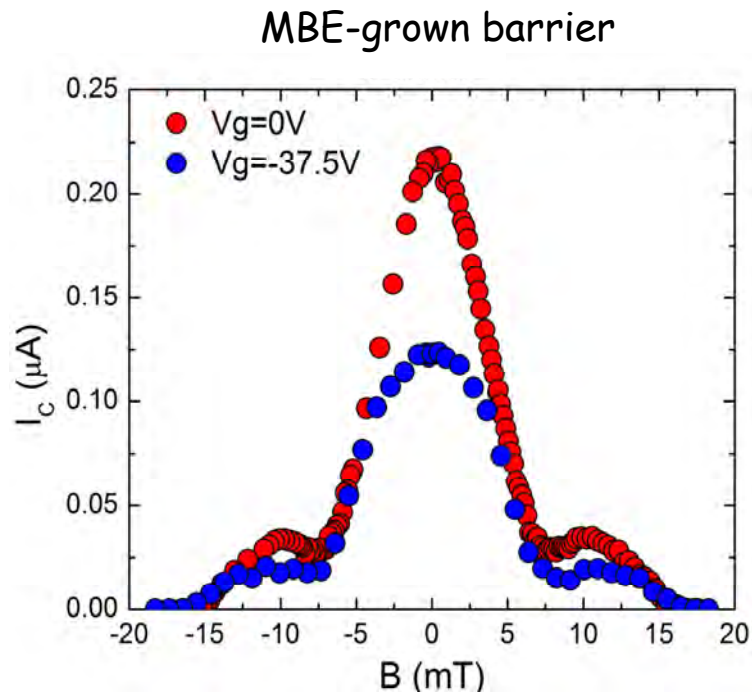
Complex system: junction transport will be affected by local switching dynamics, 2D percolation physics, and interactions/avalanches

Supercurrent diffraction patterns



- 1st minimum does not go to zero
- 2nd minimum does go to zero

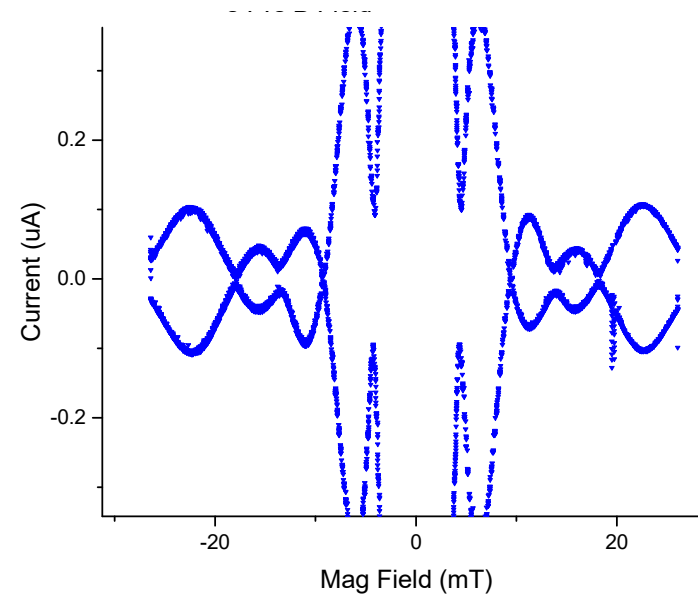
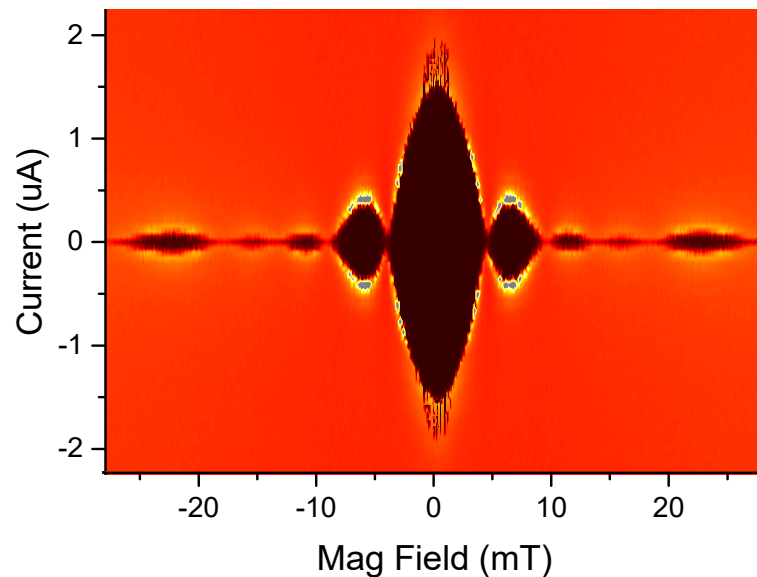
Diffraction pattern vs. gating



- Central peak drops at Dirac point
- Side lobe is nearly unchanged
- Lifting of first node persists

Diffraction pattern --- higher level nodes

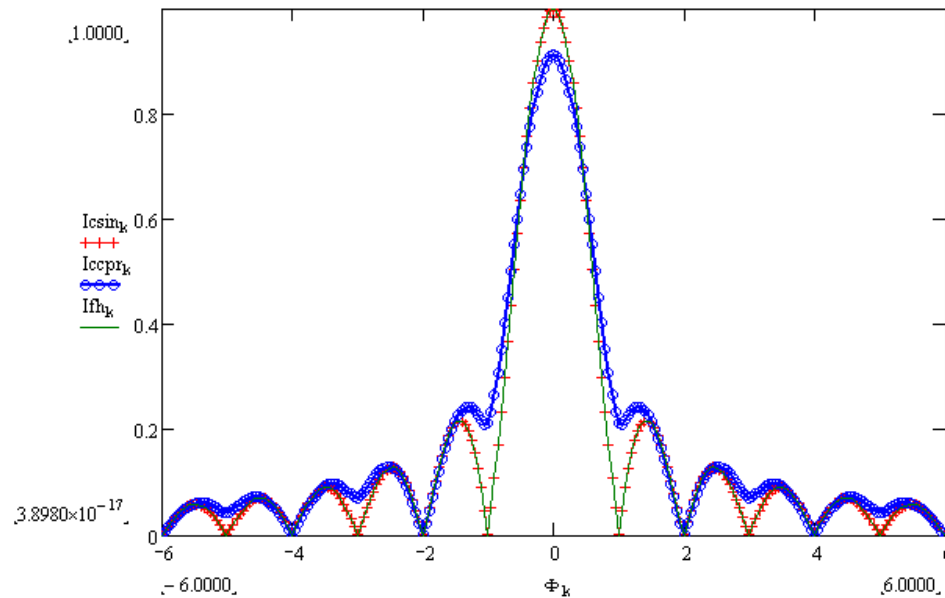
Question always arises whether higher-order nodes are lifted ---
difficult to test because the critical currents are very small



In this junction, 1st and 3rd nodes are lifted, 2nd and 4th nodes are hard
but extra bump at higher fields indicating some junction inhomogeneity

Simulations --- Hybrid Current Phase Relation

$$\text{CPR: } I(\phi, V_g) = I_{c1} \sin(\phi) + I_{c2}(V_g) \sin(\phi/2)$$



1st minimum lifted
2nd exactly nulled

Reproduces some key features

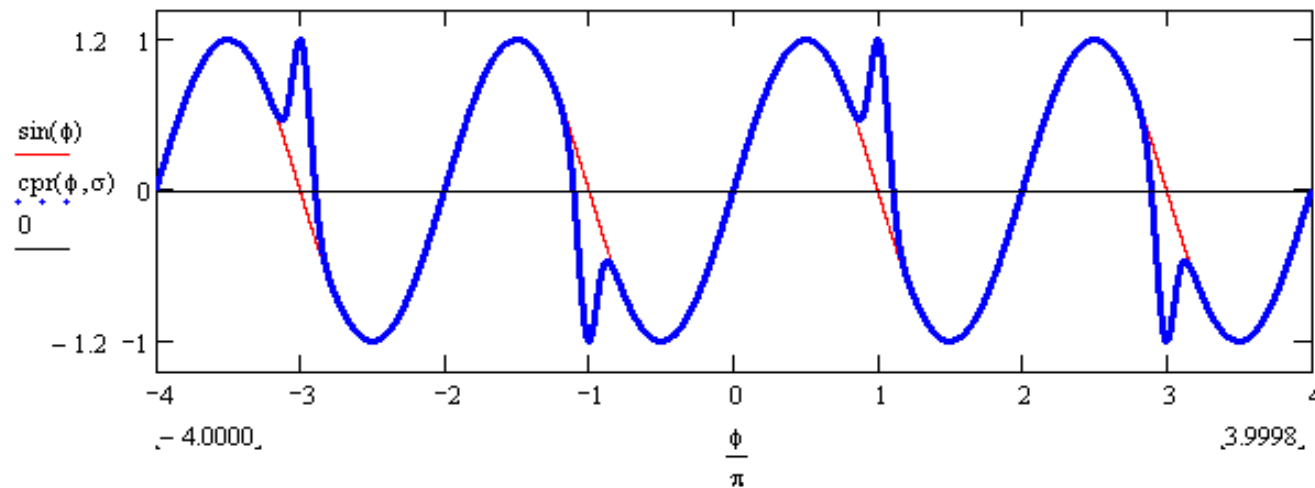
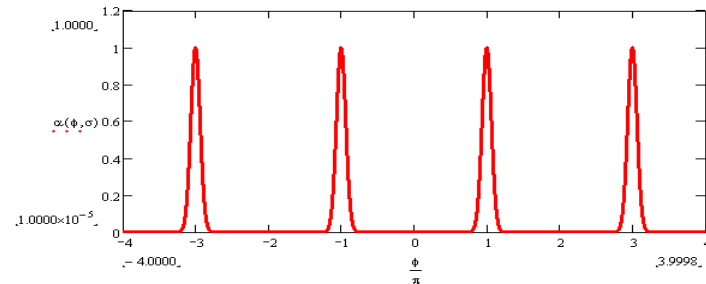
However, this assumes a uniform $\sin(\phi/2)$ -component with should not be the case for Majorana fermions --- only stable when $\phi \sim \pi$

Model --- Current-Phase Relation for S-TI-S junction

Majorana fermions nucleate when/where the phase difference is π

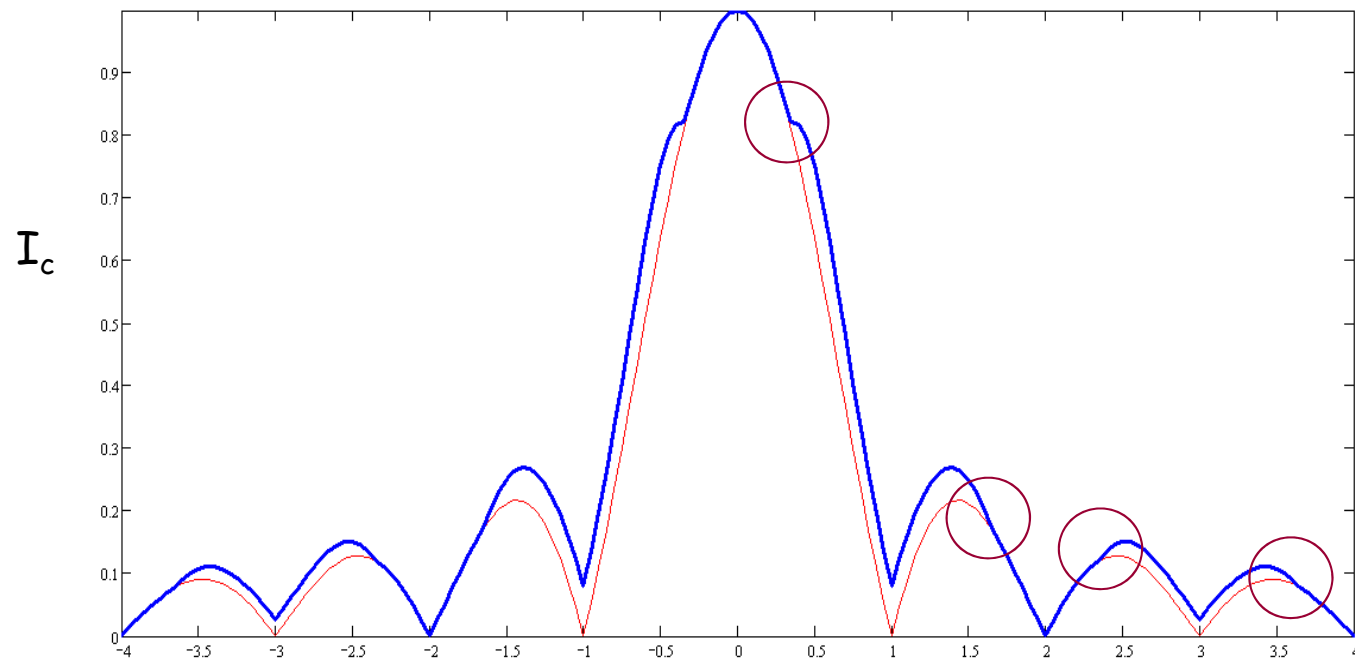
$$I_c(\phi) = (1 - \alpha) \sin(\phi) + \alpha \sin\left(\frac{\phi}{2}\right)$$

Width of the Majorana region will depend on details of the sample



Consider the junction to break up into 1D wires with a $\sin(\phi/2)$ -component

Diffraction patterns for S-TI-S junction

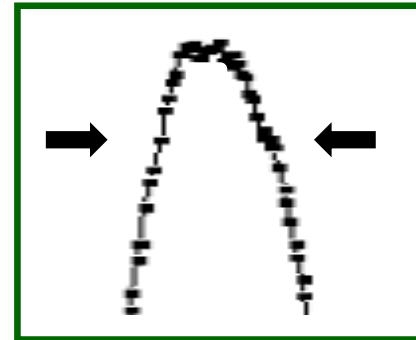
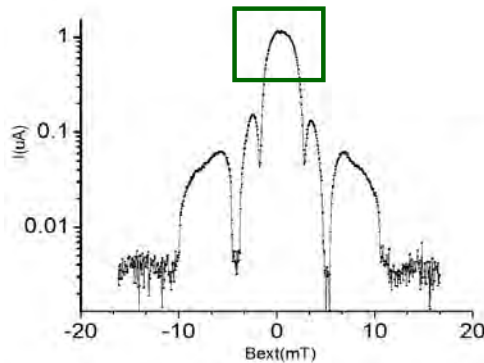
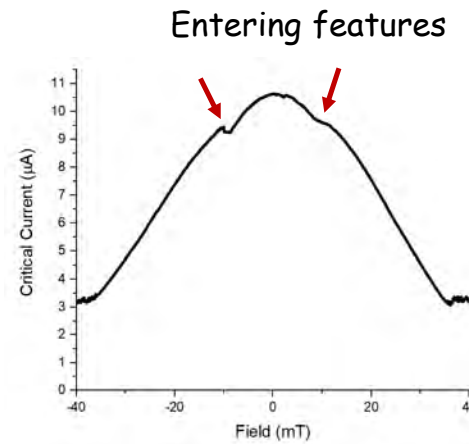


Lifting of odd nodes

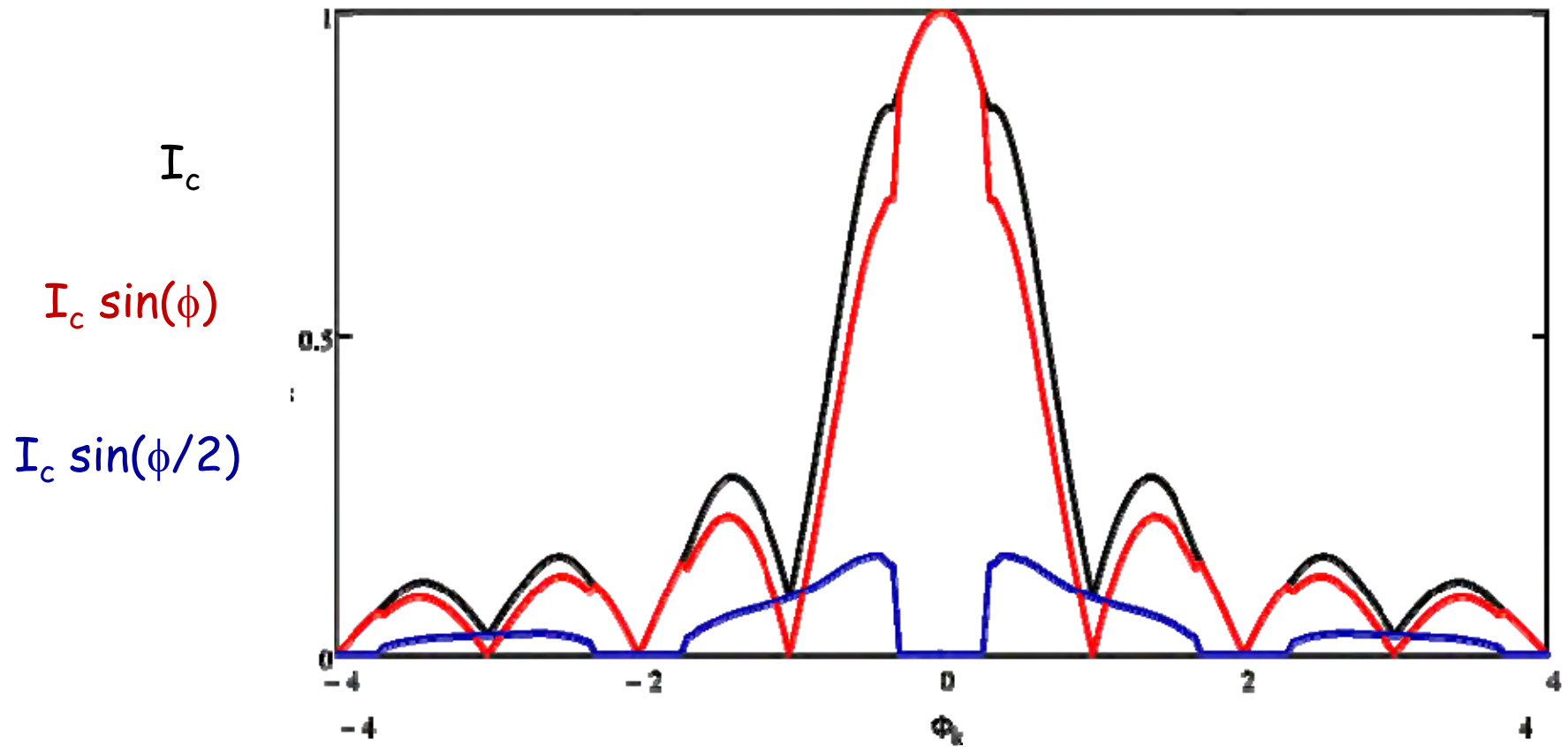
Additional structure when Majorana fermions enter the junction ---
a signature of a localized $\sin(\phi/2)$ component in the CPR

Josephson vortex + Majorana Fermion entry features

- Model predicts an increase in the equilibrium supercurrent when the first vortex/MF enters the junction
- We observe these features in the diffraction pattern
- Important feature --- indicates that MF modes are localized and when they are in the junction

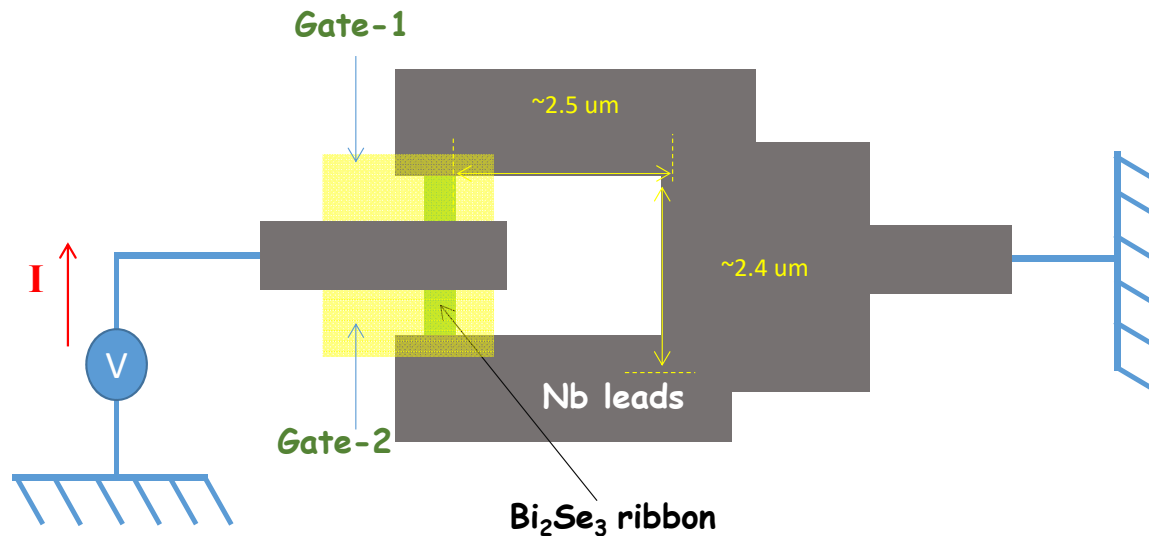


Current carried by Cooper pairs vs Majorana fermions



S-TI-S dc SQUID

Motivation: can use SQUID loop to adjust relative phase of the junctions and control the Majorana fermions



Bi₂Se₃: 19 nm thick, 4 μm long, 300 nm wide exfoliated piece

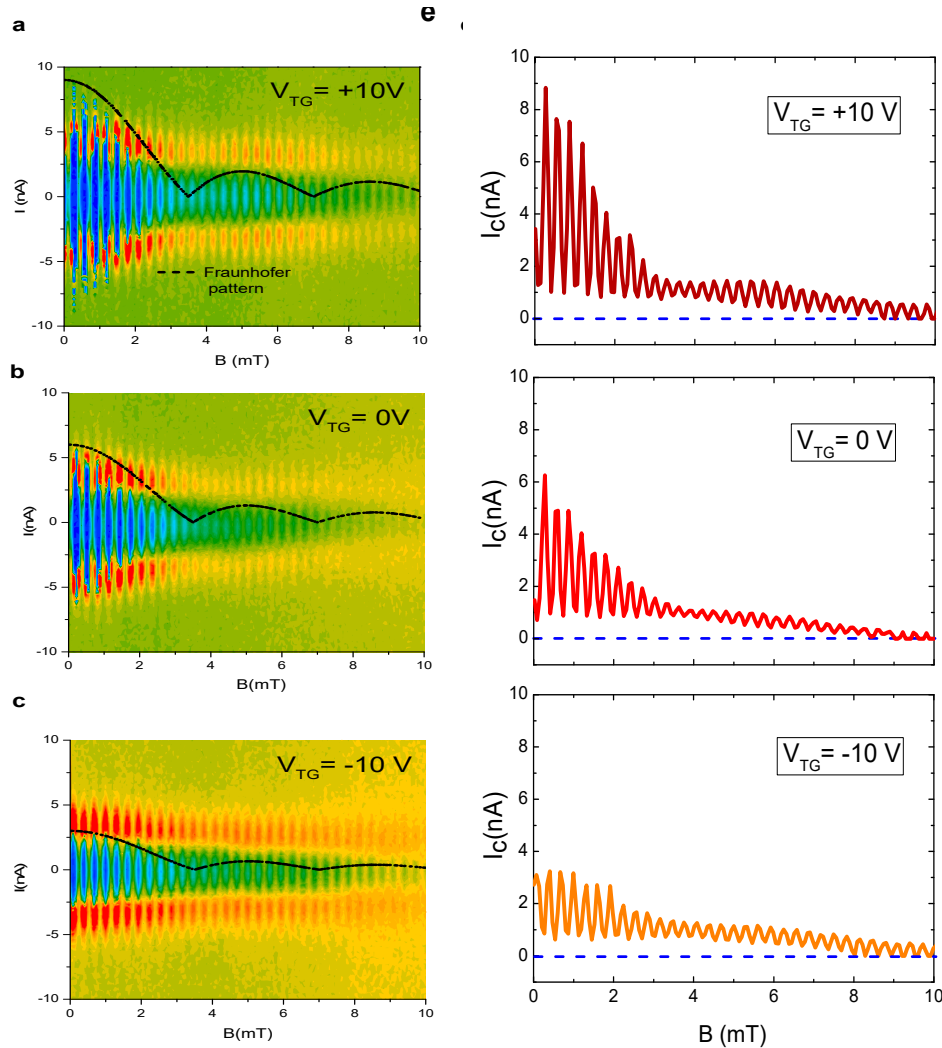
Junctions: length 300 nm, width 300 nm

Area loop ~ 6 μm²

Area junction ~ 0.9 μm²

} ratio ~ 60

SQUID oscillations --- gate dependence --- envelopes

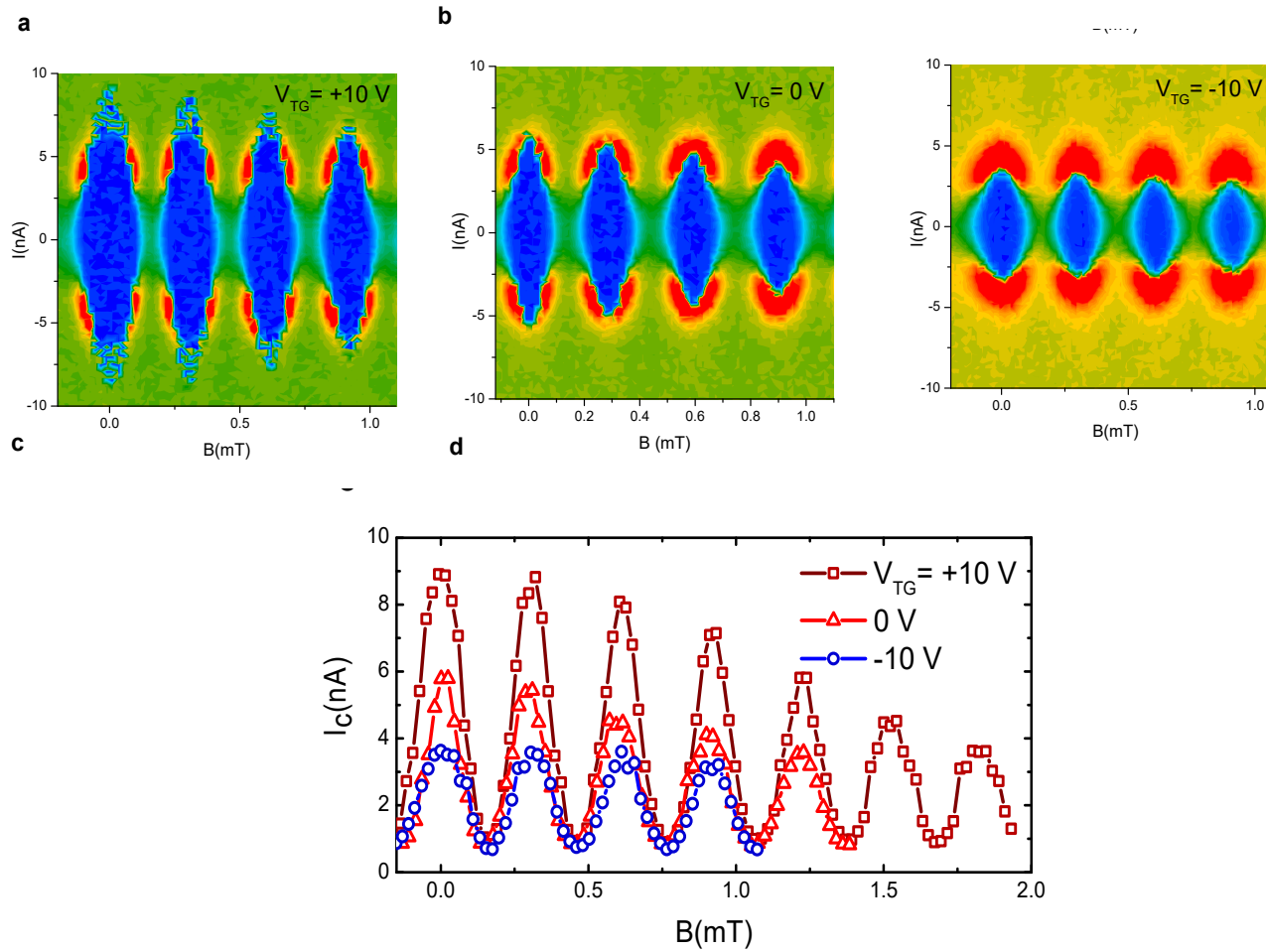


Envelopes exhibit same behavior as single junction diffraction: first node stays high, second vanishes

SQUID oscillations also do not go to zero as would be expected for $\beta \ll 1$ and a symmetric SQUID

$$\beta = 2\pi L I_c / \Phi_0$$

SQUID oscillations --- gate dependence



Modulation depth is gate-dependent: peaks drop; nodes stay constant

Node-lifting in Josephson junctions and SQUIDs

Josephson junctions:

- Inhomogeneous current distribution --- usually lifts all nodes
- $\sin(\phi/2)$ -component in the current-phase relation
- Edge currents due to MF hybridization (Potter-Fu model)

dc SQUIDs:

- $\sin(\phi/2)$ -component in the current-phase relation
- Finite inductance of SQUID loop $\beta = 2\pi L I_c / \Phi_0$
- Asymmetry in the junction critical currents $\alpha = (I_{c1} - I_{c2}) / (I_{c1} + I_{c2})$
- Asymmetry in the SQUID loop inductance $\eta = (L_1 - L_2) / (L_1 + L_2)$
- Skewness in the current-phase relation* $s = (2\phi_{\max} / \pi) - 1$

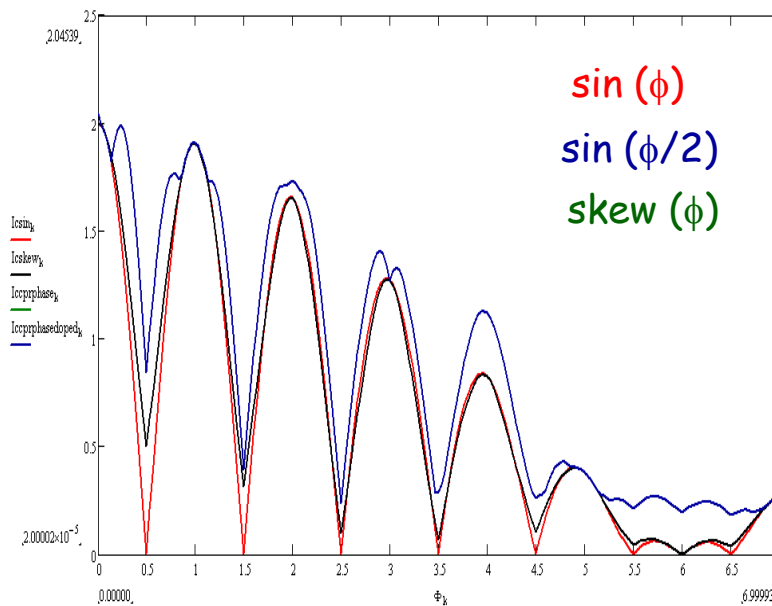
* *Skewness does NOT lift nodes in single junctions*

$$I_{c_{\min}} / I_{c_{\max}} \sim \beta \sim \alpha \sim \eta \sim s$$

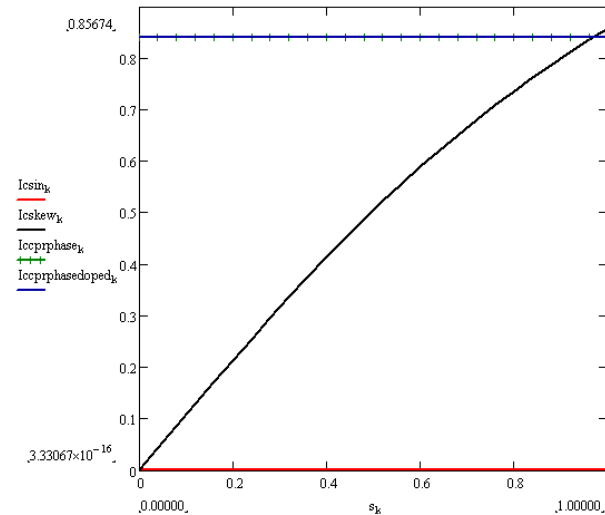
Simulations of skewed CPR on dc SQUID

Skewness arises from transport through high transparency quantized Andreev bound states

$$I_c = \frac{e\Delta}{h} \sum_{i=0}^{\infty} \frac{T_n \sin(\phi)}{\sqrt{1 - T_n \sin^2(\phi/2)}}$$



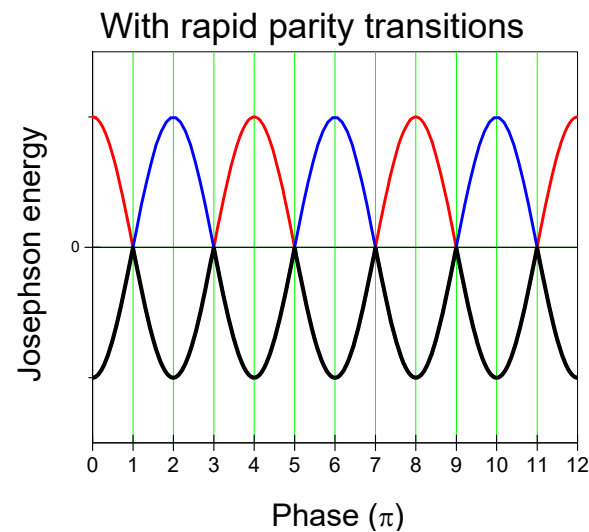
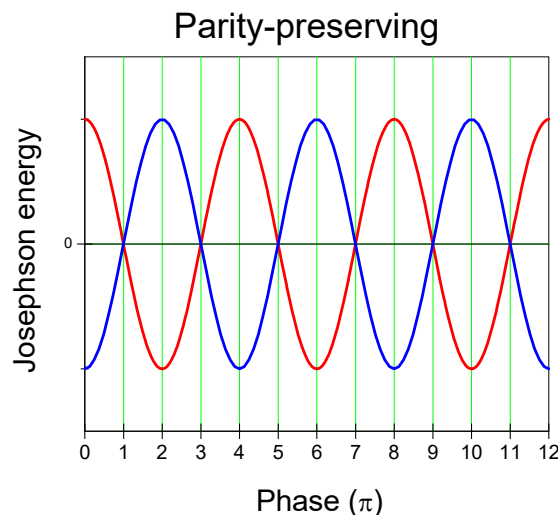
Node-lifting vs. skewness



- SQUID node requires $I(\phi) = -I(\phi+\pi) \Rightarrow$ lifted by skewness and 4π -periodicity
- Single junction node requires $I(\phi) = I(\phi+2\pi) \Rightarrow$ lifted only by 4π -periodicity

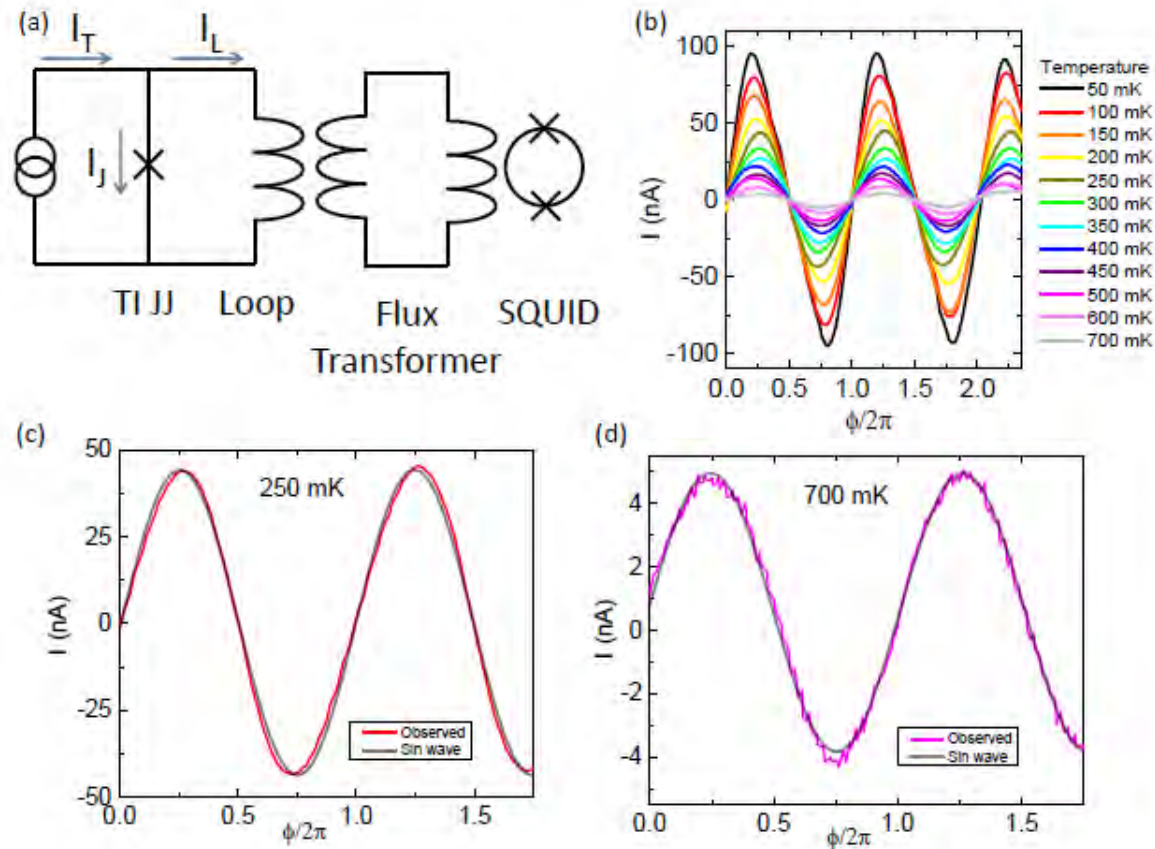
Why can we see the $\sin(\phi/2)$ -component in the CPR?

1. Cancellation of 2π -periodic component by destructive interference at nodes reduces the background \rightarrow effectively a series of 1D channels with a $\sin(\phi/2)$ CPR
2. Dynamical measurement at finite voltage so phase evolves fast enough to avoid parity transitions that suppress the 4π -periodic component. Typical Josephson frequency \sim GHz.



Slow measurements of the CPR should not see this

CPR Measurements via Interferometer technique

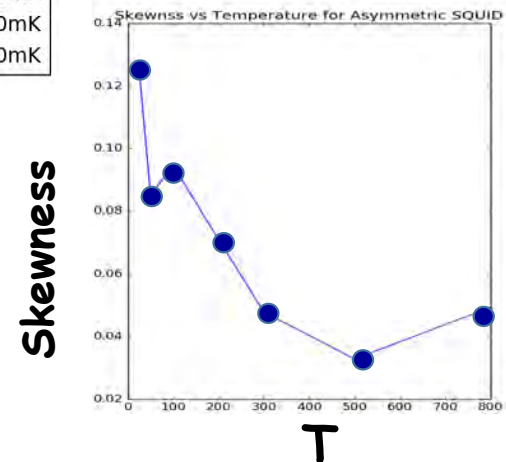
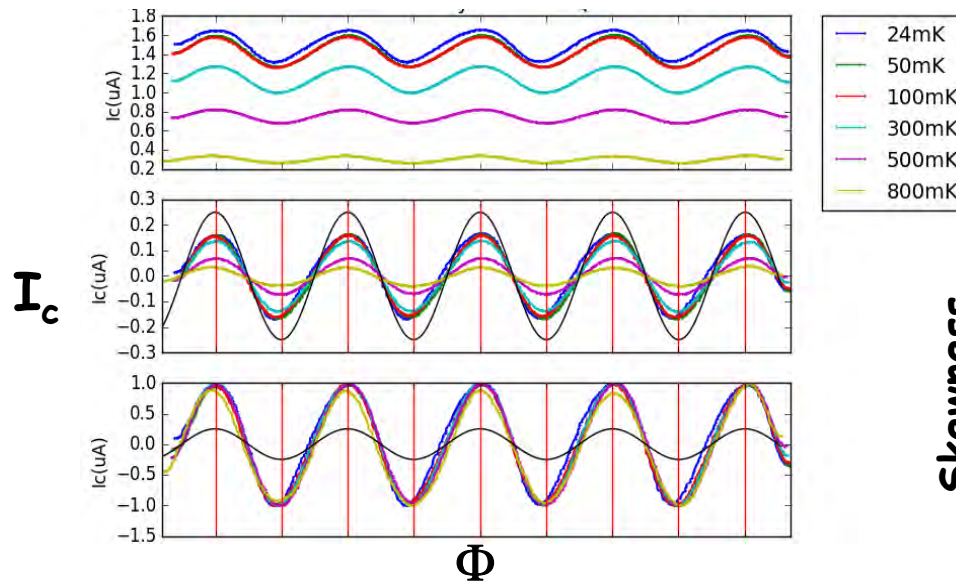
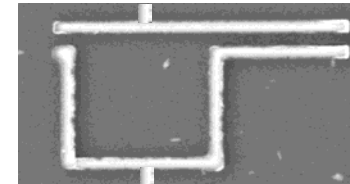
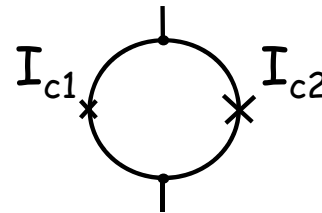


No 4π -periodicity --- expected for a static measurements
Very small skewness but need to measure when gated

Frequency-dependent CPR Measurements

Asymmetric dc SQUID technique

- Junction embedded in a dc SQUID
- Measure critical current vs. flux
- Modulation mirrors the CPR of the small junction for $I_{c1} \ll I_{c2}$

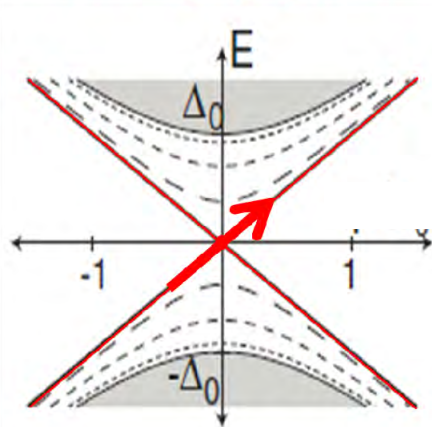


Modulation shows skewness expected from high transparency states, but no signatures of $\sin(\phi/2) \rightarrow$ not there? suppressed by qp poisoning?

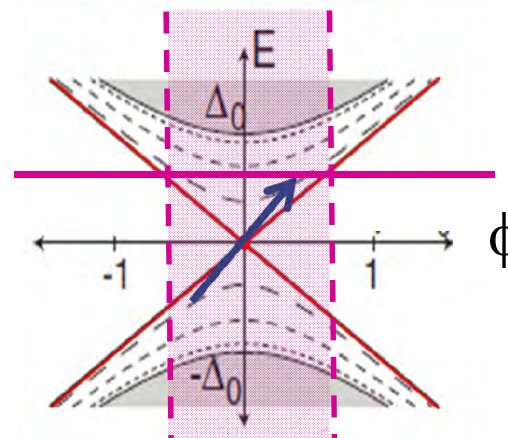
Is the $\sin(\phi/2)$ -component we see reasonable?

- estimated current from a single Majorana state is 10nA- 100nA
- in most of our samples, the node-lifting is 3nA-30nA but some are larger
- what we find to be constant is the fraction of node-lifting $\sim 10\text{-}20\%$ of $I_{c_{\max}}$
- we expect any states, even gapped ones, for which Zener tunneling can preserve the parity to exhibit the 4π -periodicity

Phase propagation



Zener tunneling



Expect a range of phase values, independent of the critical current

This is an interesting observation but perhaps not particularly good --- some states that lift the nodes may not be protected Majorana state

Moving on --- next steps

Status of experiments : Intriguing evidence for Majorana modes → most features we expect are observed

Who is convinced? **NOBODY** (as in all Majorana systems to date)

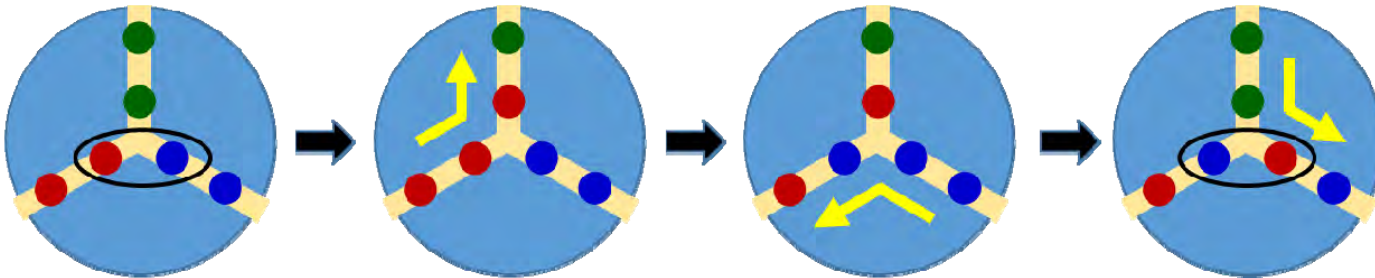
What do we do next? **BRAID**

Need to demonstrate braiding --- parity changes associated with MF exchange AND their non-Abelian statistics

1. Need to exchange MFs
2. Need to measure the parity
3. Need to measure parity lifetimes to determine how fast we need to perform braiding operations
4. Then we build a quantum computer

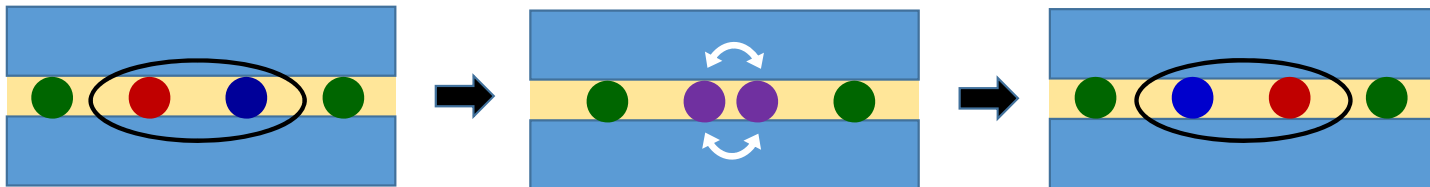
Ways to braid

Exchange braiding --- interchange MFs, change parity



Topologically-protected --- no errors

Hybridization braiding --- interact MFs, induce parity change

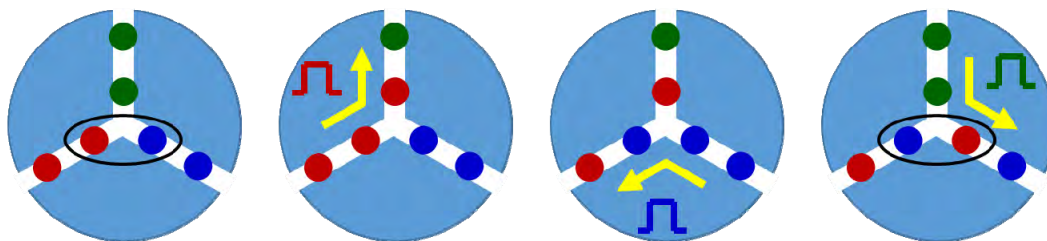
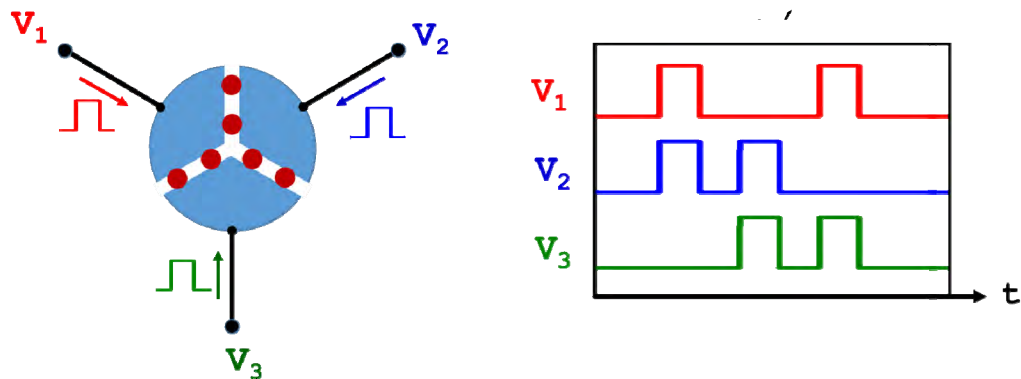


Not fully protected --- must control strength and time of coupling
Coupling depends of overlap of wavefunctions --- exponential dependence.

Exchange braiding in S-TI-S trijunctions via voltage pulses

Use voltage pulses to drive Josephson vortices/MFs

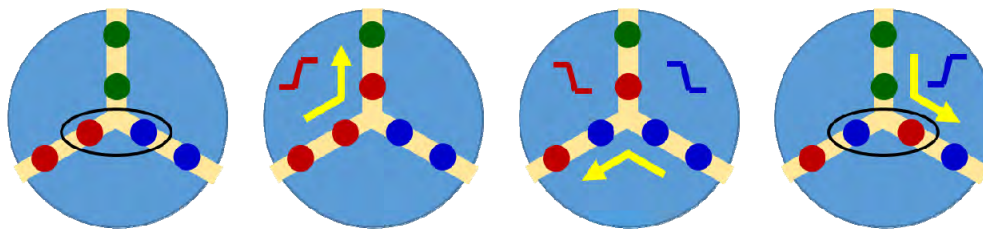
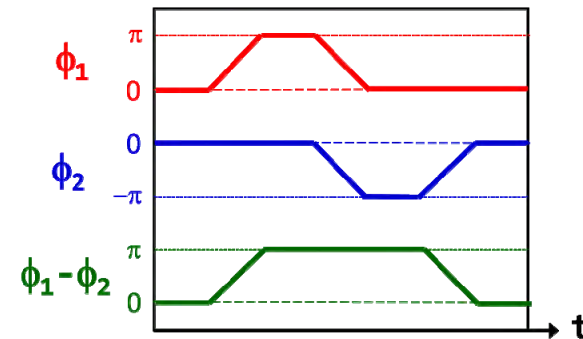
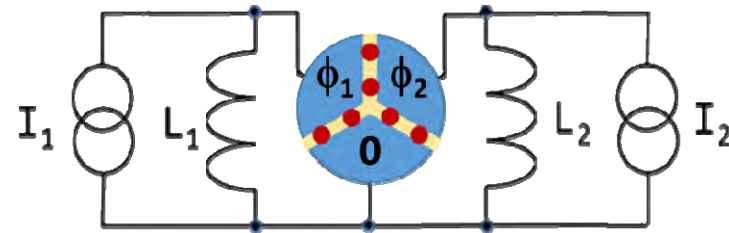
- Utilize RSFQ (Rapid Single Flux Quanta) pulses ($V \cdot \Delta t = 1 \Phi_0$)
- Apply sequences of pulses to junctions
- Exchange vortices



Can braid rapidly but voltage pulses can generate quasiparticles that enhance parity switches \rightarrow "quasiparticle poisoning"

Exchange braiding in S-TI-S trijunctions via phase shifts

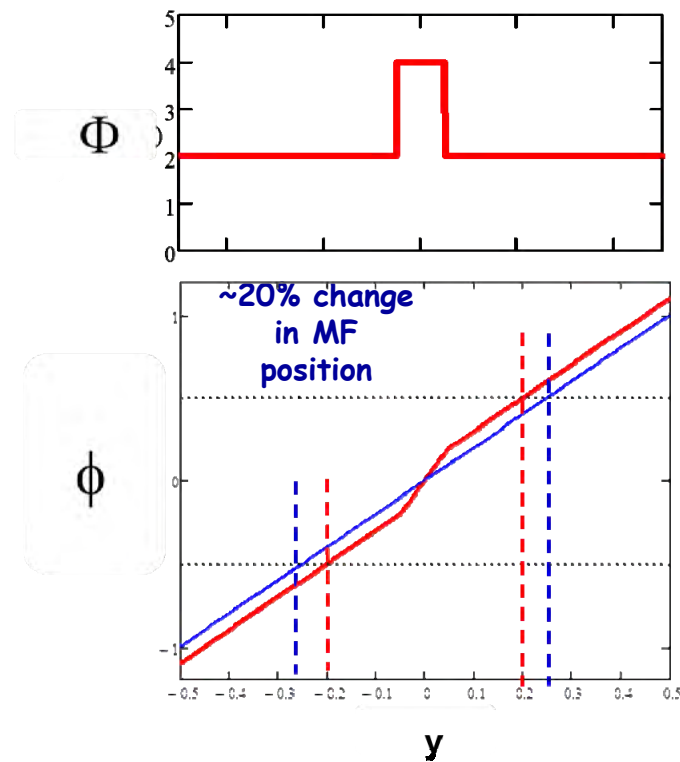
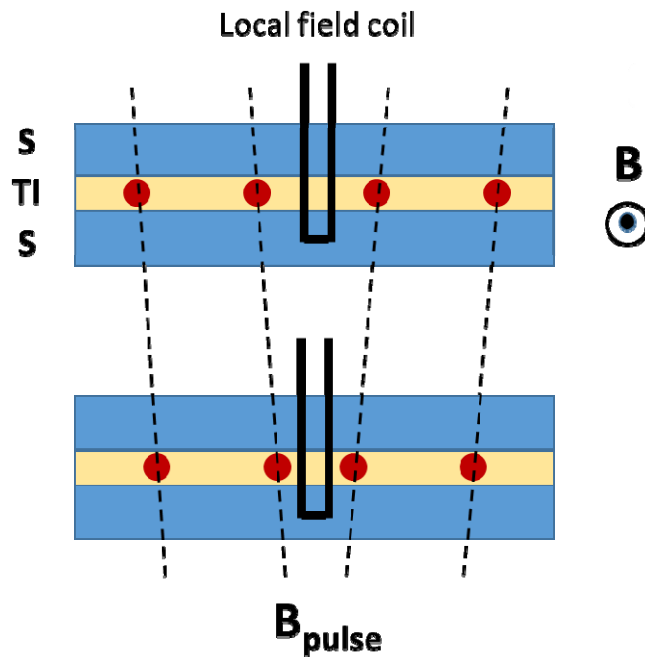
- Apply current pulses to junction arms shorted by inductors
- Response is non-linear due to supercurrent in the junction
- Allows continuous control of phases with no dissipation --- any phase configuration is accessible



Can braid rapidly but no voltages are generated, preventing quasiparticle poisoning

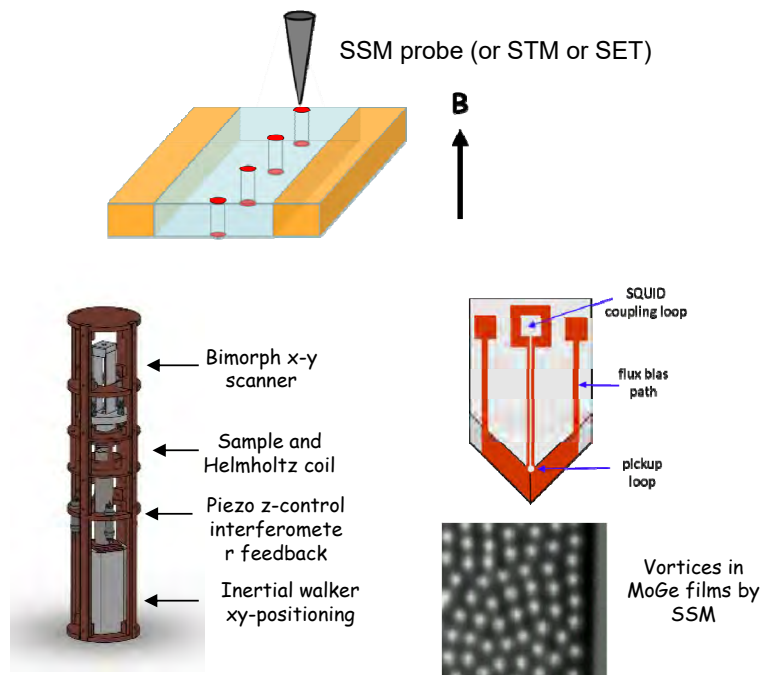
Hybridization braiding: controlling the spacing of Josephson vortices/MFs

- Current in loop creates a local field pulse
- Vortices move closer together
- Creates hybridization --- time-dependent



Hybridization braiding: measuring the effects of vortex motion

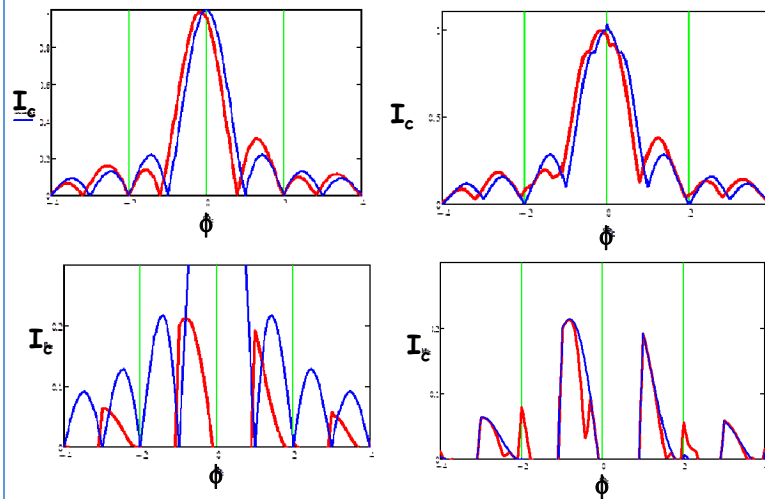
Imaging via Scanning SQUID Microscopy (SSM)



Detection via diffraction patterns

$$I_c(\phi) = \sin(\phi)$$

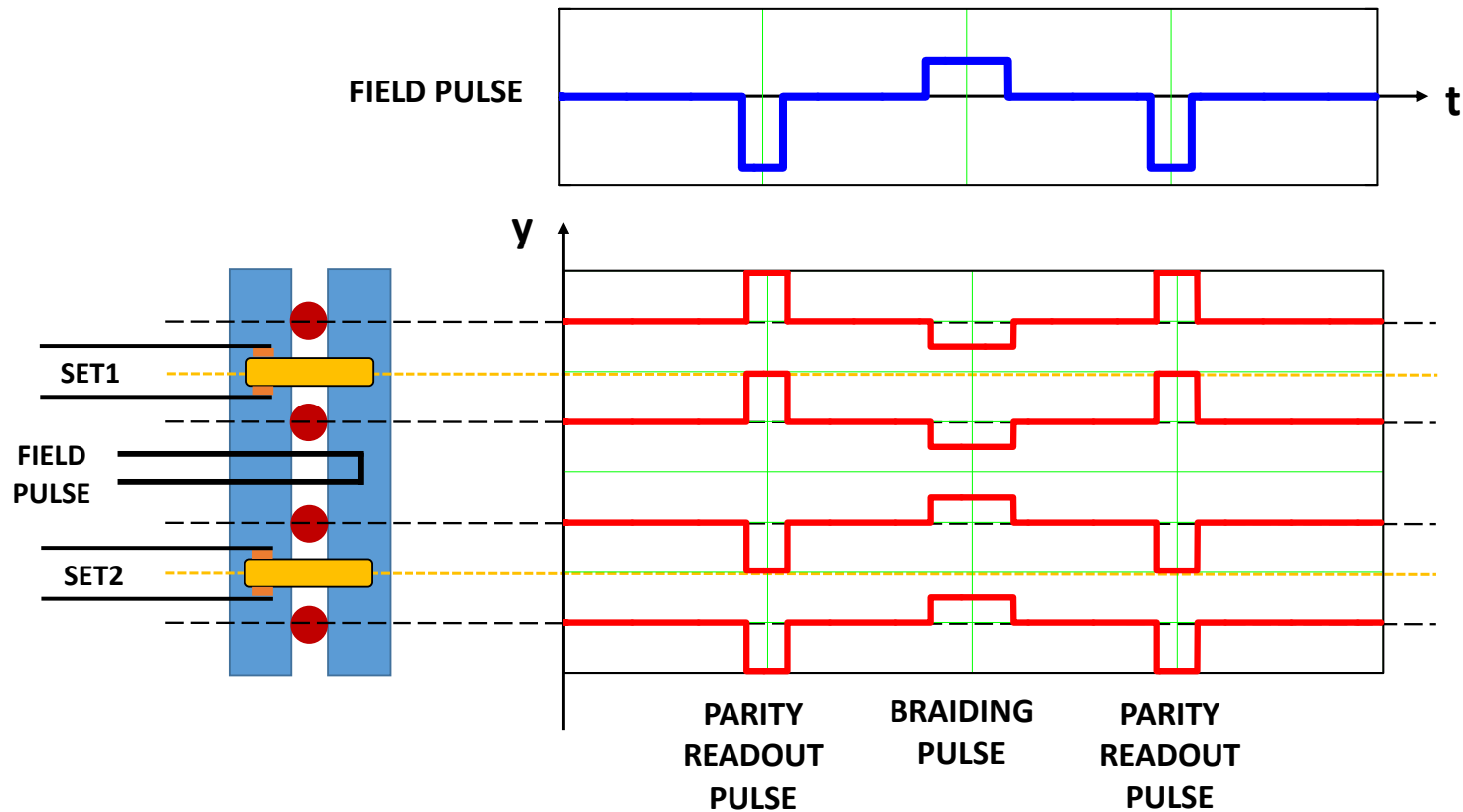
$$I_c(\phi) = \sin(\phi) + \alpha(\phi) \sin(\phi/2)$$



Significant changes in the diffraction patterns due to manipulation of vortices

Braiding by hybridization

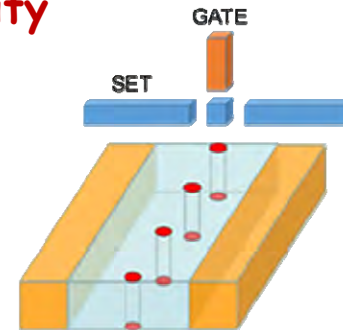
Braiding can also be effected by changing the spacing of Majorana fermions to induce phase shifts and subsequently reading out their parity



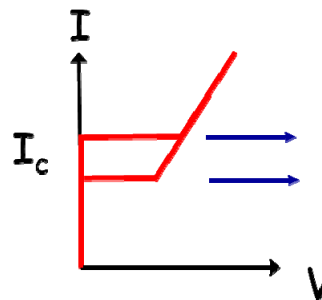
Ways to measure the parity

Many schemes have been proposed:

- couple topological device to a quantum dot
observe parity -dependent conductance
changes in a "single-electron transistor"
- incorporate topological device in a microwave cavity resonator "transmon"
observe splitting of energy levels corresponding to parity states
- measure critical current switching distribution
sensitive to the sign of the $\sin(\phi/2)$ -component which encodes the parity



measure transition
from zero voltage to
finite voltage state

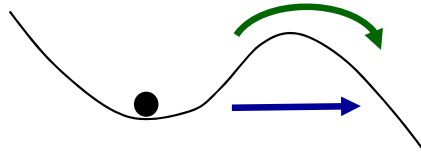


$$I_{c+} \sim \sin(\phi) + \gamma \sin(\phi/2)$$

$$I_{c-} \sim \sin(\phi) - \gamma \sin(\phi/2)$$

Expect to see switching with a bimodal distribution --- splitting would be proportional to the magnitude of the $\sin(2\phi)$ component

Critical current -- escape phase particle from the potential well



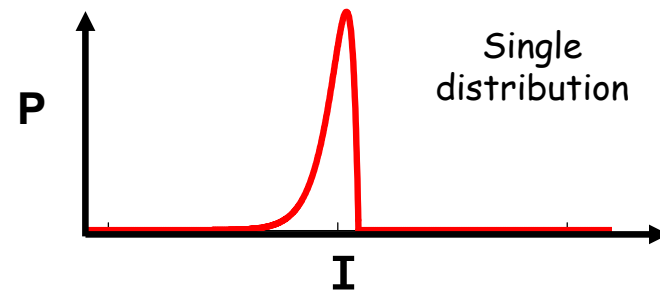
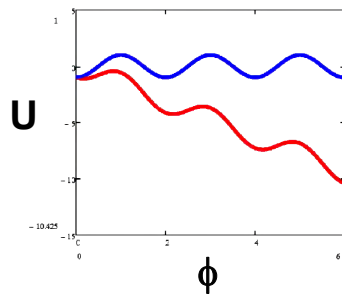
Thermal activation (over barrier)

Macroscopic Quantum Tunneling (through barrier)

S-I-S junction

$$I(\phi) = I_c \sin(\phi)$$

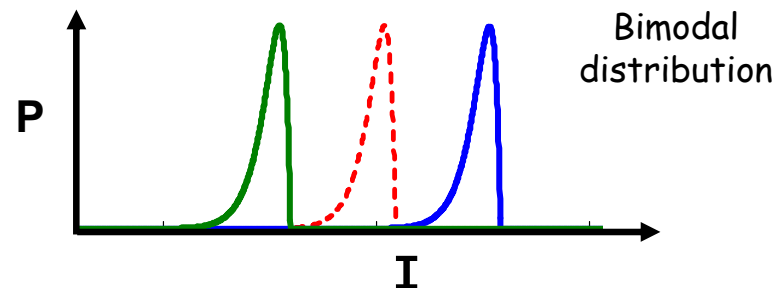
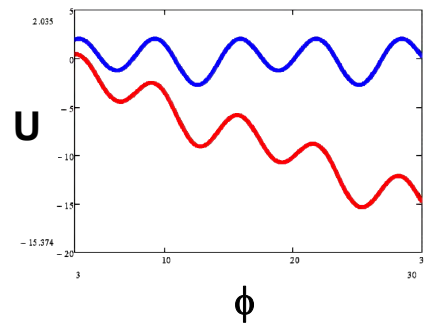
single barrier heights $\sim I_c$



S-TI-S junction

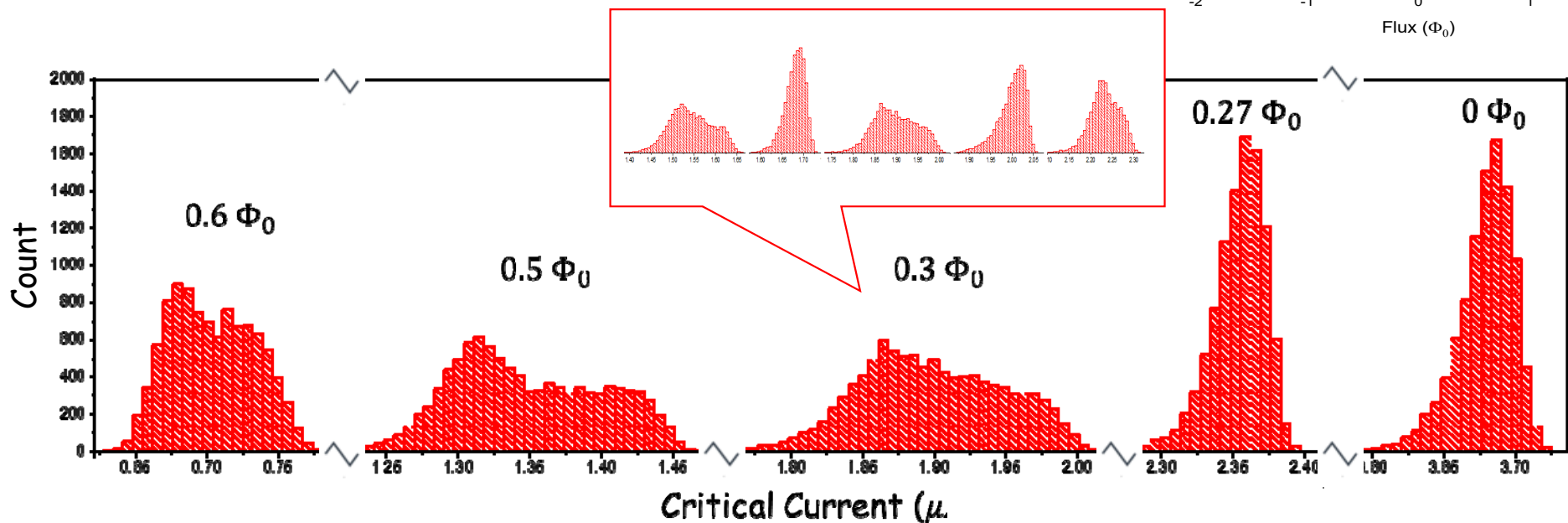
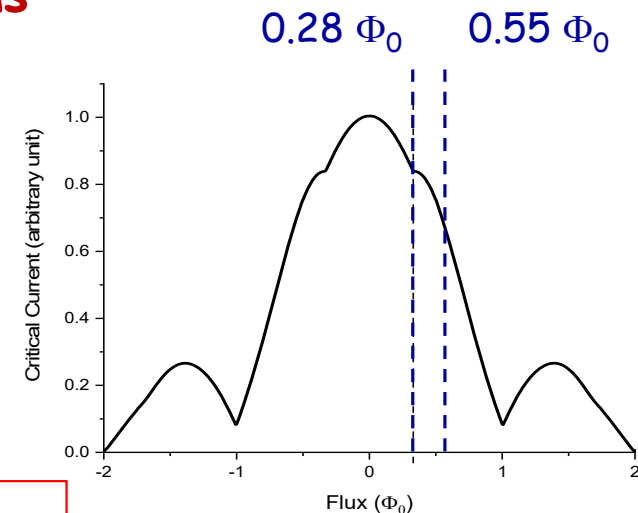
$$I(\phi) = I_{c1} \sin(\phi) + I_{c2} \sin(\phi/2)$$

Two barrier heights $\sim I_{c1} \pm I_{c2}$

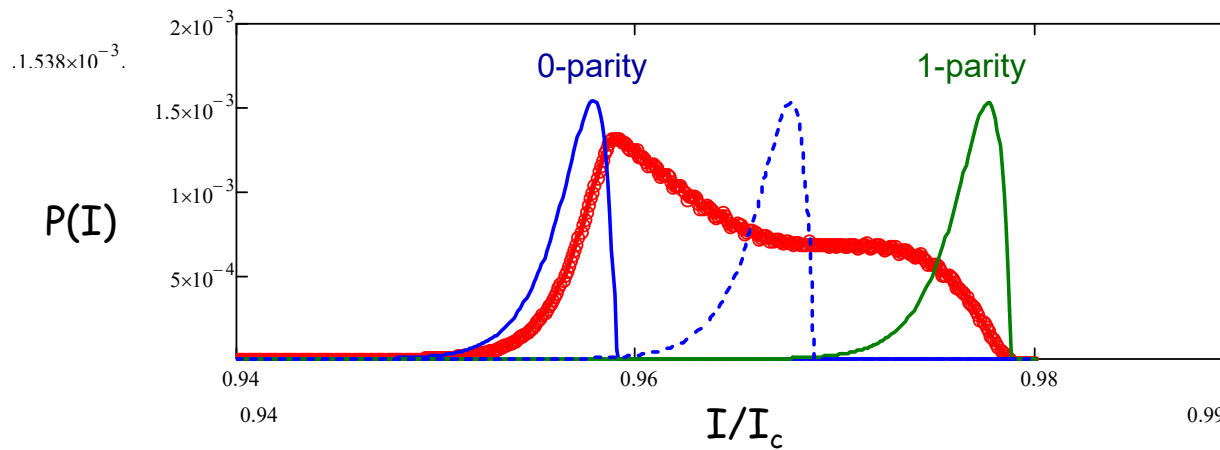


Critical current switching distributions

- Low-field distributions exhibit conventional single peak form characteristic of escape from washboard potential wells
- Above $\sim 0.28\Phi_0$, we observe a broadened distribution --- bimodal but with intermediate states
- Above $\sim 0.55\Phi_0$, distribution narrows when junction is no longer hysteretic



Modeling the switching distributions



- Low-field distributions fit MQT form --- no temperature dependence observed (expected for low-capacitance lateral junctions)
- Intermediate region fits a bimodal critical current with splitting comparable to the node-lifting --- attribute this to $\sin(\phi/2)$ contributions from MFs with different parity
- Switching between distributions arises from parity transitions --- estimate parity transition rate to be $\sim 20\text{KHz}$ \rightarrow parity readout but with low fidelity

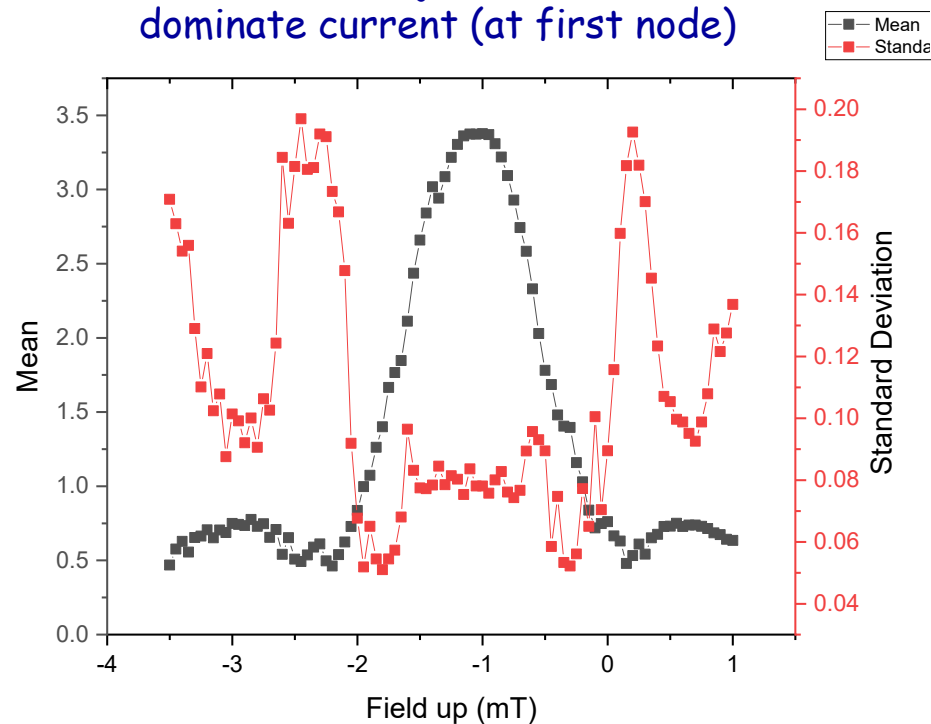


First actual parity measurements made on a Majorana fermion pair by us, and maybe by anyone.

More critical current switching distributions

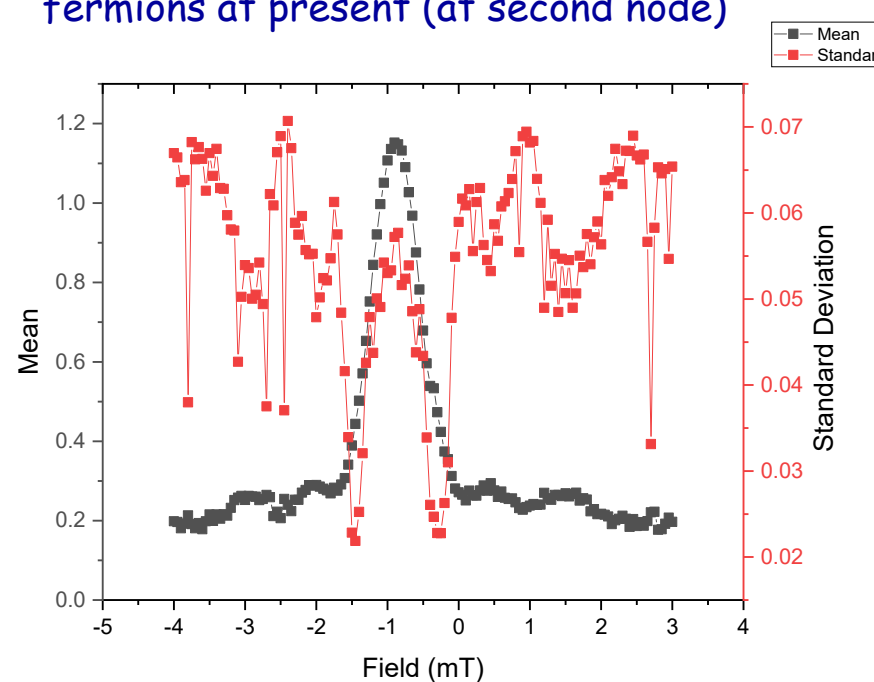
In other samples, the splitting is so small that it only shows up as a broadened single peak ---
measure distribution and the standard deviation

Peak where Majorana fermions
dominate current (at first node)



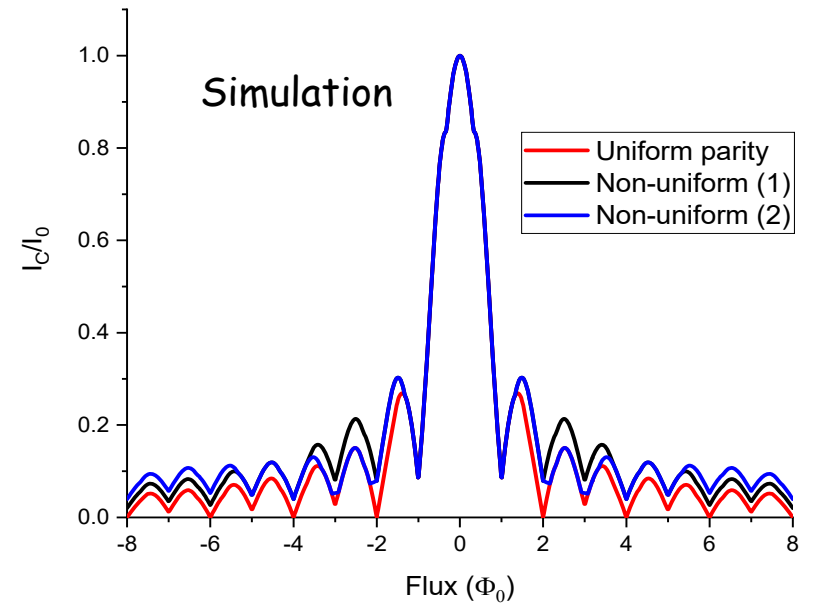
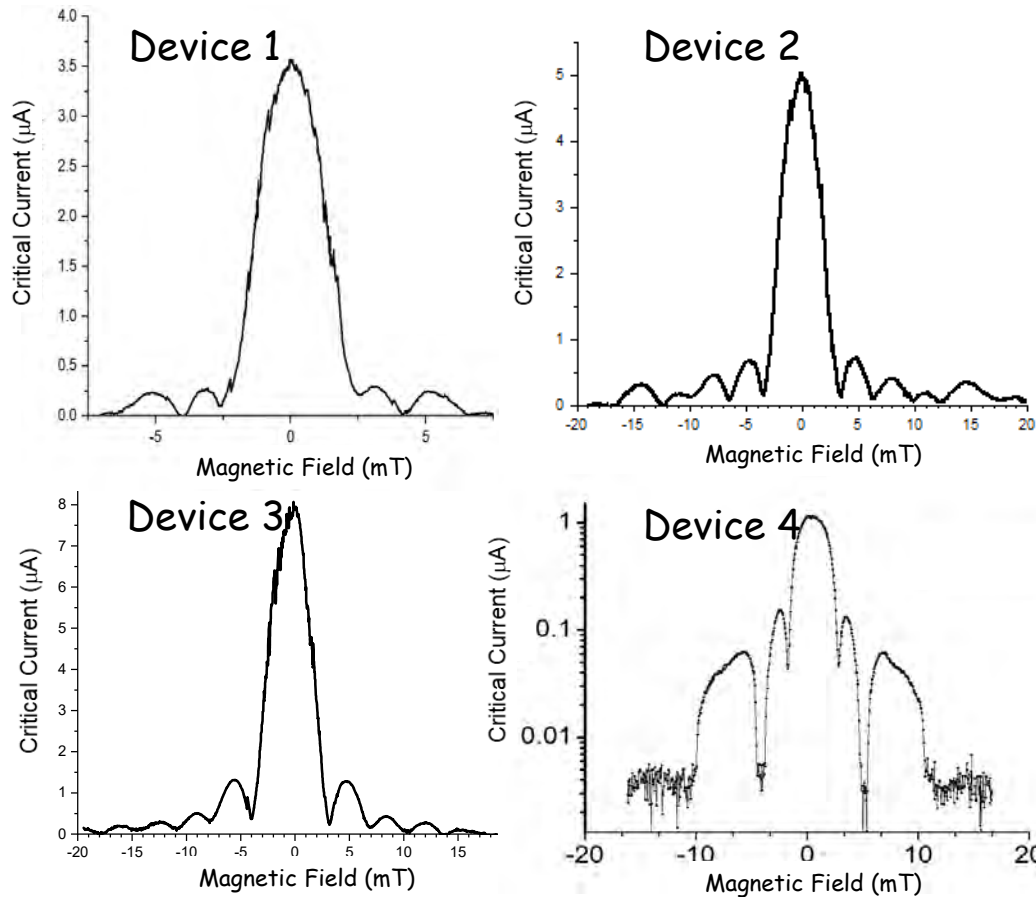
Spike at onset features

A second peak where two Majorana
fermions at present (at second node)



Drop in non-hysteretic region

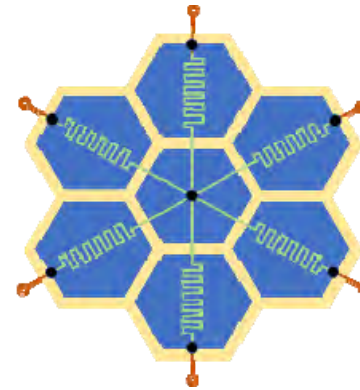
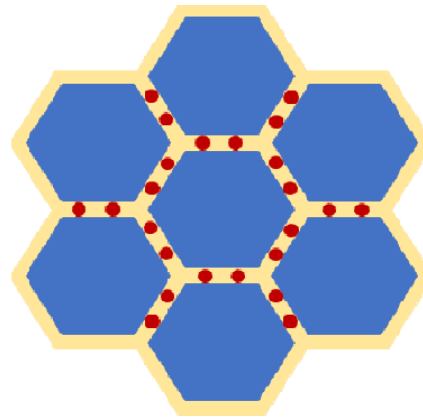
Effect of parity transitions on diffraction patterns



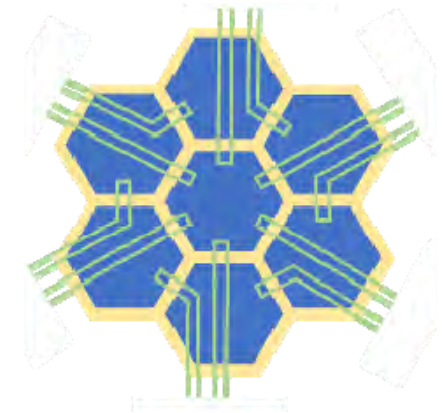
- The first node lifting is usually seen on our devices
- The second node sometime also lifts due to non-uniform parity of the MBS

Architecture for an S-TI-S quantum processor

Basic building block:
 7 hexagonal islands
 12 junctions
 6 trijunction braiding sites

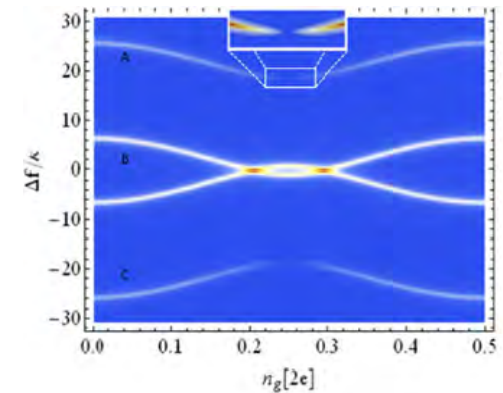
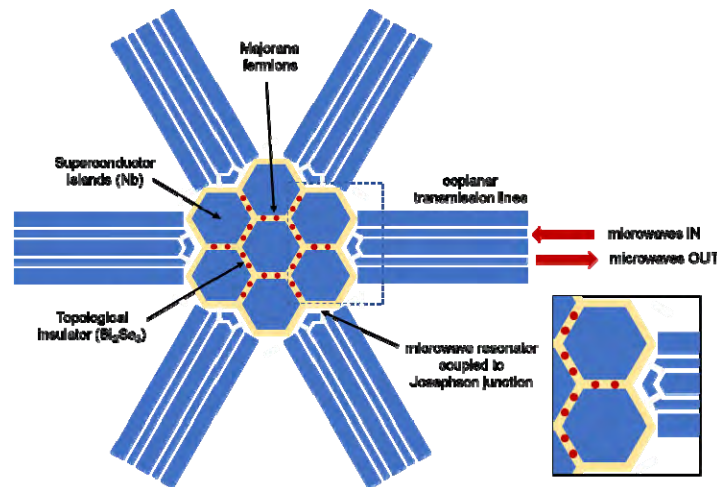


Phase control for
 exchange braiding



Field coils for
 hybridization

Transmon readout of
 MF parity at 6 sites



E. Ginossar and E. Grosfeld
 Nature Communications **5**, 4772 (2014)

Summary: the TRUTH about Majorana fermions in S-TI-S junctions

Theoretical models make specific predictions about Majorana fermions in S-TI-S Josephson junction networks. Our approach has been to:

- (1) Test those predictions as rigorously as possible via transport and Josephson experiments
 - ✓ Coherent supercurrents on the top surfaces
 - ✓ Odd node lifting
 - ✓ MF entry features
 - ✓ Skewness in CPR
 - ✓ Evidence for parity fluctuations at higher order nodes
 - Critical current distribution splitting in some junctions; broadening in all
 - × Node-lifting seems too large in some junction
 - × Splitting seems too small in some samples
 - × No observation of 4π -periodicity in direct CPR measurements
- (2) Move forward on schemes to braid and readout parity that can provide the only definitive proof of MFs with non-Abelian statistics and enable applications
- (3) Looking for talented postdocs interested in research in quantum information science